International Diversification and the Investor Horizon

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Preliminary - Comments Welcome

Abstract

In this paper, we investigate the benefits of international diversification over short- and long-run horizons. Average unconditional correlation between international equity markets is first shown to increase at long horizons, even for synchronized market data. Next, a conditional horizon-dependent correlation measure is proposed, providing evidence of a positive trend in correlation which is strongest at long horizon. To further elucidate the results, a model replicating the temporal aggregation of inter-market correlation is proposed. Finally, we investigate the impact on portfolio allocation, demonstrating decreased risk reduction benefits at long-run horizons.

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1. Introduction

Quantifying the comovement between asset returns is one of the most fundamental aspects of modern finance. In international finance, the existence of low cross-correlations between global equity markets forms the basis of risk reduction through international diversification. The risk of holding a portfolio comprised of a range of international equity markets has long been shown to be lower than the risk of the component assets (Levy and Sarnat, 1970; Grubel, 1968). While considerable work has been done on quantifying the benefits of diversification as a function of available investment opportunities (developed versus emerging markets), time (dynamic correlations between markets) and measurement method, few studies have considered the impact of time horizon on the benefits of international diversification.

In this paper, we contribute to the debate on the benefits of international diversification by considering whether these benefits accrue equally to investors with short- and long-run investment horizons. This is of particular relevance, as characteristics of financial returns such as volatility, correlation and systematic risk have been extensively shown to vary as a function of the measurement return interval. Moreover, common risk factors related to asset pricing have been shown to be priced differently over heterogeneous investment horizons (Kamara et al., 2013). Various origins for these horizon dependent characteristic changes have been proposed, including delays in the reaction of prices to systematic shocks, differential liquidity, institutional ownership and firm size. Surprisingly, given the pervasive evidence for horizon based financial characteristics, little work has been done to measure the benefits of international diversification as a function of investor horizon.

The extant literature documenting the benefits of international diversification has tended to focus on the availability of diversification opportunities between
international equity indices. However, international equity indices may have characteristics, such as non-synchronous trading hours and short horizon serial correlation, which may perturb the accurate measurement of diversification benefits. Non-synchronous trading hours may induce cross-serial correlation between markets, as each responds to common shocks at distinct times. Many studies attempt to overcome this by using weekly or monthly return intervals to reduce the amount of non-overlapping trading. In this study, we show that this may be insufficient as correlation between markets continues to alter at much longer horizons. A variety of theories have been put forward to explain the large levels of serial correlation witnessed for equity indices, most centred around partial adjustment models where shocks to the market impact one group of stocks more quickly than others (Ahn et al., 2002). In this paper, we propose a model capturing the horizon effects of international equity cross-correlations, which combines short horizon returns data with information on serial- and cross-serial correlations to replicate the long horizon temporal cross-correlation behaviour.

Barriers to foreign investment have lowered considerably in recent years, suggestive of increased levels of integration between international equity markets (Pukthuanthong and Roll, 2009). However, the evidence for a corresponding positive trend in correlation between international equity markets is mixed (Christoffersen et al., 2012). One possible explanation for this inconsistency may be variation in the pattern of dependency at different time horizons. We contribute to this discussion by developing a dynamic time-horizon dependent correlation, allowing us to examine the trend in market dependency as a function of both time and horizon.

This study contributes to the literature in several dimensions. First, we demonstrate considerable variation in cross-country international equity market correlations moving from short to long horizons. This finding is not eradicated by
the use of synchronized returns data, suggesting that forces other than non-
contemporaneous trading are at work. We then introduce a conditional, horizon-
dependent correlation measure which allows determination of the trend in inter-
national correlation at different horizons, contributing to the debate on increasing
cross-correlations in recent years. Raised long horizon cross-correlations are then
replicated using a model employing short horizon data and information on serial
and cross-serial correlations between markets. Finally, we examine the impact of
changing cross-correlations on the risk associated with international portfolio al-
location and show that the benefits of international diversification are unequally
dispersed according to time-horizon. These findings are of fundamental impor-
tance to international finance as they suggest that international diversification,
while still beneficial in terms of risk reduction, may not produce the high level of
risk reduction previously thought. A range of robustness tests are performed to
ensure the results are not a consequence of the methodology adopted, the time
period examined or the periodicity of the base data returns.

The remainder of this paper is organized as follows. In the next section we
review some literature relevant to asset dependence and international diversifica-
tion. Section 3 describes the methodology and Section 4 the data applied. Our
main results are described in Section 5, while Section 6 provides a summary and
conclusion.

2. Related Literature

International diversification has long been documented as a natural risk reduc-
tion extension of the classic Markowitz portfolio selection theory (Levy and Sarnat,
1970; Grubel, 1968; Markowitz, 1952). A variety of studies have demonstrated low
levels of cross-correlation between international equity markets and interpreted
this as an opportunity to improve portfolio risk return trade-off (Berger et al.,
2011; Goetzmann et al., 2005; Levy and Sarnat, 1970; Grubel, 1968). However, research has indicated that the low levels of correlation found between international equity markets may be deceptive, particularly when considered from a downside risk perspective (Christoffersen et al., 2012; You and Daigler, 2010). We contribute to this debate on the benefits of international diversification, examining if the measurement interval and associated investor horizon impacts the magnitude of correlation found between world equity markets.

Various methods to augment the benefits of international portfolio allocation have been proposed; Guidolin and Timmermann (2008) suggest an alteration in diversification benefits as market regimes and preferences on skewness and kurtosis are taken into account. Eun et al. (2009) demonstrate additional international diversification benefits from small-cap stocks. Systemic risk, captured though simultaneous market jumps, is shown to reduce the benefits of international diversification (Das and Uppal, 2005). Moreover, time varying market regimes are found to have significant impact on the risk management of international equity portfolios (Okimoto, 2008; Ang and Bekaert, 2002). While significant diversification benefits from international investment are demonstrated in these papers, the impact of the measurement horizon has not been considered in detail, with previous research predominantly focused on weekly or monthly return intervals.

Conditional relationships between international equity markets have demonstrated mixed results. Bekaert et al. (2009) suggest little evidence of an upward trend in the correlation between international equity markets. In contrast, Longin and Solnik (1995) demonstrate a significant positive trend in international correlation levels, while Christoffersen et al. (2012) also find increased dependency over time. In this paper, we adopt the dynamic conditional correlation (DCC) methodology to capture dynamic changes in the correlation between international equity markets (Bali and Engle, 2010; Engle, 2002). In addition, we enhance this
methodology by determining horizon-dependent dynamic conditional correlations. This further contributes to the literature, allowing us to disentangle the temporal aggregation characteristics of the correlation between world markets in a dynamic fashion. Moreover, we detail the trend in international equity market correlation at different return intervals, providing additional evidence regarding the discussion on correlation trend.

Considerable evidence exists surrounding the influence of trading frictions on the effective measurement of fundamental financial characteristics. For example, the quantification of non-diversifiable systematic risk or beta of an asset may be biased by delays in the trading process often attributed to liquidity (Kamara et al., 2013; Perron et al., 2013; Gençay et al., 2005). Various authors have derived methods linking the sensitivity of measured betas to underlying frictions using, for example, leading and lagging serial and cross-serial correlations between an asset and the market (Perron et al., 2013; de Jong and Nijman, 1997; Cohen et al., 1983). Considering international equity indices, significant serial correlation has been comprehensively observed, often with positive serial correlation at short intervals and negative serial correlation at longer return intervals, (Lewellen, 2002; Ahn et al., 2002; Poterba and Summers, 1988). Moreover, international equity markets often have non-synchronous trading hours, which induces cross-serial correlation between markets due to common information being incorporated in prices at different times (Schotman and Zalewska, 2006; Martens and Poon, 2001; Burns et al., 1998). In this paper, we develop a model capturing the horizon effects of international equity market cross-correlations by incorporating information on serial- and cross-serial correlation between markets.

The impact of the investment horizon on asset risk and return has been examined in some detail, with early research focussing on data with different return
intervals\textsuperscript{1} to disentangle how financial characteristics alter with horizon. Considering the relationship between financial assets, Epps (1979) documents an increase in the correlation between financial assets as the horizon at which price changes are measured increases.\textsuperscript{2} Recent contributions have considered the importance of time horizon in determining market predictability (Boudoukh \textit{et al.}, 2007; Barberis, 2000) and horizon-based asset pricing (Kamara \textit{et al.}, 2013).

The research outlined on time horizon effects in finance has tended to focus primarily on returns at differing intervals, explicitly ignoring information inherent in discarded returns. Another problem concerns the interpretation of the contribution of different return intervals to the aggregate, as each longer horizon explicitly incorporates information from shorter horizons. To overcome these concerns we apply the wavelet transform, as this provides an efficient means to study the multi-horizon properties of time-series, allowing the decomposition of a signal into a set of orthogonal time horizons or frequency band components. The wavelet transform has been comprehensively utilized across a range of problems in both Economics\textsuperscript{3} and Finance to understand horizon dependent characteristics. In this paper, we use the wavelet transform to disentangle the time horizon (time-frequency) charac-

\textsuperscript{1}Differing return intervals are created by sub-sampling price data or by summing over high frequency logarithmic returns and calculating long horizon or low-frequency returns. For example, daily price data may be sub-sampled every Friday to create weekly data. One of the concerns with this approach is the lack of motivation concerning sub-sample timing.

\textsuperscript{2}For this reason, the phenomena of decreasing correlation between assets at high frequency is sometimes known as the \textit{Epps effect}. While Epps (1979) considered a small number of domestic stocks at short horizons, in this study we consider a range of international equity markets and find characteristic increases in correlation at long horizons.

\textsuperscript{3}Some specific wavelet contributions to the Economics literature include tests for serial correlation in panel models (Hong and Kao, 2004), long memory estimation in time series (Fau\'th et al., 2009) and the relation between the yield curve and the Macroeconomy (Aguiar-Conrraria \textit{et al.}, 2012).
teristics of international equity indices. Moreover, we contribute to the literature on time horizon effects by demonstrating how long-run wavelet cross-correlations may be modelled using short-run data.

Considering the evidence that financial relationships may not follow the same pattern as a function of time-horizon (Kamara et al., 2013; Bandi and Perron, 2008; Campbell and Viceira, 2005), the wavelet transform provides a natural tool to understand the time-frequency properties of the process. One of the earliest applications of wavelets to Finance involved a time horizon decomposition of foreign exchange rates (Ramsey and Zhang, 1997). Illustrating the wide ranging importance of time horizon patterns in Finance, In and Kim (2006) demonstrate significantly different futures hedge ratios at different horizons, Gençay et al. (2005) illustrate that the predictions of the capital asset pricing model (CAPM) alter at different horizons, while Kim and In (2005) study the importance of horizon to the Fischer hypothesis relating inflation to nominal stock returns. Rua and Nunes (2009) develop a wavelet coherence\textsuperscript{4} characterization of international equity co-movement in the time-frequency space that is probably closest in approach to this paper. Their focus on the wavelet coherency differs from ours, since the wavelet coherency is a highly localized measure of co-dependency and it fails to incorporate well known phenomena found in financial time-series such as time-varying conditional heteroscedasticity. In this paper, we adopt a dynamic time horizon dependent correlation allowing us to explicitly determine the benefits associated with optimal portfolio allocation, not possible with wavelet coherence. We next describe the methodology adopted in this paper.

\textsuperscript{4}The wavelet coherence between two functions is a localised measure of the extent to which the functions co-move together over the business cycle. A plot of wavelet coherence reveals the level of localised relationship between two time series in both time and frequency.
3. Methodology

3.1. Wavelet Decomposition

The decomposition of Economic and Financial time series into short- and long-term contributions has long been of interest (Perron et al., 2013; Hansen and Scheinkman, 2009; Hodrick and Prescott, 1997). Low-frequency (long-term) components of a time-series may overcome short-term noise or shocks and help reveal underlying economic relationships between variables. High-frequency components may additionally be of interest, providing information regarding short-term high magnitude risks. Filtering approaches such as the Hodrick-Prescott filter inform regarding the long-term trend in a time-series but fail to distinguish between contributions from different frequencies. In finance, agents tend to operate at distinct horizons with long-term fundamental traders (pension funds) and short-term speculative traders (hedge funds, for example) all operating simultaneously. The wavelet transform provides a unique way of separating the contributions of each frequency (or time horizon) and distinguishing distinct contributors to risk. Moreover, the wavelet transform provides for exact reconstruction of the underlying time series from filtered components. The wavelet transform further benefits from an ability to associate localized features with particular frequencies, in contrast to alternative spectral filters such as the Fourier transform.

In this section, we outline the wavelet approach to time-series decomposition. Specifically, we introduce the discrete wavelet transform (DWT), a mathematical tool that projects a time-series onto a set of orthogonal basis functions (wavelets) resulting in a set of wavelet coefficients or filtered time-series associated with distinct frequencies (time horizons). A wavelet is defined as a small wave which can grow and decay in a limited time period, capturing localized features of a time-series. The DWT provides a time-frequency (time horizon) representation of a signal, detailing the frequency content of a signal as a function of time. In con-
contrast to the Fourier transform, which applies infinite sine and cosine functions as the projection basis, the wavelet transform uses a wavelet function that oscillates on a short interval in time. This results in coefficients that change over time, compared with constant coefficients associated with the Fourier transform. We provide a concise description of wavelet decomposition using the Haar wavelet, with comprehensive detail on the application of the DWT to financial time series available in the literature (In and Kim, 2006; Gençay et al., 2001).

The Haar wavelet is the most elementary wavelet filter allowing decomposition of a return series into time-series corresponding to high- and low-frequency components. The Haar wavelet filter coefficient vector of length $\tau_1 = 2^1$, corresponding to scale or horizon one, is given by $h = (h_0, h_1) = (1/\sqrt{2}, -1/\sqrt{2})$ and has the following properties:

$$
\sum_l h_l = 0, \quad \sum_l h_l^2 = 1, \quad \sum_l h_l h_{l+2n} = 0 \quad \forall \text{ integers } n \neq 0.
$$

The first property ensures the wavelet filter sums to zero and identifies changes in the data. The second guarantees that the wavelet filter has unit energy, resulting in variance preservation between the data and the decomposition. The final property provides for the orthonormality of the set of functions derived from $h$, allowing a multiresolution analysis of a finite energy signal. The wavelet filter is complemented by the Haar wavelet scaling filter $g = (g_0, g_1) = (1/\sqrt{2}, 1/\sqrt{2})$, which may be viewed as a local averaging operator and has properties:

$$
\sum_l g_l = \sqrt{2}, \quad \sum_l g_l^2 = 1, \quad \sum_l g_l g_{l+2n} = 0 \quad \forall \text{ integers } n \neq 0.
$$

Similar to the wavelet filter, the scaling filter has unit energy and is orthogonal to even shifts. The first property ensures that the scaling filter averages consecutive blocks of data, as opposed to differencing them. Applying the Haar DWT filter of length $\tau_1 = 2$ to a return series, $\{R_t\}$, produces the following wavelet coefficients:

$$
\sqrt{2} \hat{d}_{1,t} = h_0 R_{2t-1} + h_1 R_{2t}, \quad t = 1, 2, \ldots, T/2.
$$
The factor of \( \sqrt{2} \) is required to guarantee that the squared norm of the wavelet coefficients is equivalent to the squared norm of the return series. The collection of wavelet coefficients \( \tilde{d}_1 \) may be interpreted as a set of weighted differences between consecutive returns. The Haar wavelet scaling coefficient of length \( \tau_1 = 2 \) may be similarly obtained using the Haar scaling filter \( g \) as follows:

\[
\sqrt{2}\tilde{s}_{1,t} = g_0 R_{2t-1} + g_1 R_{2t}, \quad t = 1, 2, \ldots, T/2. \tag{4}
\]

In contrast to the wavelet coefficients, the vector of scaling coefficients \( \tilde{s}_1 \) are based on local averages of length two of the original returns data. Collecting both sets of coefficients into a matrix \( \tilde{w} = (\tilde{d}_1, \tilde{s}_1) \) results in a filtration of the returns data into two orthogonal vectors capturing high-frequency and low-frequency content respectively.

In order to derive longer horizon wavelet and scaling coefficients, we first need to calculate higher order filters. To derive the scale 2 filters, we first define a filter \( h' = (h_0, 0, h_1) = (1/\sqrt{2}, 0, -1/\sqrt{2}) \) which corresponds to a Haar wavelet filter with a zero between the two coefficients. Then, the scale 2 Haar wavelet filter is defined as:

\[
h_{2,i} = \{g \ast h'_{i}\} = \sum_{j=0}^{\tau_j-1} g_j h'_{i-j}, \quad i = 0, \ldots, 3. \tag{5}
\]

This allows determination of the scale 2 wavelet filter, \( h_2 = (1/2, 1/2, -1/2, -1/2) \) with length \( \tau_2 = 2^2 = 4 \). The scale 2 wavelet filter averages the first two returns and differences the following two and is associated with changes at horizon 4. The associated DWT Haar wavelet coefficient may then be found using

\[
2\tilde{d}_{2,t} = h_0 R_{4t-3} + h_1 R_{4t-2} + h_2 R_{4t-1} + h_3 R_{4t}, \quad t = 1, 2, 3, \ldots, T/4, \tag{6}
\]

where the factor of 2 ensures conservation of energy. In a similar fashion, we may define the wavelet scaling filter at scale two as \( g_2 = (1/2, 1/2, 1/2, 1/2) \), which is a simple average of four consecutive returns. The scaling coefficients are then
calculated via:

\[ 2\tilde{s}_t = g_0 R_{4t-3} + g_1 R_{4t-2} + g_2 R_{4t-3} + g_3 R_{4t}, \quad t = 1, 2, 3, \ldots, T/4. \] (7)

Scale 3 and higher order filters may be created analogously. Further detail may be found in Gençay et al. (2001).

The set of wavelet coefficients \( \tilde{d}_1 \) are associated with high-frequency content of returns \( \{R_t\} \), specifically relating to the upper half of frequencies of \( r_t \), \( 1/4 < f \leq 1/2 \). The scale 1 scaling coefficients, \( \tilde{s}_1 \) span the lower half of frequencies, \( 0 \leq f \leq 1/4 \). The scale 2 coefficients further divide the lower half of frequencies, with the detail coefficients \( \tilde{d}_2 \) associated with the frequency band \( 1/8 < f \leq 1/4 \) and scaling coefficients associated with \( 0 \leq f \leq 1/8 \). Similarly, wavelet coefficients at scales \( j = 1, \ldots, J \) are associated with frequency interval \( 1/2^{(j+1)} < f \leq 1/2^j \) (and associated horizon \( 2^j < h \leq 2^{j+1} \)). The remaining scaling coefficients span the residual frequencies \( 0 \leq f \leq 1/2^{(j+1)} \) (and associated horizons greater than \( 2^{(j+1)} \)).

The Haar DWT scaling coefficients are closely related to time-series aggregation, the process by which a set of \( \tau_j \)-day returns, \( \{R_t(\tau_j)\} \) can be created by summing over non-overlapping individual logarithmic returns,

\[ R_t(\tau_j) = \sum_{m=0}^{\tau_j-1} R_{t-m}, \quad t = 2, 4, \ldots, T. \] (8)

The DWT has an inherent downsampling that guarantees the orthogonality of the transform. Later, we use this observation to demonstrate equivalence between cross-correlation calculated for aggregated time-series and wavelet scaling coefficients. Moreover, we show how long horizon wavelet scaling correlations can be expressed as a function of correlations calculated using original data plus a correction for serial and cross-serial correlation.
3.2. Wavelet Variance, Covariance and Correlation

An important characteristic of the wavelet transform is the ability to decompose the variance and covariance of a time-series. Orthogonality of the DWT between different frequency bands (time horizons) ensures that the DWT is an energy preserving transformation with distinct information captured at each horizon. Given the structure of the wavelet coefficients, the energy in the return series $R$ is decomposed on a scale-by-scale basis via

$$
\| R \| = \sum_{j=0}^{J} \| \tilde{d}_j \|^2 + \| \tilde{s}_j \|^2
$$

(9)

where $\| \tilde{d}_j \|^2 = \sum_{t=1}^{T/2^j} \tilde{d}_{t,j}^2$ is the energy associated with scale $\tau_j = 2^j$ and $\| \tilde{s}_j \|^2 = \sum_{t=1}^{T/2^j} \tilde{s}_{t,J}^2$ corresponds to the energy associated with scale $\tau_j = 2^J$ and higher.

Unbiased estimates of the wavelet variance, $\sigma^2_{m,j}$, and wavelet scaling variance, $\tilde{\sigma}^2_{m,J}$ for asset $m$ at horizon $\tau_j$ are defined by

$$
\sigma^2_{m,j} = \text{Var}(\tilde{d}_{m,j}), \quad \tilde{\sigma}^2_{m,J} = \text{Var}(\tilde{s}_{m,J}),
$$

(10)

where $\tilde{d}_{m,j}$ and $\tilde{s}_{m,J}$ are the wavelet detail coefficients and wavelet scaling coefficients for asset $m$ at horizon $\tau_j = 2^j$ and long-run horizons greater than $\tau_j = 2^J$ respectively. Similarly, unbiased estimates of the wavelet covariance, $\gamma^2_{mn,j}$ and wavelet scaling covariance, $\tilde{\gamma}^2_{mn,J}$, between two distinct assets $m$ and $n$ at horizon $\tau_j$ are defined by

$$
\gamma^2_{mn,j} = \text{Cov}(\tilde{d}_{m,j}, \tilde{d}_{n,j}), \quad \tilde{\gamma}^2_{mn,J} = \text{Cov}(\tilde{s}_{m,J}, \tilde{s}_{n,J}),
$$

(11)

where $\tilde{d}_{m,j}$ and $\tilde{d}_{n,j}$ are wavelet coefficients for assets $m$ and $n$ at horizon $\tau_j = 2^j$ and $\tilde{s}_{m,J}$ and $\tilde{s}_{n,J}$ are associated scaling or averaging coefficients. While the wavelet covariance decomposes the covariation on a scale-by-scale basis, the empirical results in this study will focus predominantly on the wavelet correlation at different horizons,

$$
\rho_{mn,j} = \frac{\gamma^2_{mn,j}}{\sigma^2_{m,j}\sigma^2_{n,j}}.
$$

(12)
This equation relates the correlation between two series at scale $\tau_j = 2^j$ to the covariance between the assets at that scale and the variation of each. Further, we define the wavelet scaling correlation, corresponding to long-run horizons (greater than $\tau_J = 2^J$) as

$$\tilde{\rho}_{mn,J} = \frac{\tilde{\gamma}_{mn,J}^2}{\tilde{\sigma}_{m,J}^2 \tilde{\sigma}_{n,J}^2}. \quad (13)$$

This captures the level of long-run correlation between two time-series, capturing the long-run dependence between two financial assets, vital in the context of long-term asset allocation. Importantly, in the case of the Haar DWT, we can relate the scaling correlation $\tilde{\rho}_{mn,J}$ to that calculated using aggregated logarithmic return time-series at the same horizon, as demonstrated in the following proposition.

**Proposition 1.** The Haar wavelet long-run (scaling) cross-correlation between changes in two stationary, finite time-series $m$ and $n$ at horizon $\tau_J = 2^J$ is equivalent to cross-correlation between original time-series aggregated at horizon $\tau_J = 2^J$.

*Proof:* See Appendix

Wavelet decomposition can then be applied to determine short run correlations between financial time series associated with high frequency bands and a long-run correlation associated with low frequencies. Moreover the orthogonality property of the wavelet transform, means that short and long run correlations capture distinct information. The long-run wavelet correlation is equivalent to aggregated correlation, while the short-run correlation captures frequency specific information lost through aggregation.

While the wavelet scaling correlation captures the long-run behaviour between two time series this correlation will, in general, differ from that measured at short horizons. In terms of financial time series, this is usually a consequence of non-synchronous trading in addition to price adjustment delays, both of which can induce portfolio serial correlation and cross-serial correlation between assets. This
results in distinct characteristics at differing horizons. We next demonstrate how the long-run wavelet correlation can be estimated using original (1-day) data combined with information on serial and cross-serial correlation.

**Proposition 2.** The Haar wavelet long-run (scaling) cross-correlation between changes in two stationary, finite time-series of logarithmic returns $R_n$ and $R_m$ at horizon $\tau_J = 2^J$ can be expressed as a function of the original time series as follows:

$$
\rho (\tilde{s}_{m,J}, \tilde{s}_{n,J}) = \rho (R_m(\tau_J), R_n(\tau_J))
$$

$$
= \rho (1) \times \left[ \frac{\tau_j + \sum_{q=1}^{\tau_j-1} \left( \rho^q (R_m(1), R_n(1)) + \rho^{-q} (R_m(1), R_n(1)) \right)}{\left( \tau_j + 2 \sum_{q=1}^{\tau_j-1} \rho^q (R_m(1)) \right) \left( \tau_j + 2 \sum_{q=1}^{\tau_j-1} \rho^{-q} (R_n(1)) \right)} \right]
$$

where $\tilde{s}_{m,J}$ and $\tilde{s}_{n,J}$ are the wavelet scaling coefficients associated with time-series $m$ and $n$ at horizon $\tau_J$ respectively. $\rho (1)$ is the cross-correlation between the original untransformed returns $R_m$ and $R_n$ before aggregation. $\rho^q$ and $\rho^{-q}$ correspond to leading and lagging inter-temporal correlations of order $q$ between the original series.

In Section 5.5, we apply this model to capture the long horizon correlation between international equity markets using only short horizon data. Next, we introduce a new conditional approach to determine the dynamic horizon dependent correlation, combining the DWT with the dynamic conditional correlation.

### 3.3. Dynamic Conditional Correlation

To determine the dynamic structure of correlation between international equity markets, we implement the dynamic conditional correlation (DCC) approach of Engle (2002). The DCC-GARCH methodology is chosen due to parsimony in estimation, in comparison to more parameterized multivariate GARCH models. In
In this study, we allow for the possibility of horizon dependence in the dynamic correlation, by applying the DCC to components of the decomposed discrete wavelet transform series. We describe the DCC methodology and the horizon dependent extension below.

In the first stage of the DCC estimation, standardized residuals are estimated:

\[ r_{m,t} = \gamma_0 + \gamma_1 R_{m,t-1} + \gamma_2 R_{n,t-1} + \varepsilon_{m,t} \]  \hspace{1cm} (16)

where \( R_{m,t} \) is the return of the equity index of country \( m \) at time \( t \). Univariate GARCH models are estimated for each country and estimates of \( \sigma_{mm,t}^2 \) are found,

\[ \sigma_{mm,t}^2 = c_m + \alpha_m \varepsilon_{m,t-1}^2 + \beta_m \sigma_{mm,t-1}^2, \]  \hspace{1cm} (17)

and standardized residuals given by \( \eta_{m,t} = \varepsilon_{m,t}/\sqrt{\sigma_{mm,t}^2} \). Next, a multivariate conditional variance is specified,

\[ H_t = D_t V_t D_t \]  \hspace{1cm} (18)

with \( V_t \) the symmetric conditional correlation of \( \varepsilon_t \) and \( D_t = \text{diag} [\sqrt{\sigma_{mm,t}^2}]_{[2,2]} \).

The bivariate conditional correlation follows the process,

\[ Q_{mn,t} = (1 - \alpha - \beta)Q_{mn} + \alpha \eta_{m,t-1} \eta_{n,t-1} + \beta Q_{mn,t-1} \]  \hspace{1cm} (19)

where \( Q_{mn} = E[\eta_{m,t-1} \eta_{n,t-1}] \) and \( \alpha \) and \( \beta \) are scalar parameters such that \( \alpha + \beta < 1 \). This model ensures a positive-definite quasi correlation matrix but needs to be scaled to obtain a proper correlation matrix with ones on the diagonal,

\[ V_{mn,t} = \frac{Q_{mn,t}}{\sqrt{Q_{mm,t} Q_{nn,t}}} \]  \hspace{1cm} (20)

In this paper, we estimate a horizon dependent dynamic conditional correlation, using the coefficients from the DWT transform at each horizon as inputs to the DCC GARCH model. The DWT coefficients at each horizon are orthogonal by
construction, which means that unique information is captured at each horizon. The wavelet coefficients are then treated in the same fashion as the original time series, first producing standardized residuals and then estimating univariate GARCH models for each country. Using wavelet transformed coefficients allows us to determine the unique horizon dependent dynamic correlation associated with a horizon $\tau_j$ at time $t$,

$$V_{mn,t,j} = \frac{Q_{mn,t,j}}{\sqrt{Q_{mm,t,j}Q_{nn,t,j}}}.$$  \hspace{1cm} (21)

While the wavelet coherence provides localized information relating to the cross-dependency between international markets (Rua and Nunes, 2009), the DCC-DWT correlation can be interpreted as a stochastic process and captures the stylized properties associated with correlation dynamics.

4. Data

Data for the study was obtained from DataStream, a division of Thompson Reuters. The data consists of 8,193 daily prices for a range of international equity indices from January 1, 1980 through May 26, 2011.\footnote{The data selected ends at this point to ensure a dyadic sample. This reduces problems with boundary coefficients associated with the wavelet transform but still results in an extensive data set.} For each country, the index chosen represents a broad coverage of equities, with data available over the entire sample period. In total, equity indices from 21 countries were utilized for the study, chosen to reflect a diverse range of developed and emerging markets with geographical diversity. The countries examined in the study are listed in table 1 along with details of the equity index used.

[Table 1 about here.]
To remove the impact of exchange rate fluctuations from the study, each local index is converted into a central currency.\textsuperscript{6} In this study we choose to measure the benefits of international diversification to a U.S. investor and thus select the U.S. dollar as the common currency. Throughout the study, the base data considered are daily logarithmic returns. Various studies considering international finance have considered daily (Berger \textit{et al.}, 2011; Pukthuanthong and Roll, 2009), weekly (Christoffersen \textit{et al.}, 2012) and monthly (Rua and Nunes, 2009) base data. In this study we will use the wavelet transform to disentangle the impact of these differing data horizons, with asynchronicity captured by the highest frequencies and long-term relationships by low frequency data.\textsuperscript{7}

5. Empirical Results

5.1. Summary Statistics

Descriptive summary statistics for the set of international equity markets studied can be found in table 1. There are considerable cross-sectional differences in the characteristics of the raw returns. The MSCI Denmark index had the highest mean return over the extended period examined, albeit one of the lowest median returns. The Malaysian Kuala Lumpur Composite Index (KLCI) was found to have the lowest mean return. The average standard deviation of daily returns is 1.44\%, with Asian markets of Hong Kong, Malaysia and South Korea found to have standard deviations much greater than average. The lowest daily returns of \(-37.01\%\), \(-30.31\%\) and \(-26.48\%\) were experienced by Singapore, Australia and Malaysia.

\textsuperscript{6}This is standard in international finance studies; Pukthuanthong and Roll (2009) suggest that “such conversions represent a ubiquitous practice in empirical studies of international financial markets”.

\textsuperscript{7}Robustness checks are also performed using monthly base data, with consistent results found, section 5.7.
The majority of indices demonstrate negative skewness and positive excess kurtosis, with the Jacquie Bera statistic rejecting the hypothesis of normally distributed returns for all. Serial 1st order correlation is significant for fifteen indices at a 1% level. The majority of indices display positive serial correlation, with only indices from the USA and South Korea having significant negative serial correlation. The Ljung-Box test rejects the hypothesis that the first 20 autocorrelations in absolute returns are zero.

5.2. Unconditional Correlations

We begin our study of horizon effects on international diversification by measuring the unconditional cross-correlation between representative international equity markets in two ways. First, as we are interested in measuring the benefits of international diversification to a U.S. investor, we measure the average unconditional correlation between the U.S. (S&P 500) and each world market. Since bivariate correlations between the U.S and other world markets only contribute partially to overall diversification benefits, we also investigate the average correlation between each market excluding the U.S.. Considering the correlations in this way allows us the opportunity to distinguish between the various contributors to international diversification risk reduction.

The impact of time-horizon on the unconditional correlation between markets is examined in a selection of eight time-cohorts between 1980 and 2011 and for the whole period. The Haar wavelet transform is employed to decompose the time-series data for each equity index into a series of components corresponding to different time horizons.\textsuperscript{8} Then, the unconditional wavelet correlation between

\textsuperscript{8}In the remainder of the analysis we refer to time horizons, where each time horizon is taken as the corresponding average across a frequency band. For example, the first time horizon corresponds to frequency band $1/4 < f \leq 1/2$, corresponding to horizons between 2 and 4 days,
these components at each horizon is calculated using equation 12 and the long-run correlation using equation 13.

Average horizon dependent unconditional correlations between the U.S. (S&P 500) and each world market are shown in table 2. Considering first the average correlation between 1980 and 2011 at each distinct horizon, this is found to differ considerably across the various horizons studied. The long-run correlation of 0.58 is over five times greater than the correlation found at the shortest horizon. This suggests that short horizon investors appear to reap further benefits from international diversification than their long horizon counterparts. However, correlation measured at short horizons may be biased downwards by a lack of contemporaneous trading. To overcome this, studies on international diversification often resort to weekly or monthly data to increase the amount of trading overlap. Our results suggest that this may be insufficient to eradicate all friction related biases, as correlation continues to increase at horizons much greater than monthly, as evidenced by our results.\(^9\)

[Table 2 about here.]

The dynamic nature of asset correlation has been well documented. In order to examine the consistency of our results at differing points in time, we examine the correlation between international markets in different cohorts. Table 2 details average correlation within each cohort for a range of differing horizons. The results are consistent across all cohorts with increasing average correlation between the S&P 500 and each world market moving from short to long horizons. Moreover, the results are suggestive of increasing cross-correlation between the U.S. and

\(^9\)We further demonstrate in a robustness test, section 5.7, that international equity correlation increases to at least a twelve-month horizon.
world markets over time, an issue we investigate in further detail using a horizon dependent DCC-GARCH approach later.

Average inter-market correlations between each of the world indices (Ex. U.S.) are shown in table 3. Consistent with our earlier results, the average correlation is found to increase moving from short to longer horizons. However, the divergence between short and long horizon correlations is lower than found for bivariate U.S. correlations. Considering the differing cohorts detailed, considerable time variation in unconditional correlation is further demonstrated.

[Table 3 about here.]

The increasing level of dependency detailed between world markets at long horizons suggests that differing diversification benefits accrue to investors with heterogeneous investment horizons. The lowest level of dependence between international equity markets is found at short horizons, suggesting increased opportunity for diversification. In contrast, at long horizons markets were shown to have much increased interdependence, reducing the potential for diversification. While a short horizon investor may regularly adjust their portfolio weights, a long horizon investor may be a “buy-and-hold” investor, who adjusts their weights irregularly. In other words, to increase the benefits of diversification it may be necessary to adjust portfolio weights regularly, in turn increasing transaction costs. In section 5.5 we develop a model to try to understand the drivers behind increased long horizon international equity market correlations.

5.3. Synchronized Returns

In order to isolate the impact of non-contemporaneous market trading and price transmission delays on inter-market correlations, we now consider a reduced set
of contemporaneously measured international markets. As suggested earlier, the use of non-synchronized data may induce cross serial-correlation between markets, in turn downward biasing the estimation of cross-correlation, (Martens and Poon, 2001; Burns et al., 1998). We now determine whether the increasing long-horizon correlations previously detailed are purely a consequence of non-synchronous markets or whether there are additional frictions at work.

Average horizon dependent unconditional correlations for both synchronized and unsynchronized markets are shown in Table 4. Considering first the average correlation between the USA (S&P 500) and the range of other markets, we find substantial increases in cross-correlation moving from short to long horizons, in keeping with our previous findings. If non-contemporaneous market trading hours are the main driver of raised long horizon correlations, then we would expect this effect to be mitigated using synchronized data. However, we find that the differential between long and short horizons, while reduced considerably, is still strongly evident. Short horizon average correlations are measured at 0.733 which is similar to the average market cross-correlation calculated using original daily returns of 0.756. However, when measured at long horizons of 96 days the correlation is 0.827. Moreover, the long-run correlation is 0.881, an increase of 20% from short horizon levels. The findings for synchronized markets suggest that the increased correlation at long horizons detailed is not purely a consequence of gaps in the timing of index measurement.

We next consider the average inter-market correlation between the range of

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10 The indices were selected due to the availability of a common index price measured daily at 16:00 GMT. Data was sourced from Datastream and stretches from July 2003 to May 2011, a total of 2,048 daily returns. The choice of representative index for the previous unconditional analysis was driven by availability of data over the entire period 1980–2011, resulting in different indices.
markets excluding the USA. The remaining markets are all pan-European, suggesting that the timing gaps between market trading hours should be minimal. This is in keeping with our findings, with little evident difference between synchronized and unsynchronized correlations at a particular horizon. However, for both synchronized and unsynchronized markets we continue to find a consistent increase in correlations from short to long horizons. The long term correlation between markets using synchronized data is 0.841 compared to the daily return correlation of 0.739, a 14% increase. This suggests that the benefits of international diversification are strongly mitigated at long horizons. We further consider this shortly, when we consider the impact on portfolio optimization at differing horizon.

While the variation from short to long horizons for markets measured contemporaneously is found to be less than for non-contemporaneous markets, the remaining intertemporal dependence suggests the existence of additional frictions that alter correlation measurement. These findings are in keeping with previous studies investigating the temporal characteristics of markets with synchronized trading hours but potential lags in information transmission due to liquidity differentials (Schotman and Zalewska, 2006; Cohen et al., 1983). In particular, for equity indices the latest index price may reflect shocks to the largest stocks with smaller stocks lagging due to information transmission delays. Informed trading is transmitted from large liquid stocks to smaller more illiquid stocks with a lag, inducing cross-autocorrelations between stocks (Chordia et al., 2011). Moreover, the observed high levels of serial correlation between equity indices is partially a consequence of microstructure frictions and information transmission delays from large to small stocks (Ahn et al., 2002; Lo and MacKinlay, 1990). In this paper, we model the increased long horizon correlation using short horizon data by similarly incorporating serial- and cross serial-correlations between international markets to
model raised long horizon cross-correlations (Section 5.5).

[Table 4 about here.]

5.4. Conditional Correlations

The results on unconditional correlations detailed in both sections 5.2 and 5.3 indicated increased correlation at long time horizons, even for synchronized markets. However, it is possible that these results are a consequence of the specific time cohort chosen. To capture the dynamic nature of covariance and associated correlation we now propose a dynamic horizon-dependent conditional correlation and apply this to the full set of markets. As before, the Haar wavelet transform is used to decompose the time-series data for each country into a set of orthogonal horizon dependent coefficients. Then the DCC-GARCH model (section 3.3) is applied individually to each set of wavelet coefficients to measure the bivariate correlation between each pair of international markets. As before, we consider the correlation between the U.S. and world markets and world inter-market correlations separately to distinguish between contributors to diversification.\textsuperscript{11}

The average dynamic correlation between the U.S. (S&P 500) and each market is displayed in figure 1 (i-ii) across a variety of horizons. Contrasting the differing horizons detailed we find evidence of distinct correlation dynamics at each horizon. Considering the short-term (3 day) correlation and the longer-term (48 day) correlation we find no evidence of overlap between the level of measured correlation at each horizon. Moreover, the short-term correlation tends to have negligible magnitude for the majority of the time periods examined. Confidence bounds for the shortest and longest horizons studied are shown in figure 1 (iii-iv). These reinforce

\textsuperscript{11}Due to the vast quantities of information generated, the estimation results from the DCC-GARCH model between each pair of indices at each horizon are not tabulated. However, detailed results are available upon request from the authors.
our findings, with average correlation at the 1.5 horizon of 0.04 and at the longest 48 day horizon of 0.49, a tenfold increase.

[Figure 1 about here.]

Figure 2 captures the dynamic horizon dependent correlations between world markets excluding the US. While correlation differences are evident across horizons, the divergence between long and short horizons is not found to be as considerable as documented for U.S. markets. In particular the average correlation at the shortest horizon is 0.32 compared to 0.56 at the longest, an increase of 68%. This increase may be due to less asynchronous market timing between world markets than in for US bivariate correlations.

[Figure 2 about here.]

We further consider this issue in tables 5 and 6 where we detail the dynamic correlation characteristics across all horizons. Findings are consistent with the unconditional analysis, with increased average correlation at longer horizons, continuing beyond the monthly interval often adopted in empirical research into the benefits of international diversification. Moreover, measuring the correlation using the original unfiltered daily data, lower correlation levels are found than for the majority of horizons. This suggests that correlation measured using high frequency data may be an imperfect measure of the long-term relationship between assets and may overstate the true diversification benefits. As demonstrated later, this is a consequence of the autocorrelation structure of and between each international market, in turn partly a result of differences in trading hours.

[Table 5 about here.]

[Table 6 about here.]

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Tables 5 and 6 also detail the deterministic linear time trend in the correlation time-series associated with at each time horizon (Kearney and Poti, 2008). The estimated model has form $\rho_t = \alpha + \delta t + \beta_{t-1}\rho_{t-2} + \beta_{t-2}\rho_{t-2} + \epsilon_t$, where $\beta_{t-1}$ and $\beta_{t-2}$ are the coefficient on the previous correlation values $\rho_{t-1}$ and $\rho_{t-2}$, and $\delta$ the coefficient on the time-trend. Durbin’s h-stat suggests that the model residuals are not first-order serially correlated at any of the horizons examined. For both bivariate U.S. Correlations and world inter-market correlations we find a significant positive trend across all horizons. This trend is further found to increase moving from short to long horizons. This suggests that while short-term correlation between world equity markets has experienced an increase, the benefits of international diversification to a long-term investor have altered significantly in recent years. This is in keeping with previous findings of increased world market dependence over time (Christoffersen et al., 2012; Longin and Solnik, 1995).

5.5. Why do international equity correlations increase at long horizons?

In previous sections, we have demonstrated considerable increase in average cross-correlation between international equity markets at long horizons, even for synchronous markets. In this section, we show how raised long-term correlations may be a result of serial- and cross-serial correlation between markets. To this end, we model long-term correlations using only short horizon (daily) data and information on leading and lagging dependencies for and between markets. These intertemporal equity market dependencies are firstly a consequence of non-synchronous trading between markets, resulting in markets impounding common information at differing times. Further, the dependencies may be a result of price-transmission delays between markets, previously linked to observed index serial correlation (Ahn et al., 2002). While our findings on increased correlation for synchronized markets suggest price transmission delays may be possible between international equity markets, we leave a detailed examination of the mechanism
underlying this for further study.

In proposition 2, we demonstrated how the long-run wavelet scaling correlation between two stationary, finite time series can be expressed as a function of the original time series plus a correction for serial and cross-serial correlation between the original series. In this section, we apply this link to model the long horizon cross-correlation using original one day interval returns. To model the long-run $\tau_J$-day cross-correlation using original untransformed (daily) data, we need to measure the serial correlation and all lagging and leading cross-serial correlations up to interval $\tau_J$ between each market.

Table 7 details modelled $\tau_k$ horizon long-run correlations found using original one-day returns. In each case, the average over all pairs of international equity markets is shown. Long-run correlations are modelled for a range of different horizons. For example, the $> 8$ day horizon corresponds to the long-run wavelet correlation at horizon $\tau_3 = 2^3$. When no serial or cross-serial corrections are accounted for, proposition 2 reduces to the cross-correlation between the original time-series, which underestimates the long-horizon correlation by 17.5%. Incorporating information on serial and cross-serial correlation for up to 8 lags increases the modelled correlation to 0.473, consistent with the average measured correlation of 0.468.

When modelling long-run correlation at longer horizons, similar results are found. Considering the scale $\tau_6 = 128$ day horizon, we demonstrate a consistent increase in the modelled correlation as further intertemporal lags are included in the model. For example, incorporating information on serial and cross-serial correlation for up to 4 lags results in an increase of 22.6% in average modelled cross-correlation relative to daily. While this captures almost 50% of the total increase required to replicate the measured correlation, 32 intertemporal lags, corresponding to approximately a two month horizon, are required to accurately model the
average long-run measured correlation.

[Table 7 about here.]

Previous research examining the intertemporal behaviour of financial characteristics such as systematic risk have suggested that short horizon measurements may be biased (Gençay et al., 2005; Cohen et al., 1983). This is consistent with our findings, where measured short horizon correlations are downward biased due to serial and cross-serial correlations between international equity markets. The results are further consistent with literature demonstrating the prevalence of serial correlation in equity indices (Lewellen, 2002; Ahn et al., 2002). The above findings further suggest that the measured benefits of international diversification may be biased downwards at short horizons, a theory we investigate in the next section.

5.6. Implications for International Diversification

The evidence presented thus far indicates that correlation between international financial markets is a function of the measurement horizon. This suggests that the diversification opportunities available to investors are a function of the return interval and associated horizon selected. In this section, we investigate the potential benefits of international diversification in two ways. First, considering an equally weighted portfolio, we determine the level of risk reduction achieved at different horizons as the number of portfolio assets increases. Next, we examine whether the mean-variance efficient frontier alters as function of time horizon. This helps to determine whether the benefits of international diversification accrue equally to short- and long-term investors.

In figure 3 we investigate the level of risk reduction achieved by international investors as a function of the quantity of international equity market investments held. Figure 3 measures the level of variance risk reduction as a proportion of the average risk of a single randomly chosen asset. A simulation approach is
used, with the level of risk reduction measured for portfolios made up of randomly chosen assets. Risk reduction is then averaged across 10,000 simulations. Results demonstrate the level of risk reduction achievable is a function of horizon. For example, for a diversified portfolio made up of 15 equally weighted international markets, an investor with a short horizon of 1.5 days removes 37% of the risk involved in holding a single randomly chosen asset, while a long-horizon investor only removes 22%. Robustness tests for these results are provided in section 5.7, where the results are shown to be independent of the wavelet methodology.

One potential problem with measuring only the level of risk reduction, is the potential for higher returns as compensation for the higher risk associated with long horizons. We address the risk-return trade-off now by determining the efficient frontier across the range of time horizons studied, figure 4. The top figure displays the efficient frontier measured using all data from 1980 – 2011. For a given level of return, we see that the level of risk increases from short to long horizons. For example, for a 10% annualized expected return, the associated risk is 11.4% at the shortest horizon studied but 14.9% at the longest horizon, an increase of 31%. Investors with short time-horizons appear to achieve much lower levels of risk for a given level of return than long horizon investors, given the same investment set. This explicitly demonstrates the impact on portfolio allocation of the high correlation between international equity markets at long horizons, with greater associated levels of risk due to negated diversification benefits.

Figure 4 also shows a series of mean-variance efficient frontiers associated with the different time cohorts studied in tables 2 and 3. Results across the cohorts

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\[\text{A risk aversion level of 3 was used for the mean-variance allocation. Results were found to be consistent for differing aversion levels.}\]
are consistent with findings for all data, with short horizon investors experiencing substantially lower risk than long horizon investors across the majority of cohorts. The exception is between 1992 and 1995, which was the cohort shown earlier to have lowest correlation differential between short and long horizons. The recent cohort, 2007−2011, is worth focussing on, as the difference between short and long term risk is found to be considerable. For example, for a 5% annualized expected return, risk is shown to increase from 14.95% to 21.65% moving from the shortest to longest horizon, an increase of 45%. This suggests that long horizon investors might have struggled to reduce risk during the global market crisis, contrary to market folklore.

[Figure 4 about here.]

5.7. Robustness

To ensure that our results are not simply an artefact of the estimation procedure, we test our results without the wavelet transform, using sub-sampled return intervals. Table 8 displays the average correlation between the U.S. (S&P 500) and each international equity market for a range of sub-sampled return intervals. The trend in correlation is found to be consistent with previous wavelet results, showing increased correlation at long return intervals. The distinct advantage of using wavelets is the ability to discriminate between the contributions of each frequency band (horizon) to the overall. In contrast, sub-sampled return intervals explicitly incorporate high frequencies.

[Table 8 about here.]

\[13\]Similar results were obtained for the average correlation between world markets (Excluding U.S.).
Throughout this study, we have focussed on daily base data, as the quantity of available data allows for study of the dynamic characteristics of international correlation. In table 9, we examine the robustness of our results using monthly base data, to determine if our findings are a result of the base return interval chosen. The reduced data quantities mean that results are only available over the entire 1980 – 2011 period. In keeping with earlier findings, correlations are shown to increase moving from short to long horizon. Consistent with previous results, the average U.S correlation with world markets increases from 0.238 at the original daily horizon to 0.50 at a 3 month horizon, with a further increase to 0.629 at a 12 month horizon. Moreover, the long-run correlation is measured at 0.593, substantially above the weekly or monthly correlations commonly applied in the literature. Similarly, the inter-market correlation excluding the USA is shown to have a long-run level of 0.574, much greater than the daily or monthly measured correlations. This further supports our hypothesis that long-term buy and hold investors achieve much lower diversification benefits than short-term investors and that measurement of international diversification benefits using weekly or monthly data is insufficient.

[Table 9 about here.]

Optimal asset allocation decisions have been found to differ considerably during bull and bear regimes (Okimoto, 2008; Guidolin and Timmermann, 2008). In order to ensure that our main results relating to increased unconditional correlation at long horizons are robust to varying market conditions, we explore the horizon dependence during NBER categorized economic expansion and contraction cohorts. In table 10 we detail the average correlation between all world markets across short and long-horizons during both recessionary and expansionary periods. Results are shown to be consistent with earlier findings, with increasing correlation between
short and long horizons found across all cohorts. This suggests that the results documented are not a consequence of asymmetric dependence structure during bull and bear market regimes.

[Table 10 about here.]

Building on the risk reduction analysis in section 5.6, we further consider the benefits of international diversification using sub-sampled return intervals, figure 5. The impact of adding additional assets to a portfolio made up of a randomly chosen selection of international equity markets is examined at different sub-sampled time horizons. The results suggest that our previous findings were not a consequence of the wavelet method adopted. As found previously, long horizon investors achieve significantly less diversification benefits than short horizon counterparts.

[Figure 5 about here.]

6. Conclusions

In this paper we investigate the benefits of international diversification over short- and long-run horizons. To disentangle the horizon related effects, the discrete wavelet transform is applied, allowing the separation of short-term seasonality from long-term trends. The wavelet transform provides a natural tool to understand the time-frequency properties of a financial time series, providing insight into the distinct contributions of individual horizons (frequency bands) to the aggregate.

Unconditional correlations are first measured between international equity markets, with a monotonic increase in correlation detailed moving from short to longer horizons. Considering the average correlation between the U.S. and World markets from 1980 to 2011, a five-fold increase in unconditional correlation is found at the longest horizon examined (96 days) compared to the shortest (3 days).
increase in correlation is further demonstrated to be robust to the methodology adopted and the return interval of the base data. Moreover, the observed increase in correlation is found to persist for synchronously measured equity indices. The later finding suggests that frictions other than non-synchronous trading hours contribute to the observed long horizon correlation increases.

Wavelet transformed coefficients are then applied as inputs to a DCC-GARCH model, resulting in a horizon dependent conditional correlation. Consistent with the unconditional results, evidence of increasing long horizon correlation is demonstrated over time. Moreover, a trend in correlation is found at all horizons, with evidence of a positive trend strongest at long horizons. This suggests that while the benefits of international diversification have decreased in recent years, long horizon investors have been particularly impacted.

In order to shed light on why long horizon international equity correlation are found to increase, a model accounting for frictions in the trading process is described. Incorporating delays in information transmission and non-synchronous trading between international markets using lagged and leading intertemporal correlations, we detail how elevated long horizon correlations may be generated using short horizon data. Our findings are in keeping with previous studies on serial correlation in equity indices and on the measurement of systematic risk. While finds may in interpreted as increased long horizon correlation, an alternative interpretation may be that short-horizon correlations are downward biased by frictions.

The impact of increased correlation between international equity markets at long horizons is then investigated from a portfolio allocation viewpoint. Assessing the level of risk mitigation achievable for differing asset numbers, we demonstrate a differential in the risk reduction found by investors with differing horizon. In particular, short horizon investors are shown to achieve further risk reduction than
their long horizon counterparts for all asset numbers.

Considering the risk-return trade-off, we then determine the mean-variance efficient frontier associated with different horizons and cohorts. Our results indicate that a long horizon investor experiences much higher risk, for a given level of return, than a short horizon counterpart. For a 10% annualised expected return, the associated risk is measured as 11.4% at the shortest horizon but 14.9% at the longest, an increase of 31%. Consistent results are then shown for a range of cohorts.

In summary, our results show that the benefits of international diversification are not equally dispersed across heterogeneous investors. When measured at short horizons, the benefits of international diversification may appear to be large but this may be a function of characteristics specific to international indices such as serial correlation. For long horizon investors international diversification is shown to reduce portfolio risk, but perhaps not to the extent previously thought.
Appendix

Proof of Proposition 1.

We demonstrate that determination of the Haar wavelet scaling correlation at horizon $\tau_J = 2^J$ is equivalent to finding the correlation between aggregated data at the same dyadic horizon. Considering first horizon $\tau_1 = 2^1$, the Haar scaling filter of length 2 is given by $g = (g_0, g_1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$. Applying the Haar scaling filter to a set of returns $\{R_t\}$ and rearranging the formula for the scaling coefficient (equation 2), we get

$$2\tilde{s}_{1,t} = R_{2t-1} + R_{2t}, \quad t = 1, 2, 3, \ldots, T/2. \quad (22)$$

The orthogonality property of the DWT and associated decimation results in $T/2$ scaling coefficients associated with horizon $\tau_1$, in a similar fashion to non-overlapping time series aggregation.

Next, represent the aggregated time series at horizon $\tau_1 = 2^1$ as a summation over consecutive non-overlapping returns,

$$R_{m,2t}(\tau_1) = R_{2t-1} + R_{2t}, \quad t = 1, 2, 3, \ldots, T/2. \quad (23)$$

The common terms on the right hand sides of equations 22 and 23 are now exploited to determine the relationship between the wavelet and aggregated correlations at horizon $\tau_1$. Given two time-series $R_m$ and $R_n$, the aggregated covariance and variance may be written in terms of wavelet scaling coefficients $\tilde{s}_{m,1}$ and $\tilde{s}_{n,1}$ associated with assets $m$ and $n$ at horizon $\tau_1 = 2^1$,

$$\text{Cov} (R_m (\tau_1), R_n (\tau_1)) = 2^2 \text{Cov} (\tilde{s}_{m,1}, \tilde{s}_{n,1}) = 4\gamma_{mn,J}^2 \quad (24)$$

$$\text{Var} (R_m (\tau_1)) = 2 \text{Var} (\tilde{s}_{m,1}) = 2\sigma_{m,j}^2 \quad (25)$$

$$\text{Var} (R_n (\tau_1)) = 2 \text{Var} (\tilde{s}_{n,1}) = 2\sigma_{n,j}^2 \quad (26)$$
In turn, this relates the aggregated correlation between \( R_m(\tau_1) \) and \( R_n(\tau_1) \) to the wavelet scaling correlation

\[
\rho(R_m(\tau_1), R_n(\tau_1)) = \frac{4 \text{Cov}(\tilde{s}_{m,1}, \tilde{s}_{n,1})}{4 \text{Std}(\tilde{s}_{m,1}) \text{Std}(\tilde{s}_{n,1})} = \frac{4 \bar{\gamma}_{m,n}^2}{4 \bar{\sigma}_{m,J}^2 \bar{\sigma}_{n,J}^2} = \tilde{\rho}(\tilde{s}_{m,1}, \tilde{s}_{n,1}) \tag{27}
\]

Similarly, Haar DWT scaling coefficients, representing the average or long run trend at horizon \( \tau_2 = 4 \) can be rearranged as

\[
2^{3/2} \tilde{s}_{2,t} = x_{4t-3} + x_{4t-2} + x_{4t-1} + x_{4t}, \quad t = 1, 2, 3, \ldots, T/4 \tag{28}
\]

Aggregated returns at horizon \( \tau_2 = 2^2 \) can similarly be written as

\[
R_{m,4t}(\tau_2) = x_{4t-3} + x_{4t-2} + x_{4t-1} + x_{4t}, \quad t = 1, 2, 3, \ldots, T/4 \tag{29}
\]

As previously shown for horizon \( \tau_1 = 2^1 \), this allows us to write the aggregated variance and covariance as a function of the wavelet scaling coefficients at horizon \( \tau_2 = 2^2 \). The aggregated correlation at horizon \( \tau_2 = 2^2 \) can then be written in terms of wavelet scaling coefficients,

\[
\rho(R_m(\tau_2), R_n(\tau_1)) = \frac{2^3 \text{Cov}(\tilde{s}_{m,2}, \tilde{s}_{n,2})}{2^3 \text{Std}(\tilde{s}_{m,2}) \text{Std}(\tilde{s}_{n,2})} = \frac{2^3 \bar{\gamma}_{m,n}^2}{2^3 \bar{\sigma}_{m,2}^2 \bar{\sigma}_{n,2}^2} = \tilde{\rho}(\tilde{s}_{m,2}, \tilde{s}_{n,2}) \tag{30}
\]

Adopting a similar approach, we may demonstrate that the wavelet scaling correlation is equivalent to the aggregated data correlation at any horizon \( \tau_J = 2^J \),

\[
\rho(R_m(\tau_J), R_n(\tau_J)) = \tilde{\rho}(\tilde{s}_{m,J}, \tilde{s}_{n,J}) \tag{31}
\]


Proof of Proposition 2.

The estimation of cross- and serial-correlations from non-synchronous data has long been of interest de Jong and Nijman (1997); Lo and MacKinlay (1990). Following Cohen et al. (1983), we first outline how cross-correlation at horizon \( \tau_j \) may be related to the cross-correlation measured at the original (1 day) interval with
a correction for serial and cross-serial correlation. Applying proposition 1, the long-run wavelet scaling correlation can then be related to original (1 day horizon) correlation with correction terms.

Assume that changes (returns) in two sets of stationary, finite variance time series of returns \( \{R_m\} \) and \( \{R_n\} \) are additive. In other words, by summing over non-overlapping individual returns \( R_{m,t} \) and \( R_{n,t} \), we achieve a \( \tau_j \)-day return,

\[
R_{\{m,n\},t}(\tau_1) = \sum_{i=0}^{\tau_j-1} R_{\{m,n\},\tau_j-i}
\]  

(32)

The aggregated \( \tau_j \) day covariance between \( R_m \) and \( R_n \) is then given by

\[
\sigma_{mn}(\tau_j) = Cov[R_m(\tau_j), R_n(\tau_j)]
= Cov\left[\sum_{i=0}^{\tau_j-1} R_{m,\tau_j-i}, \sum_{k=0}^{\tau_j-1} R_{n,\tau_j-k}\right]
= \sum_{i=0}^{\tau_j-1} \sum_{k=0}^{\tau_j-1} Cov[R_{m,\tau_j-i}, R_{n,\tau_j-k}]
\]  

(33)

The diagonal elements of the time-covariance matrix \( \sigma_{mn}(\tau_j) \) are each equal to the contemporaneous covariance between \( R_m \) and \( R_n \) at the base horizon (1 day in our analysis). The off-diagonal covariances are leading and lagging cross-covariances. Assuming stationarity, all covariances where the lead and lag, \( s = i - k \), are the same are equal and there are \( (\tau_j - s) \) of these (Similarly, for \( -s = i - k \)). Considering the various component elements of \( \sigma_{mn}(\tau_j) \), we can write

\[
i = k, \quad Cov\left[R_{m,\tau_j-i}, R_{n,\tau_j-k}\right] = Cov[R_m, R_n]
\]

\[
i > k, \quad Cov\left[R_{m,\tau_j-i}, R_{n,\tau_j-k}\right] = Cov\left[R_{m,\tau_j-i}, R_{n,\tau_j-(i-s)}\right]
= \rho_{mn}^s \sigma_m \left(R_{m,\tau_j-i}\right) \sigma_n \left(R_{n,\tau_j-(i-s)}\right)
= \rho_{mn}^s \sigma_m (1) \sigma_n (1)
\]

\[
k > i, \quad Cov\left[R_{m,\tau_j-i}, R_{n,\tau_j-k}\right] = \rho_{mn}^{-s} \sigma_m (1) \sigma_n (1)
\]
Substituting each of these into equation 33, we get

\[ \sigma_{mn}(\tau_j) = \tau_j \sigma_{mn}(1) + \sum_{s=1}^{\tau_j-1} (\tau_j - s) \rho_{mn}^s \sigma_m(1) \sigma_n(1) + \sum_{s=1}^{\tau_j-1} (\tau_j - s) \rho_{mn}^s \sigma_m(1) \sigma_n(1) \]  

(34)

Since \( \sigma_m(1) \sigma_n(1) = \frac{\sigma_{mn}(1)}{\rho_{mn}} \), we can write the covariance between \( R_m \) and \( R_n \) at horizon \( \tau_j \) as

\[ \sigma_{mn}(\tau_j) = \sigma_{mn}(1) \left[ \tau_j + \sum_{s=1}^{\tau_j-1} \rho_{mn}^s + \rho_{mn}^s (\tau_j - s) \right] \]  

(35)

Similarly, we show how the variance of an aggregated time-series at horizon \( \tau_j \) can be written as

\[ \sigma_m(\tau_j) = \sigma_m(1) \left[ \tau_j + 2 \sum_{s=1}^{\tau_j-1} \rho_{mn}^s (\tau_j - s) \right] \]  

(36)

Combining equations 36, 35 and 31, the aggregated correlation between returns \( R_m \) and \( R_n \) at horizon \( \tau_j = 2^j \) can be expressed as a function of the original (one day) time-series cross-correlation plus a correction as follows:

\[ \rho_{mn}(\tau_j) = \rho_{mn}(1) \times \left[ \frac{\tau_j + \sum_{s=1}^{\tau_j-1} \left( \rho_{mn}^s + \rho_{mn}^s \right)}{\tau_j + 2 \sum_{s=1}^{\tau_j-1} \rho_{mn}^s} \right] \]  

(37)

\[ \bar{\rho}(s_m, \tau_j, s_n, J) = \rho(R_m(J), R_n(\tau_j)) \]  

(38)

Proposition 1 demonstrated that the wavelet long-run cross-correlation between two time series is equivalent to the aggregated correlation at the same horizon. Applying proposition 1, we can write the wavelet scaling correlation in terms of correlation between the original (one day) time-series plus a correction for leading and lagging serial and cross-serial correlation,
References


Figure 1: Dynamic Horizon Dependent Correlation between USA and World Markets (1980-2011).

The average correlation between the US market (S&P 500) and the range of world markets studied is reported across a variety of time horizons. (i) Average Correlation is shown at a 3, 12 and 48 day horizon. (ii) Average Correlation is shown at a 6, 24 and 96 day horizon. (iii) The correlation range (10th and 90th percentiles) is reported for a 3 day horizon. (iv) The correlation range (10th and 90th percentiles) is reported for a 96 day horizon. The data is decomposed into the underlying time horizons using the HAAR wavelet transform and a DCC GARCH model is applied at each horizon to calculate the conditional correlation between each pair of markets.
Figure 2: Dynamic Horizon Dependent Correlation All Markets (1980-2011).
The average correlation between each of the world markets (Ex. USA) studied is reported across a variety of time horizons. (i) Average Correlation is shown at a 3, 12 and 48 day horizon. (ii) Average Correlation is shown at a 6, 24 and 96 day horizon. (iii) The correlation range (10th and 90th percentiles) is reported for a 3 day horizon. (iv) The correlation range (10th and 90th percentiles) is reported for a 96 day horizon. The data is decomposed into the underlying time horizons using the Haar wavelet transform and a DCC GARCH model is applied at each horizon to calculate the conditional correlation between each pair of markets.
Figure 3: Risk Reduction at a Range of Time Horizons for Differing Portfolio Sizes (1980 – 2011).
The average risk of a portfolio made up of a number of randomly chosen assets is measured as a proportion of the average variance of a single randomly chosen asset. The Haar Wavelet transform is used to decompose the returns data into the associated horizons. Portfolio risk is determined using all available data for each asset from 1980 – 2011. A simulation approach is applied, with risk averaged over 10,000 randomly generated portfolios for each portfolio size.
Figure 4: Mean Variance Efficient Frontiers at different Horizons (1980-2011).

Figure 5: Risk Reduction at a Range of Sub-Sampled Time Horizons for Differing Portfolio Sizes (1980 – 2011).
The average risk of a portfolio made up of a number of randomly chosen assets is measured as a proportion of the average risk of a single randomly chosen asset. Subsampled returns are created by sampling the original data. Portfolio risk is determined using all available data for each asset from 1980 – 2011. A simulation approach is applied, with risk averaged over 10,000 randomly generated portfolios for each portfolio size.
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<th>Country</th>
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<th>Median</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
<th>1st Order Auto Correlation</th>
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Table 1: Summary Statistics for Daily International Equity Index Returns (1980-2011)

Notes: Each country and associated equity index is listed along with summary statistics denominated in USD. Mean, median, minimum, maximum and standard deviation are in terms of daily returns. The period is from January 1980 to December 2011. LB(20) is the Ljung-Box test for autocorrelation of order 20, JB is the Jacque-Bera test for normality based on excess skewness and kurtosis. ‘DS Market’ corresponds to the Datastream compiled representative equity index for the country. ** indicates significance at a 5% level, while *** indicates significance at a 1% level.
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Table 2: Average Unconditional Correlation between US and World Markets (1980-2011) using Wavelet Transformed Data.
Notes: The average unconditional correlation between the US (S&P 500) and a range of 20 world equity markets is calculated at different time horizons using data from 1980-2011 and within 4 year cohorts. The Haar Wavelet transform is used to decompose returns data into the range of horizons detailed, and correlations are then calculated at each horizon. Correlations found using the original untransformed data are also shown. 95% confidence intervals are shown beneath each correlation.
Table 3: Average Unconditional-Correlation between World Markets (Ex. USA) 1980-2011 using Wavelet Transform.

Notes: The average correlation between World markets excluding the USA is calculated at different time horizons using data from 1980-2011 and within 4 year cohorts. The Haar Wavelet transform is used to decompose returns data into the range of horizons detailed, and correlations are then calculated at each horizon. Correlations found using the original untransformed data are also shown. 95% confidence intervals are shown beneath each correlation.
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<td>24</td>
<td>0.737</td>
<td>0.755</td>
</tr>
<tr>
<td></td>
<td>[0.717, 0.756]</td>
<td>[0.736, 0.773]</td>
</tr>
<tr>
<td>48</td>
<td>0.768</td>
<td>0.775</td>
</tr>
<tr>
<td></td>
<td>[0.749, 0.785]</td>
<td>[0.758, 0.792]</td>
</tr>
<tr>
<td>96</td>
<td>0.773</td>
<td>0.788</td>
</tr>
<tr>
<td></td>
<td>[0.755, 0.790]</td>
<td>[0.771, 0.804]</td>
</tr>
<tr>
<td>Long Term</td>
<td>0.841</td>
<td>0.854</td>
</tr>
<tr>
<td></td>
<td>[0.828, 0.853]</td>
<td>[0.842, 0.866]</td>
</tr>
<tr>
<td>Original Daily</td>
<td>0.739</td>
<td>0.747</td>
</tr>
<tr>
<td></td>
<td>[0.719, 0.758]</td>
<td>[0.727, 0.765]</td>
</tr>
</tbody>
</table>

Table 4: Unconditional Synchronized and Unsynchronized Correlations 2003-2011 using Wavelet Transform.
Notes: Correlations calculated using synchronized data measured at 16:00 GMT at different horizons and using unsynchronized data measured at market close. The Haar Wavelet transform is used to decompose the returns data into the associated horizons. 95% confidence intervals are shown below each correlation. Equity market indices for each country were selected as follows: Austria - ATX Index, Denmark - KFX Index, France - CAC 40, Germany - DAX 30, Holland - AEX Index, Ireland - ISE Index, Switzerland - SWX Index, USA - S&P 500.
<table>
<thead>
<tr>
<th></th>
<th>Original Data</th>
<th>3 Days</th>
<th>6 Days</th>
<th>12 Days</th>
<th>24 Days</th>
<th>48 Days</th>
<th>96 Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.190</td>
<td>0.044</td>
<td>0.242</td>
<td>0.299</td>
<td>0.376</td>
<td>0.412</td>
<td>0.494</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.098</td>
<td>0.076</td>
<td>0.089</td>
<td>0.093</td>
<td>0.120</td>
<td>0.107</td>
<td>0.123</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.888</td>
<td>1.231</td>
<td>1.046</td>
<td>0.485</td>
<td>-0.027</td>
<td>0.317</td>
<td>0.290</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.342</td>
<td>1.745</td>
<td>0.840</td>
<td>-0.664</td>
<td>0.274</td>
<td>-0.936</td>
<td>-0.946</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.002</td>
<td>-0.142</td>
<td>0.060</td>
<td>0.139</td>
<td>0.035</td>
<td>0.163</td>
<td>0.275</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.478</td>
<td>0.300</td>
<td>0.507</td>
<td>0.516</td>
<td>0.705</td>
<td>0.624</td>
<td>0.790</td>
</tr>
</tbody>
</table>

**Correlation Trend - Mean Equation**

<table>
<thead>
<tr>
<th></th>
<th>Original Data</th>
<th>3 Days</th>
<th>6 Days</th>
<th>12 Days</th>
<th>24 Days</th>
<th>48 Days</th>
<th>96 Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.0002</td>
<td>-0.0055**</td>
<td>0.0018***</td>
<td>0.0069***</td>
<td>0.0126**</td>
<td>0.0295***</td>
<td>0.0455***</td>
</tr>
<tr>
<td>Time Trend</td>
<td>1.844E-07***</td>
<td>1.948E-07***</td>
<td>4.761E-07***</td>
<td>1.603E-06***</td>
<td>2.438E-06***</td>
<td>5.408E-06***</td>
<td>6.795E-06**</td>
</tr>
<tr>
<td>(\beta_{-1})</td>
<td>0.810***</td>
<td>0.885***</td>
<td>0.964***</td>
<td>0.887***</td>
<td>0.836***</td>
<td>0.764***</td>
<td>0.665***</td>
</tr>
<tr>
<td>(\beta_{-2})</td>
<td>0.185***</td>
<td>0.109***</td>
<td>0.021</td>
<td>0.069**</td>
<td>0.104***</td>
<td>0.112**</td>
<td>0.181***</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.969</td>
<td>0.994</td>
<td>0.985</td>
<td>0.972</td>
<td>0.944</td>
<td>0.935</td>
<td>0.864</td>
</tr>
<tr>
<td>h-statistic</td>
<td>2.009</td>
<td>2.005</td>
<td>2.003</td>
<td>1.994</td>
<td>1.988</td>
<td>1.992</td>
<td>2.000</td>
</tr>
<tr>
<td>h-stat. sig.</td>
<td>(0.639)</td>
<td>(0.541)</td>
<td>(0.496)</td>
<td>(0.417)</td>
<td>(0.382)</td>
<td>(0.385)</td>
<td>(0.3631)</td>
</tr>
</tbody>
</table>

Table 5: **Dynamic Correlation Characteristics and Time-Trend between World Markets and USA (1980-2011) at different horizons.**

Notes: The average correlation between the US market (S&P 500) and the range of world markets studied is detailed across a variety of time horizons. The data is decomposed into the underlying time horizons using the Haar wavelet transform and a DCC GARCH model is applied to the wavelet coefficients at each horizon to calculate the conditional correlation. The mean, standard deviation, skewness, kurtosis, minimum and maximum of the correlation are calculated over time at each horizon. The time-trend in average correlation is also examined at each time horizon using a linear model with Newey-West standard errors and significance levels. The time-trend regression model is given by \(\rho_t = \alpha + \delta t + \beta_{-1}\rho_{t-1} + \beta_{-2}\rho_{t-2} + \epsilon_t\), where \(\beta_{-1}\) and \(\beta_{-2}\) are the coefficients on the previous correlation values \(\rho_{t-1}\) and \(\rho_{t-2}\), and \(\delta\) the coefficient on the time-trend. Durbin’s h-statistic and associated significance tests the null that the model residuals are not first order serially correlated. ** indicates significance at a 5% level, while *** indicates significance at a 1% level.

Notes: The average correlation between the range of world markets studied (Ex. USA) is detailed across a variety of time horizons. The data is decomposed into the underlying time horizons using the Haar wavelet transform and a DCC GARCH model is applied at each horizon to calculate the conditional correlation. The mean, standard deviation, skewness, kurtosis, minimum and maximum of the correlation are calculated over time at each horizon. The time-trend in average correlation is also examined at each time horizon using a linear model with Newey-West standard errors and significance levels. The time-trend regression model is given by $\rho_t = \alpha + \delta t + \beta_{t-1}\rho_{t-1} + \beta_{t-2}\rho_{t-2} + \epsilon_t$, where $\beta_{t-1}$ is the coefficient on the previous correlation value $\rho_{t-1}$, and $\delta$ the coefficient on the time-trend. Durbin’s h-statistic and associated significance tests the null that the model residuals are not first order serially correlated. ** indicates significance at a 5% level, while *** indicates significance at a 1% level.
(i) Imputed $\tau$ horizon Correlations

<table>
<thead>
<tr>
<th>No of Intertemporal Lags</th>
<th>Original</th>
<th>$&gt; 4$</th>
<th>$&gt; 8$</th>
<th>$&gt; 16$</th>
<th>$&gt; 32$</th>
<th>$&gt; 64$</th>
<th>$&gt; 128$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.386</td>
<td>0.386</td>
<td>0.386</td>
<td>0.386</td>
<td>0.386</td>
<td>0.386</td>
<td>0.386</td>
</tr>
<tr>
<td>4</td>
<td>0.448</td>
<td>0.460</td>
<td>0.467</td>
<td>0.470</td>
<td>0.472</td>
<td>0.473</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.473</td>
<td>0.488</td>
<td>0.496</td>
<td>0.500</td>
<td>0.502</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>0.495</td>
<td>0.514</td>
<td>0.523</td>
<td>0.527</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
<td>0.523</td>
<td>0.547</td>
<td>0.559</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>64</td>
<td></td>
<td></td>
<td>0.546</td>
<td>0.552</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>128</td>
<td></td>
<td></td>
<td></td>
<td>0.567</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(ii) Measured Long-Run Correlations

<table>
<thead>
<tr>
<th>Return Interval</th>
<th>Original</th>
<th>$&gt; 4$</th>
<th>$&gt; 8$</th>
<th>$&gt; 16$</th>
<th>$&gt; 32$</th>
<th>$&gt; 64$</th>
<th>$&gt; 128$</th>
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<tbody>
<tr>
<td>Average Correlation</td>
<td>0.386</td>
<td>0.438</td>
<td>0.468</td>
<td>0.501</td>
<td>0.535</td>
<td>0.540</td>
<td>0.558</td>
</tr>
</tbody>
</table>

Table 7: Modelled $\tau$-Horizon Long-Run Correlations for Different Intertemporal Lags.

Notes: Long horizon average $\tau$-Day correlations between each pair of international equity indices is modelled using equation 15. The wavelet transform is employed to construct two orthogonal components, the wavelet rough and smooth. The imputed correlation incorporates the contemporaneous correlation between each market in additional to a range of lagged and leading correlations, calculated using wavelet smooth. The number of intertemporal lags indicates the number of lagged and leading intertemporal variances and covariances used to impute the $\tau$-Horizon correlation. Measured Correlations are found using wavelet scaling coefficients at different horizons. Correlations are modelled for each pair of international equity markets individually and then averaged cross-sectionally.
### Table 8: Average Unconditional-Correlation between US and World Markets 1980-2011 Calculated using Sub-Sampled Data.

Notes: The average correlation between the S&P 500 and world markets is calculated at different time horizons. The original data is subsampled and associated correlations found at each horizon. 95% confidence intervals are shown in square brackets below each average correlation value.
### Table 9: Average Unconditional Correlation between International Markets (1980-2011) using Wavelet Transformed Data for Monthly Base Data.

<table>
<thead>
<tr>
<th>Horizon (Months)</th>
<th>Average U.S. Correlation</th>
<th>Average World Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.501</td>
<td>0.485</td>
</tr>
<tr>
<td></td>
<td>[0.423, 0.571]</td>
<td>[0.407, 0.556]</td>
</tr>
<tr>
<td>6</td>
<td>0.581</td>
<td>0.527</td>
</tr>
<tr>
<td></td>
<td>[0.513, 0.643]</td>
<td>[0.452, 0.594]</td>
</tr>
<tr>
<td>12</td>
<td>0.587</td>
<td>0.544</td>
</tr>
<tr>
<td></td>
<td>[0.518, 0.649]</td>
<td>[0.471, 0.610]</td>
</tr>
<tr>
<td>24</td>
<td>0.629</td>
<td>0.599</td>
</tr>
<tr>
<td></td>
<td>[0.564, 0.687]</td>
<td>[0.531, 0.659]</td>
</tr>
<tr>
<td>Long Run</td>
<td>0.593</td>
<td>0.574</td>
</tr>
<tr>
<td></td>
<td>[0.527, 0.653]</td>
<td>[0.505, 0.636]</td>
</tr>
<tr>
<td>Original Daily</td>
<td>0.238</td>
<td>0.396</td>
</tr>
<tr>
<td></td>
<td>[0.218, 0.258]</td>
<td>[0.378, 0.413]</td>
</tr>
</tbody>
</table>

Notes: The average unconditional correlation between a range of world equity markets is calculated at different time horizons using data from 1980-2011. Monthly sub-sampled data is used as the base-data set upon which the wavelet transform is applied. The Haar wavelet transform is used to decompose returns data into the range of horizons detailed, and correlations are then calculated at each horizon. Correlations found using the original untransformed data are also shown. 95% confidence intervals are shown beneath each correlation.
Table 10: Average Unconditional-Correlation between World Markets 1980-2011 using NBER Recession Data.
Notes: Unconditional correlations between 21 world equity markets are calculated at differing horizons for cohorts corresponding to NBER economic recession data. Approximate recession and expansion timings are given in each case. The Haar Wavelet transform is used to decompose returns data into the range of horizons detailed, and correlations are then calculated at each horizon. Correlations found using the original untransformed data are also shown. 95% confidence intervals are shown beneath each correlation.