Econ 302: Microeconomics II - Strategic Behavior

Problem Set #5 – June13, 2016

1. T/F/U? Explain and give an example of a game to illustrate your answer. A Nash equilibrium requires that all players are maximizing their payoffs irrespective of the decisions of the other players.

a) A Nash equilibrium requires that all players are maximizing their payoffs irrespective of the decision of the other players.

False. A Nash equilibrium requires that all players are maximizing their payoff **given** *the (equilibrium) decisions of the other players. See, e.g., Game 2 in Question 2, where Row plays Top and gets 3 in the NE (Top, Right) even though Row's maximal payoff is 6, which is attained if she plays Bottom (and if the other player plays Left).*

b) An equilibrium in dominant strategies usually maximizes joint payoffs.

False. A player has a dominant strategy if al l other strategies yield a lower payoffs regardless of what other players do. This does not imply joint utility maximization. Just take the Prisonner's Dilemma game as an example. The dominant strategy for each player is to confess, yet the joint payoff from (confess, confess) is lower that from (not confess, not confess).

2. Check the following two games for dominance solvability. Find the Nash equilibria. Use Game 3 to explain why it is important to describe an equilibrium by using the strategies employed by other players, not merely the payoffs received in equilibrium.

		Column: Left	Center	Right
Row	Top	$4, 3$	$2, 7$	$0, 4$
	Bottom	$5, 5$	$5, -1$	$-4, -2$

Game 1

This game is dominance solvable: player Column's strategy Right is dominated by Middle. Eliminating Right from the matrix, we see that now player Row has a dominated strategy: Top (dominated by Bottom). Eliminating Top, Player Column has the choice between Left and Middle, and chooses Left. There is thus a unique equilibrium (Bottom, Left) which can be found by iterated elimination of dominated strategies. This is also a Nash equilibrium. Suppose Row plays Bottom. Column's best response to Bottom is to play Left. And Row's best response to Left is Bottom, so we have found the Nash equilibrium. Also, there can be no others: if Row plays Top, Column's best response to Top is Center. But if Column plays Center, Row should play Bottom, not Top. There is therefore no Nash equilibrium where Row *plays Top. Thus, the equilibrium is unique since we've exhausted all possibilities (either Row plays Top or Row plays Bottom).*

		Column: Left	Column: Right
Row	Top	$2,1$	$3,3$
	Bottom	$6,5$	$2,2$

Game 2

This game is not dominance solvable. Neither player has a strategy that is worse than some other strategy, regardless of what the other player does: for instance, if Column plays Left, Row is better off playing Bottom than Top, but the other way around if Column plays Right. The same argument can be made for Column. The game has two NE's: (Bottom, Left) and (Top, Right). To see that, e.g., (Top, Right) is a NE, notice that if Column plays Right, Row should play Top, and if Row plays Top, Column should play Right. So (Top, Right) are mutually best responses. The same is true for (Bottom, Left). Also note that (Top, Right) is still a NE even though both players are better of in the (Bottom, Left) equilibrium.

		Column: Left	Column: Center	Column: Right
Row	Top	$1,2$	$2,1$	$1,0$
	Middle	$0,5$	$1,2$	$7,4$
	Bottom	$-1,1$	$3,0$	$5,2$

Game 2

This game is dominance solvable: in this game, Column's strategy Center is dominated by playing Left. We can thus eliminate Center for Column. In the remaining game, we can eliminate Bottom which is dominated for Row by Middle. Finally, we can eliminate Right for Column which is dominated in the remaining game by Left. So Column will play Left, and best response to Left is for Row to play Top. (Top, Left) is the unique NE. Note that in this NE, Row gets 1 and Column gets 2. There are other potential outcomes such as (Middle, Right) or (Bottom, Right) which give a higher payoff to ***each*** *player. Still, these outcomes will not materialize in equilibrium because if* ***Row were to play Middle, Column will play Left, not Right*** *and if* ***Column were to play Right, Row will play Middle, not Bottom****.*

3. The following game was actually played between the US and Japan in Word War II. In 1943 the Japanese admiral Imamura was ordered to move a convoy of ships to New Guinea; he had to choose between a stormy northern route and a sunnier southern route, both of which required 3 days sailing time. The American admiral

Kenney knew about the convoy and wanted to send bombers, but he did not know which route Imamura would take. He had to send reconnaissance planes, but they only had enough planes to explore one route at a time. If the convoy was on the route first explored by Kenney, he could send bombers right away; if not one day of bombing would be lost. Poor weather of the northern route would also hamper bombing. If Kenney sent the reconnaissance planes the northern route and found the Japanese there, they could expect only 2 out of 3 good bombing days. If they found the Japanese had gone south, they could also expect 2 bombing days. If Kenney went the southern route first, he could expect 3 days of bombing if the Japanese were there, but only 1 day if they instead had taken the northern route.

a) Illustrate the situation in a matrix zero-sum game.

Named after the place where the actual bombing took place, this game is called "The Battle of the Bismarck Sea". Recall that zero-sum means that the sum of the entries in each cell equals zero. Let's represent payoffs by 'days of bombing'. Given the story above, we get:

		Imamura	
		North	South
Kenney	North	$2, -2$	$2, -2$
	South	$1, -1$	$3, -3$

The Battle of the Bismarck Sea

b) Find any equilibria (dominated strategies, Nash). *Note that for the Japanese, going North is a **weakly dominant** strategy (see also below for a definition of weak dominance): it is strictly better than going South if the Americans go South, and just as good if the Americans go North. However, there are no (strictly) dominated strategies for either player, so we can't eliminate any rows/columns from the game matrix – there is no equilibrium in dominant strategies. To find the NE's, assume first Imamura goes South, then Kenney will go South, too. But then going South is not a best response for the Japanese, so there is no NE were Imamura goes South. Next, suppose Imamura goes North, then Kenney should go North, too. And given Kenney goes North, Imamura might as well go North, too: he is indifferent between North and South, so him choosing North is **one** response to Kenney's going North. We thus have a unique NE where the Japanese choose the Northern route and where the Americans sent their reconnaissance planes on the Northern route, too. As a side remark, this is indeed what happened in 1943.*

4.* (Contributing to a Public Good) Each of n people chooses whether to contribute a fixed amount toward the provision of a public good. The good is provided if and only if at least k people contribute, where $2 \leq k \leq n$. If the good is not provided, contributions are not refunded. Each person ranks outcomes from best to worst as follows: (i) any outcome in which the good is provided and she does not contribute (ii) any outcome in which the good is provided and she contributes, (iii) any outcome in which the good is not provided and she does not contribute, (iv) any outcome in which the good is not provided and she contributes.

a) formulate this situation as a strategic game. Is the game solvable by iterated elimination of dominated strategies?

There are n players, each of which has two strategies: Contribute (C) or don't contribute (D). The payoffs for player i are as follows, given that α other players contribute. First, if $n \geq \alpha \geq k$, player i is better off not contributing (the good will be provided without her contribution). If $\alpha = k - 1$ people contribute, player i is better off contributing (the good will only be provided with her contribution which she values higher than the good not being provided). If $0 \leq \alpha \leq k - 2$ people contribute, player i is better off not contributing (the good won't be provided anyway). Clearly, player i best response/optimal strategy does depend what the other players are doing, so there is no dominant strategy/dominated strategy.

b) find all Nash equilibria of this game. Is there one where more than k people contribute? One where k people contribute?

To answer this question, we just have to go through the various possibilities. First, is there a NE where more than k people contribute? Answer: no, since this means that from the point of view of some player i, $\alpha \geq k$ others already contribute, so the best response for this player is to play D. Second, is there a NE where exactly k people contribute? Yes: from a contributing player i's point of view $\alpha = k-1$ people play C so he should play C as well. This is true for all players who play C. For all players who play D, $\alpha = k$, so playing D is optimal as well. Note that we cannot say who contributes. ***Each*** *possible combination of contributing players/defecting players is a NE. Finally, are there NE were strictly less than k people contribute? Yes, there is one NE where no one contributes: if nobody else contributes $a = 0$ and it doesn't pay to contribute for player i either. So D for all players is a NE. Are there any NE where a positive number of people, say $\beta > 0$ contribute but $\beta < k$? No. If $\beta = k - 1$, then each non-contributing player would have an incentive to deviate and contribute instead. If $\beta < k - 1$, then each contributing player would want to deviate and play D.*

Further questions for review:

a) how do the payoff entries (a,b,c,d,e,f,g,h) have to be in relation to each other if
1. Consider the following matrix game:

		Column: Left	Column: Right
Row	Top	a, b	c, d
	Bottom	e, f	g, h

(Top,Left) is a dominant strategy equilibrium? *$a > e$, $c > g$, $b > d$ and $f > h$.*

b) how do the payoff entries (a,b,c,d,e,f,g,h) have to be in relation to each other if (Top,Left) is a Nash equilibrium? $a > d$ and $b > d$

2. An old lady is looking for help to cross the street. Only one person is needed to help her; more are okay but no better than one. You and I are the only two people in the vicinity who can help; we have to choose simultaneously whether to do so. Each of us will get the pleasure worth 3 from her success (no matter who helps her). But each one who goes to help will bear a cost of 1. Set this up as a game. Write the payoff table, and find all Nash Equilibria.

The Payoff Table is:

		I: Help	I: Don't
You	Help	$2, 2$	$2, 3$
	Don't	$3, 2$	$0, 0$

Helping the old Lady

As can easily be verified, the game has two Nash equilibria (in pure strategies). The first NE is (Help,Don't) and the second NE is (Don't, Help). In both NE's, one of us helps the lady. Each of us prefers the other to help her, but given the other is not doing anything, each of us prefers helping her. Helping the lady is providing a public good. In equilibrium, only one of us contributes. Note, however, that in this particular case this is also efficient: the joint surplus is lower if both of us help her.

3.* In any game, we say that a player's strategy **weakly dominates** another strategy if the first strategy is at least as good as the second strategy, no matter what other players do, and is strictly better than the second strategy for some actions of the other players. In other words, weak dominance is like the (strict) dominance we discussed in class (where a player does strictly better with one strategy, regardless what the other player does), except that the player may sometimes be indifferent.

Recall the guessing game we played in class, assuming that players can name any real number, not just integers (so fractions are allowed). Argue carefully that it is solvable by iterated elimination of weakly dominated strategies. Give the unique equilibrium outcome one arrives at by this procedure, and argue that this outcome is also the unique Nash equilibrium of this game.

The figure below describes the process of iterated elimination of weakly dominated strategies. A rational player should never choose a number strictly above $\frac{2}{3} \times 100 = 66.6$*, since any such number it is weakly dominated by choosing* 66.6 *(because regardless of what the other players do, there can never be a winning number above* 66.6*). So we can eliminate any choice above* 66.6 *from the game matrix. But given the new game matrix, nobody should pick number above* $\frac{2}{3}66.6 = \frac{2}{3}^2 \times 100 = 44.4$*. In words: if a player is rational, and beliefs that others are rational, he should not expect anyone to pick a number above* 66.6*, so he should pick nothing above* 44.4 *because it is weakly dominated by picking* 44.4*. Eliminating any picks above this number from the game matrix, the new matrix will have everything above* $\frac{2}{3}44.4 = \frac{2}{3}^3 \times 100 = 29.6$ *and so on until numbers but zero are eliminated.*

Equilibrium	Iteration					
...	E(4)	E(3)	E(2)	E(1)		E(0)
0	13.17	19.75	29.63	44.44	66.66	100

E(0) is the area of dominated choices, E(1) is the area of one iteration of elimination and so on.

The outcome where everybody picks 0 *is also the unique NE. To show this, we first need to argue that it is a NE: if everybody else picks* 0 *any one player should pick* $\frac{2}{3} \times 0 = 0$ *as well, so we do have a NE at* 0*. We also have to show that there are no other NE. Again, this is very simple: given everybody else's choice, any one player should pick* $\frac{2}{3}\times$*average of that choice, but then everybody should lower their choice again, so there is no equilibrium in which the average is different from zero.*