Econ 302: Microeconomics II - Strategic Behavior
Final Exam - Aug 13 2915, 12:00pm - 2:00 pm in RCB IMAGTH RC Brown

1. (10 points) True/False/Uncertain? Explain your answer briefly.
a) In a public good contribution game, each player will choose his or her contribution so as to achieve the same utility as the other player(s).
b) Market participants who hold private information about the quality of the goods they sell generally benefit from this informational advantage.
2. (8 points, game theory) The following game is played between two animals fighting over food.

a) Write down all strategies of animal A and animal B. Determine the subgame perfect equilibrium.
b) Show that the outcome where A is peaceful and B is aggressive can be supported as a Nash equilibrium in the sequential game. Explain carefully why this is a Nash equilibrium, but not subgame perfect.
3. (8 points, risk aversion and insurance) Sally has $\$ 100$ and thinks about buying a share in a sunglasses company that will pay her $\$ 225$ if it is sunny but only $\$ 36$ if it rains. Both states are equally likely. Sally's utility function is $v(y)=\sqrt{y}$ where $y$ is her income/wealth.
a) Sally is an expected utility maximizer. If the share costs $\$ 100$, should she buy it?
b) What would be the actuarially fair price per dollar coverage if an insurance company offers insurance against rain? How much coverage would full insurance entail? Show that Sally would be happy to buy this insurance.
4. (12 points, price discrimination) Greedy Inc is a cellphone service monopoly with no marginal cost and no fixed cost. The company uses two-part tariffs: it charges customers a monthly fee $F$ and a price $p$ per call. Greedy has two types of customers, lovebirds and regular folks, in equal numbers. Regular folks have a demand function $p=12-q$ each, lovebirds have a demand function $p=10-q / 4$, where $q$ is the number of calls per month.
a) Assume first that Greedy can tell customers apart: all lovebirds are easily recognizable by the blissful smile on their faces. Let $\left(F_{R}, p_{R}\right)$ and $\left(F_{L}, p_{L}\right)$ be the contracts for the "regular package" and the "lovebird package", respectively. Determine the profit-maximizing values of $F_{i}$ and $p_{i}$, and the number of calls $q_{i}$ for each group of customers $i=R, L$.
b) When the first cellphone bills arrive in the mail, lovebirds become grumpy and stop smiling. Greedy can no longer tell them apart from regular folks. What happens if Greedy continued to offer the packages $\left(F_{R}, p_{R}\right)$ and $\left(F_{L}, p_{L}\right)$ as calculated in a)? Is this a problem of adverse selection?
c) Greedy thinks they can still indirectly price discriminate by making the regular package less attractive: they increase the price $p_{R}$ [from a)] by $\$ 2$, while keeping $p_{L}$ at its previous level [from a)]. Of course, the monthly fees may have to be adjusted as well. Write down conditions for $F_{R}$ and $F_{L}$ so that Greedy
i) will not loose any customers
ii) keeps the market separated, i.e., lovebirds purchase the $\left(F_{L}, p_{L}\right)$ package and regular folks purchase the ( $F_{R}, p_{R}$ ) package. How should Greedy optimally set $F_{R}$ and $F_{L}$ ?
(12 points, repeated games). Consider the following game that is played weekly between two roommates, Arnie and Bert:

|  | Bert |  |
| :---: | :---: | :---: |
|  | Clean | Dirty |
| Arnie Clean | 2, 2 | 0, 3 |
| Dirty | 3, 0 | 1,1 |

Cleaning the appartment...or not
a) Is this game solvable in dominant strategies? Explain. Find the equilibrium.
b) Now suppose this game is repeated infinitely often. Let $\delta>0$ be the common discount factor. Carefully describe the grim trigger strategy that, if adopted, allows Arnie and Bert to overcome the one-shot outcome. Determine the critical value of $\delta$ for which these grim-trigger strategies form a subgame perfect equilibrium in which both choose Clean every week.
c) Arnie and Bert decide not to adopt grim-trigger (too harsh!) but instead play tit-for-tat, which goes as follows: each starts out with Clean in the first week and then, in subsequent weeks, mimics the action that his roommate chose the week before - play Clean if he played Cleaned, and play Dirty if he chose Dirty. Write down the condition for $\delta$ that has to hold to ensure that a player who considers choosing Dirty in one week (but continues to stick to tit-for-tat thereafter) would be better off playing Clean, so that the subgame perfect equilibrium outcome is once again that the rooms are clean every week. Note: $1+\delta^{2}+\delta^{4}+\ldots=1 /\left(1-\delta^{2}\right)$.
d) Bonus Question (5 points). Under which condition on $\delta$ is tit-for-tat actually a best response to tit-for-tat? In other words, assuming the opponent plays tit-for-tat, when is it better for a player to continue playing tit-for-tat following a deviation rather than switching to Dirty forever after?

