Econ 302: Microeconomics II - Strategic Behavior

Problem Set #6 – June 21, 2016

1. T/F/U? If firms in a Bertrand oligopoly collude (set prices so as to maximize joint profits), the incentive to cheat increases as the number of member firms in the cartel increases.

True. The joint profit maximizing price in a cartel is equal to the monopoly price, which is independent of how many members the cartel has. Individual profit is decreasing in the number of firms n, because if all firms set the same price, each firm gets $1/n$'th of the market. Since the profit one can make if one deviates is again independent of how many firms are in the cartel (why?), the incentive to deviate increases.

2. Two firms in a Cournot duopoly produce a homogeneous product, the demand of which is described by the inverse demand function $p = 100 - \frac{1}{2}q_1 - \frac{1}{2}q_2$, where q_i is the output of firm $i = 1, 2$. Each firm has a different cost structure, however. Firm 1's cost function is $C(q_1) = 19q_1$. Firm 2's cost function is $C(q_2) = \frac{1}{2}q_2^2$.

a) Derive the best response functions for each firm, and determine Nash equilibrium price and quantities. *To find each firm's best response function, we have to maximize profit, taking rival output as given. The first-order conditions equate marginal revenue with marginal cost:*

$$\text{Firm 1}: \qquad 100 - \frac{1}{2}q_2 - q_1 = 19 \qquad \Rightarrow \qquad q_1 = 81 - \frac{1}{2}q_2.$$

$$\text{Firm 2}: \qquad 100 - \frac{1}{2}q_1 - q_2 = q_2 \qquad \Rightarrow \qquad q_2 = 50 - \frac{1}{4}q_1.$$

Substituting firm 2's best response into firm 1's best response (or vice versa) gives the NE quantities $q_1^* = 64$ and $q_2^* = 34$. The price is $p^* = 51$.

b * Now suppose the two firms collude, i.e., they choose q_1 and q_2 so as to maximize joint profit. Determine the new equilibrium quantities and price. (Note: you have to be careful about how the cartel will allocate production across firms because marginal cost of production differ). *If the firms collude and maximize joint profits, they will allocate output to each firm so as to minimize total production costs. The problem is similar to a multiplant monopolist. The production cost are minimized at a point where the marginal cost of each firm (plant) are equal, i.e., $MC_1 = MC_2$. Since $MC_1 = 19$ and $MC_2 = q_2$, we have $q_2 = 19$ in the cost minimizing production (this makes sense, quantities above $q_2 = 19$ are cheaper produced by firm 1). Total output is found by equating marginal revenue to marginal cost of each firm (=19):*

$$100 - q = 10 \quad \Rightarrow \quad q = 81,$$

so we get $q_2 = 19$ *and* $q_1 = 81 - 19 = 62$. *The price is* 59.5. *Note the same results are obtained if one simply sets up the joint profit maximization program*

$$\max_{q_1,q_2}(100 - \frac{1}{2}q_1 - \frac{1}{2}q_2)(q_1 + q_2) - 19q_1 - \frac{1}{2}q_2^2.$$

.

3.* Consider a duopoly where firms compete in prices but their products are <u>differentiated</u>, not homogeneous. If firm 1 chooses price p_1 and firm 2 chooses price p_2, the demand functions for firm 1's product and firm 2's product are, respectively,

$$q_1 = 24 - 5p_1 + 2p_2 \qquad \text{and} \qquad q_2 = 24 - 5p_2 + 2p_1.$$

Note that each firm's demand is decreasing in its own price, but increasing in its rival's price.

a) Determine each firm's best response function and illustrate the functions in a diagram. How do they differ from the response functions in Cournot competition where firms compete in quantities? Explain. *The best response functions can be determined in the usual fashion: each firm chooses its own price to maximize its profit, taking the rival's price as given. The first order condition of the respective optimization programs give the best response functions:*

$$p_1 = \frac{1}{10}(24 + 2p_2) \qquad \textit{and} \qquad p_2 = \frac{1}{10}(24 + 2p_1).$$

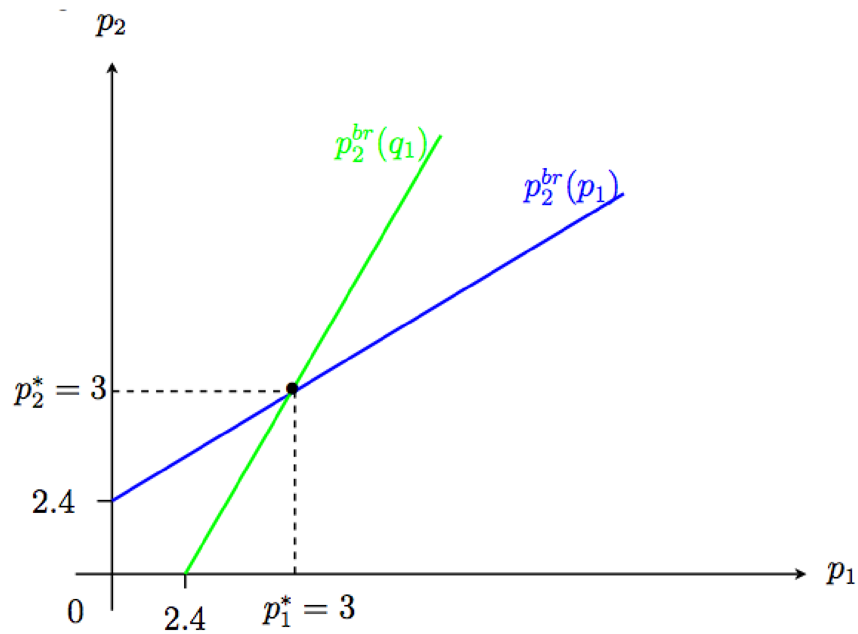
Graphically, we get:

We see that reaction functions are upward (rather than downward) sloping: each reaction function is ***increasing*** *in the rival firm's strategy (price): prices are* ***strategic complements****, which is very different from the Cournot model where reaction functions are* ***decreasing*** *in the rival firm's strategy (quantity) and quantities are* ***strategic substitutes****. Intuitively, this is because a firm's marginal revenue increases as more customers switch to its own product if the other firm's price increases.*

b) Calculate for the Nash equilibrium in prices. What are the firm's profits?

Solving for the equilibrium yields $p_1^* = p_2^* = 3$ *for prices and* $\pi_1^* = \pi_2^* = 45$ *for profits.*

c) Suppose now the firms collude (or merge), i.e., they jointly determine prices to maximize their joint profit. Determine the optimal prices and total profit, and



Price Competition in Differentiated Products

compare the results to the answer in b). Give an intuitive explanation for the difference.

Maximizing joint profits gives $p_1 = p_2 = 4$ *for prices. Profits are* $\pi_1 = \pi_2 = 48$. *Compared to b) these prices are higher than in the Nash equilibrium, which is different from Cournot competition where quantities are lower in the joint profit max outcome than in the Nash equilibrium. Intuitively, increasing one's prices here exerts a positive externality on one's rival (as opposed to Cournot where increasing one's quantity exerts a negative externality). Because this positive externality isn't taking into account in the Nash equilibrium, prices in the NE are too low from a joint profit maximizing point of view.*

d) Is there an incentive for either firm to deviate from the collusive agreement? If so, what would it's price be? *Yes, just like in Cournot competition, each firm would deviate from the joint profit maximizing price. If the other firm sticks to* $p_j = 4$, *firm i's optimal price would be* $p_i = \frac{1}{10}(24 + 2 \times 4) = 3.2$. *This gives a profit of* $\pi_i = 51.2$ *which is better than* 48.

4. Harry and Sally are the only two people at a fundraising event. The host of the event has set up a raffle draw, where one can win a prize by purchasing raffle tickets at a price of \$ 1 a ticket. All the purchased tickets are placed in a bag, from which one ticket is drawn. The owner of that ticket wins the price. If Sally purchases x_S tickets,

and Harry purchases x_H, tickets, Sally wins the price with probability $x_S/(x_S + x_H)$ and Harry wins the price with probability $x_H/(x_S+x_H)$. Harry and Sally have identical utility functions:

$$u_S = u_H = 144(\text{Prob of winning prize}) + \text{money}.$$

a) Derive the best response functions. *Note: The derivative of the function $f(y) = y/(y+k)$ is $f'(y) = 1/(y+k) - y/(y+k)^2$.*

Sally maximizes her utility taking x_H as given.

$$\max_{x_S} \; u_S = 144\frac{x_S}{x_S + x_H} - x_S.$$

The first order conditions give

$$\begin{aligned} \frac{\partial u_S}{\partial x_S} = 144\left(\frac{1}{x_S + x_H} - \frac{x_S}{(x_S + x_H)^2}\right) &= 1 \\ \Rightarrow \qquad (x_S + x_H)^2 &= 144x_H \\ \Leftrightarrow \qquad x_S(x_H) &= 12\sqrt{x_H} - x_H \end{aligned}$$

By symmetry, $x_H(x_S) = 12\sqrt{x_S} - x_S$.

b) Assuming Harry and Sally move simultaneously, how many tickets x_H^* and x_S^* do they each purchase in the Nash equilibrium? What is the probability that each of them wins the prize?

In the Nash equilibrium, $x_S^* = x_S(x_H^*)$ and $x_H^* = x_H(x_S^*)$. Plugging one response function into the other gives

$$\begin{aligned} x_S^* &= 12\sqrt{12\sqrt{x_S^*} - x_S} - 12\sqrt{x_S^*} + x_S^* \\ \Leftrightarrow \qquad \sqrt{12\sqrt{x_S^*} - x_S^*} &= 12\sqrt{x_S^*} \\ \Leftrightarrow \qquad 12\sqrt{x_S^*} - x_S &= x_S^* \\ \Leftrightarrow \qquad 6\sqrt{x_S^*} &= x_S^* \qquad \Rightarrow x_S^* = 36 \qquad \text{(or zero)} \end{aligned}$$

By symmetry, $x_H^* = 36$. The probability that each gets a ticket is 1/2.

c) Determine the number of tickets that each should purchase from a Pareto optimal (joint surplus maximizing) point of view, and compare it to your answer in b). Explain!

The Pareto optimal number of tickets maximizes joint surplus:

$$\max_{x_S, x_H} \; u_S + u_H = 144\frac{x_S}{x_S + x_H} - x_S + 144\frac{x_H}{x_S + x_H} - x_H = 144 - x_S - x_H.$$

By inspection, this function is decreasing in x_S and x_H, i.e., the Pareto optimal thing to do is to buy as few tickets as possible. Assuming the prize can be split, that means Harry and Sally buy just one ticket, and split the prize. If the prize cannot be split, there is another Pareto optimum where each buys one ticket, and gets the prize with probability 1/2.

5. A neighborhood association is considering building a park. The value of the park for each user depends on its size, G and the number of users n: a users utility is

$$u(G,n) = 4\sqrt{\frac{G}{n}} + \text{money}.$$

It costs G dollars to built a park of size G

a) Calculate the Pareto optimal size of the park G^{eff}. Show that i) G^{eff} increases in n, but ii) any single users' value of a park of efficient size will **not** depend on n. Explain these properties intuitively. *The park is a public good, the pareto optimal quantity of which is found by equating the sum of the marginal benefits with the marginal cost. The marginal cost is equal to 1. The marginal benefit of each user is*

$$\text{MB} = 4\frac{1}{2}\left(\frac{G}{n}\right)^{-\frac{1}{2}}\frac{1}{n} = \frac{2}{n\sqrt{\frac{G}{n}}}$$

. There are n users, so the condition of sum of MB's equals MC becomes

$$n\frac{2}{n\sqrt{\frac{G}{n}}} = \frac{2}{\sqrt{\frac{G}{n}}} = 1 \quad \Leftrightarrow \quad G = 4n.$$

So $G^{eff} = 4n$ which is an increasing function of n. Plugging this value back into individual utility gives a benefit of $4\sqrt{\frac{G^{eff}}{n}} = 8$, independent of n. Intuitively, the efficient park size increases in n because of its public good character: increasing G simultaneously benefits all users, and the more users there are, the higher (the sum of) these benefits. At the same time, though, there is a congestion effect: each individual's utility is decreasing in the number of users n for a given park size. Under the efficient quantity provided, both effects just offset each other, and individual utility is independent of n.

b) Consider a single individual who contributes x dollars and expects all others to contribute X in total. The size of the park would then be $G = x + X$. Show that users gain from contributing if the expect park size to be $G < 4/n$ and will stop contributing if they expect park size to be $G > 4/n$.

Under private provisions, users choose x to maximize their utility $U(x) = 4\sqrt{\frac{x+X}{n}} - x$ taking X as given. Taking derivatives, we find that the change in individual utility from a small increase in x is

$$\frac{\partial U}{\partial x} = 4\frac{1}{2}\left(\frac{x+X}{n}\right)^{-\frac{1}{2}}\frac{1}{n} - 1 \gtreqqless 0 \quad \Leftrightarrow \quad x + X \lesseqqgtr \frac{4}{n}.$$

So marginal utility is increasing in x (the individual wishes to contribute more) if $x + X = G < 4/n$ and decreasing in x (the individual gains from reducing his or her contribution) if $x + X = G > 4/n$.[1]

c) Conclude that under voluntary contributions, the equilibrium park size will be $G^* = 4/n$. Thus, under voluntary contributions, the realized park will be smaller the larger the neighborhood. Explain! *Note that due to symmetry, each individual's maximization problem is exactly the same. So we cannot have positive marginal utility for some and negative marginal utility for other users, implying that each users marginal utility must be zero. The equilibrium park size is thus $G^* = 4/n$ (this is the same as the problem in the lecture if Harry and Sally have the same utility functions – there are many equilibria where people contribute different amounts; however, the total contribution is always the same and equal to G^*). Intuitively, the equilibrium park size is decreasing in n because of the congestion effect and the free rider problem. The more users there are, the less is individual benefit and the more does the individual expect others to contribute. Both effects reduce individual willingness to contribute.*

[1] At the max, marginal utility is zero, and we get the best response functions $x^{br}(X) = 4/n - X$. Since contributions x cannot be negative, however, this is only valid $X \leq 4/n$; otherwise $x^{br}(X) = 0$.

Further questions for review:

1. Consider a Cournot duopoly with two identical firms. Market demand is $p = 90 - 2q_1 - 2q_2$, where q_i is the output of firm $i = 1, 2$. The firms have a marginal cost of production of \$ 30.

   a) Derive the best response functions for each firm, and determine Nash equilibrium price, quantities, and profits. *Taking the first derivative of profits gives the best response function* $q_i = 15 - \frac{1}{2}q_j$. *The Nash equilibrium quantities are* $q_1^* = q_2^* = 10$, *the price is* $p^* = 50$ *and profits are* $\pi_i^* = 200$.

   b) Firm 1 could invest in R&D, which would decrease its marginal production cost to zero. Calculate the new Cournot Nash equilibrium (price, profits, quantities). Compared to the hypothetical situation where firm 1 is a monopolist in the market, is its incentive to invest in process innovation (cost reduction) higher or lower in the Cournot case? Discuss, and give an intuitive explanation.

   The equilibrium quantities are now $q_1^* = 20$ *and* $q_2^* = 5$, *i.e., firm one optimally increases its production as a result of the reduction in marginal cost, which in turn prompts firm 2 to decrease production. The new equilibrium price is* $p^* = 40$ *and profits are* $\pi_1 = 800$ *and* $\pi_2 = 50$.
   If firm 1 was a monopoly, it would produce $q^m = 15$ *for a price of* $p^m = 60$ *and a profit of* $\pi^m = 900$ *before the cost reduction and* $q_m = 22.5$ *for a price of* $p^m = 45$ *and a profit of* $\pi^m = 1025$ *after the cost reduction. The rise in profits as a consequence of the process innovation is therefore* **smaller** *under a monopoly than under a duopoly. Intuitively, there are two opposing effects: first, a duopoly firm produces less than the monopoly firm; thus, it's total increase in profit is lower even though the cost savings per unit produced are the same. The second effect is an indirect strategic effect. By lowering its cost, the duopoly firm gets an advantage in the market because its opponent reduces quantity. This has an additional positive impact on profits, beyond the mere cost saving effect. In this example, the latter dominates. However, it doesn't have to - competitive pressure does not always equals higher incentives to innovate.*

2. Harry (H) and Sally (S) are the only two inhabitants of the tiny town of Boring, BC. To fight their boredom, they consider building an ice-rink. If an ice-rink of y m^2 is built, their utilities are $u_H = -\frac{a_H}{y} + \text{money}$ and $u_S = -\frac{100}{y} + \text{money}$, where a_H is a parameter in Harry's utility function. It costs 4 units of money to built $1m^2$ of ice rink. Answer the following questions for $a_H = 96$ and for $a_H = 125$.

   a) Calculate the Pareto efficient size of the ice rink.

The ice rink is a public good. To find the Pareto efficient amount, we add up the marginal benefits and equate the sum to the marginal cost. This gives

$$MB_H + MB_S = \frac{a_H}{y^2} + \frac{100}{y^2} = 4, \quad \Rightarrow \quad y = \frac{1}{2}\sqrt{a_H + 100}.$$

So $y = \sqrt{196}/2 = 7$ *if* $a_H = 96$ *and* $y = \sqrt{225}/2 = 7.5$ *if* $a_h = 125$.

b) Harry and Sally decide to set up a collection box, in which each of them can put some money to finance the rink. If each of them expects the other to contribute nothing, how much should each contribute? If each of them expects the other to contribute enough money for the Pareto efficient amount of y, how much should each contribute? Determine the best-response functions and calculate the contributions in the Nash equilibrium. *Each of these questions can be answered by calculating the best responses first. If H contributes* y_H *and S contributes* y_S *units of money, the size of the ice rink will be* $(y_H + y_S)/4$. *Taking the other person's contribution as given, Harry chooses* y_H *to maximizes his own utility* $-\frac{4a_H}{y_H+y_S} - y_H$. *The first order conditions give*

$$\frac{a_H}{(y_H + y_S)^2} = 4 \quad \Rightarrow y_H = \frac{1}{2}\sqrt{a_H} - y_S,$$

and analogously for Sally,

$$\frac{100}{(y_H + y_S)^2} = 4 \quad \Rightarrow y_S = 5 - y_H$$

for the best-response functions. Thus, if each expects the other to contribute nothing, Harry will contribute $y_H = \frac{1}{2}\sqrt{a_H}$ *and Sally will contribute such that* $y_S = 5$. *In both cases, this is less than the Pareto efficient amount (why? because contributing to a public good causes a positive externality which people don't take into account, there is thus too little of it in equilibrium). If each expects the other to contribute* $y = \frac{1}{2}\sqrt{a_H + 100}$, *neither will contribute anything (just plug this amount into the response functions – you'll get a negative number, but since one cannot take money out of the box by assumption, the contribution is zero). In the NE, the two best response functions have to intersect. If you calculate their intersection analytically, you will find that Harry's optimal contribution is negative – again, we therefore have a 'corner' solution where H contributes nothing,* $y_H^* = 0$ *and S contributes such that* $y_S^* = 5$. *You can also check the solution again by drawing these functions graphically!*

3. * The Hotelling Model of Electoral Competition. Consider the following simple model of two politicians competing for voters (votes). There is a continuum of voters,

each with a favorite policy/position x, which can be described on a left-right scale and can be represented by real numbers from 1 to 100. There are two candidates in an electoral race, which is determined by majority vote, i.e., whoever gets more than 50 percent of the votes is the winner; the other is the looser. In the event of a tie, each has an equal chance of winning.

Candidates A and B simultaneously pick policy platforms x_A and x_B. Voters vote in favor of the candidate that is closes to their most preferred policy. For instance, if candidate B adopts platform $x_B = 75$ and candidate A adopts platform $x_B = 40$, then candidate A gets all the votes from voters closer to him than to candidate B, i.e., for which $x \leq 40 + (75 - 40)/2 = 57.5$. Candidate B gets all the votes from people whose preferred policy is closer to his position than to candidate B's, which is everybody with $x \geq 57.5$. So A gets 57.5 percent of the vote, and B gets 42.5 percent of the vote.

Candidates care only about winning the race, not about political platforms or policies. Each prefers winning to a tie, and prefers a tie to loosing.

a) Find the best-response functions. Note: candidate i's best response will depend on whether his rival's position is larger, smaller, or equal to the **median** position $x_m = 50$.

   To find the best response functions for candidate 1, fix the position x_2 of candidate 2 and consider the best position for candidate 1. Suppose first $x_2 < x_m$. Clearly, if candidate 1 takes a position to the left of x_2 ($x_1 < x_2$), then candidate 2 will win. If candidate 1 takes a position to the right of x_2, then he/she will win as long as the dividing line between her supporters and those of candidate 2 is less than x_m (if candidate 1 goes to far to the right, however, he/she will loose). So the set of best responses is the set of positions that causes the midpoint $x_2 + \frac{1}{2}(x_1 - x_2)$ of the line segment from x_2 to x_1 to be less than x_m (if this midpoint is equal to x_m, there would be a tie). The condition $x_2 + \frac{1}{2}(x_1 - x_2) < x_m$ is equivalent to $x_1 < 2x_m - x_2$, so candidate 1's best response is the the set of all positions between x_2 and $2x_m - x_2$, excluding the points x_2 and $2x_m - x_2$ (note that if candidates would maximize votes, candidate 1 should set $x_1 = x_2 + \epsilon$). A symmetric argument applies to the case where $x_2 > x_m$. In this case, the set of candidate 1's best responses is all positions between $2x_m - x_2$ and x_2, again excluding the points x_2 and $2x_m - x_2$ (if candidates would maximize votes, candidate 1 would set $x_1 = x_2 - \epsilon$). Finally, suppose $x_2 = x_m$. In this case, candidate 1's unique best response is to choose $x_1 = x_m$ as well: if he/she chooses any other position, she looses, whereas at $x_1 = x_2 = x_m$, at least there is a tie.
   The same is true by symmetry for candidate 2's best response to x_1.

b) Find the unique Nash equilibrium (x_A^*, x_B^*).

If you draw the best-responses, you will see that they have only one point in common, which is $x_1 = x_2 = x_m = 50$. *But we can also make a direct argument for why both candidate's will choose the median position in the unique Nash equilibrium. First, it is a Nash equilibrium: if the other candidate chooses* $x_j = x_m$, *my best response is to do likewise. Second, there is no other Nash equilibrium: if the other candidate chooses some other* $x_j \neq x_m$, *I can always win by positioning myself in between* x_j *and* x_m *because I thereby attract more than half the votes.*

The conclusion is that competition between candidates (or parties) to secure a majority of votes drives them to converge on the same political position – the 'political center', which equals the median of the citizens' favorite positions. Hotelling (1929), the originator of this model, writes that this outcome is " strikingly exemplified". He continues, "the competition for votes between the Republican and Demogratic parites [in the United States] does not lead to a clear drawing of issues, an adoption of two strongly contrasted positions between voters may choose. Instead, each party strives to make its platform as much like the other's as possible."