

Tests of the risk premium on foreign currency futures implied by the intertemporal asset pricing theory

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In earlier literature on futures exchange rates, some evidence against the martingale hypothesis is not accompanied by an explicit alternative hypothesis derived from the asset pricing theory. This paper utilizes the asset pricing theory to show that the rate of growth of futures price is a function of the rate of growth of the per capita consumption, the logarithm of the gross risk-free rate of return and the time varying risk premium. The form of risk premium implied by the asset pricing theory is examined and its statistical significance is tested with the British Pound and German Deutschemark futures. Consequently, the specification of the test equation implied by the asset pricing theory is tested against *ad hoc* test equations of the martingale hypothesis. For both currency futures, there is strong evidence against the *ad hoc* test equations of the martingale hypothesis.

I. INTRODUCTION

The testing of the existence of a risk premium in futures asset prices has been a major focus of attention in empirical financial economics. Samuelson (1965) originally argued that futures prices in an efficient market would be a martingale. A martingale hypothesis postulates

$$f_t = E_t(f_{t+1}) \quad (1)$$

where f_t is today's futures price, f_{t+1} is tomorrow's futures price and E_t is the expectation conditioned on the information available at time t , I_t . The martingale hypothesis states that today's futures price is the best guess for predicting tomorrow's futures price, given the information available at time t .

On the other hand, the asset pricing paradigm predicts that the martingale hypothesis does not hold due to the existence of a risk premium which separates today's price from its expected price tomorrow:

$$f_t = E_t(f_{t+1}) + R(z_{t+1}|I_t) \quad (2)$$

where R is a risk premium function and z_{t+1} is a vector of relevant economic and financial variables at $t+1$. Under rational expectations, $f_{t+1} = E_t(f_{t+1}) + \varepsilon_{t+1}$ and ε_{t+1} is the

forecast error which is orthogonal to information set at t . Hence Equation 2 yields

$$f_{t+1} - f_t = -R(z_{t+1}|I_t) + \varepsilon_{t+1} \quad (3)$$

Therefore the two hypotheses are:

$$H_0: R(z_{t+1}|I_t) = 0$$

$$H_1: R(z_{t+1}|I_t) \neq 0.$$

Evidence against the martingale hypothesis with currency futures has developed along two different paths. The first path is followed by McCurdy and Morgan (1987, 1988), Hodrick and Srivastava (1987) and Taylor (1986). McCurdy and Morgan (1987) investigated the martingale hypothesis for futures prices for five currencies with daily and weekly data. Although they found evidence against the martingale hypothesis for all currencies with the daily data, the martingale hypothesis was rejected for only Deutschemark with weekly data. McCurdy and Morgan (1988) examined Deutschemark futures price data in a different time period. They rejected the martingale hypothesis with daily data, but retained with weekly data. Hodrick and Srivastava (1987) rejected the martingale hypothesis with daily exchange rate futures but reported that the rejection is much weaker with

monthly data. The shortcoming of the testing procedure¹ employed in these papers is that the asset pricing theory (e.g. Equation 2) is not utilized to derive a test equation for the alternative hypothesis. One of the main reasons is that some of the arguments of function $R(z_{t+1} | I_t)$ are unobservable. The shortcoming of this approach is that the testing does not indicate the potential sources of rejection as it does not utilize a specific equilibrium model as an alternative hypothesis. Without such a link it is not possible to empirically distinguish between an inefficient market and a time-varying risk premium.

The second path is to construct such a link by deriving a test equation from an equilibrium model.² Extensions of intertemporal asset pricing models to foreign currencies are done by Hodrick (1981), Stulz (1981), Lucas (1982) and Hodrick and Srivastava (1984). Domowitz and Hakkio (1985) utilize the model of Lucas (1982) which is extended by Hodrick and Srivastava (1984) to derive a test equation with time-varying risk premium that is a function of the conditional variance of domestic and foreign currency. Their results indicate some evidence of a non-zero risk premium for monthly currencies. In two recent papers McCurdy and Morgan (1992a, 1992b) utilize the rational expectations general equilibrium model of Richard and Sundaresan (1981) to derive test equations for foreign exchange futures. With nonexpected utility explanations of prices and under the assumption that the first differences of the logs of consumption and wealth are jointly normally distributed, McCurdy and Morgan (1992a) express the expected logarithm of real total return of an asset as a linear function of the covariances with the logarithm of real total return of wealth and the change in the logarithm of consumption. By constructing a benchmark portfolio for the consumption component and using a world index for the wealth component, they design a test equation which is linear in mean and estimate it by autoregressive conditional heteroscedasticity³ (ARCH) of Engle (1982). They find no evidence of risk premia when the price of covariance risk is constant, but when the price of the covariance risk is replaced by the conditional expectation of excess returns from the benchmark portfolios and their conditional variances, they obtain evidence of risk premia. McCurdy and Morgan (1992b) measure the covariance risk relative to excess returns from a broadly diversified international portfolio of equities

and find strong evidence of risk premia with five weekly currencies.

Hodrick and Srivastava (1987) extended the two-country intertemporal asset pricing model of Lucas (1982) to foreign currency futures. Their development, however, does not discuss how money is formally introduced into the model in order that it will not be an asset whose rate of return is dominated. In the present work, a Svensson's (1985) style cash-in-advance economy with currency futures markets is constructed which is an extension of Richard and Sundaresan's (1981) intertemporal asset pricing model with money introduced as a cash-in-advance constraint. It is shown that the rate of growth in a futures price is a function of the rate of growth of the per capita consumption and a time varying risk premium. The form of the time varying risk premium is not parametrically constrained by the theory. The time varying risk premium function is estimated by non-parametric kernel estimation as suggested by Pagan and Ullah (1988). Non-parametric estimation techniques have been successful in applications to models with risk terms. Pagan and Hong (1989) estimated an equity premium reflected in the variance of stock returns by non-parametric kernel and flexible fourier techniques and found these techniques preferable to parametric ones. Pagan and Schwert (1990) applied these techniques to alternative models of conditional stock volatility. Diebold and Nason (1990) estimated the conditional mean functions of ten major nominal dollar spot rates by nearest-neighbour technique, but did not find any out-of-sample improvement over a simple random walk model. McCurdy and Stengos (1992) obtain similar results by showing that the parametric estimator of the risk premia has more out-of-sample forecastability⁴ than does the kernel estimator. Pagan and Ullah (1988) provide an integrated approach to the estimation of models with risk terms.

Section II is on the theoretical model leading to the test equation implied by the asset pricing theory. Section III details the modelling strategy and the empirical findings. Conclusion follows thereafter.

II. THE MODEL

A representative firm in a closed economy produces a stochastic nonstorable output $\{y_t\}$. The random supply of

¹This testing procedure mainly involves construction of an *ad hoc* alternative hypothesis from the null model. The test results indicate whether the martingale hypothesis holds or does not hold for this particular alternative hypothesis. However, if an *ad hoc* test equation leads to the rejection of the null hypothesis, the researcher cannot assess what this means for the risk premium implied by an asset pricing theory. Consequently, the rejection of the null hypothesis with an *ad hoc* test equation does not necessarily imply that the risk premium implied by an asset pricing theory is statistically significant or vice versa.

²The first application of a single period capital asset pricing model (CAPM) to commodity futures was by Dusak (1973), who found no evidence of risk premium. Black (1976) derived testable alternative specifications of futures commodity contracts with single period CAPM.

³An excellent survey of the application of the ARCH models to finance is presented in Bollerslev, Chou and Kroner (1992).

⁴This evidence supports the findings of Hsieh (1989) that the temporal dependence in exchange rates is not in the conditional mean but in the conditional variance.

money in period t is given by

$$m_{t+1}^s = w_t m_t^s \quad (4)$$

where w_t is the stochastic gross rate of monetary expansion. $s_t = (y_t, w_t)'$ is the state at time t which follows a Markov process with the probability distribution of s_{t+1} given by the distribution function $F(s_{t+1}, s_t)$, i.e. the distribution of s_{t+1} depends only on the realization of s_t .

The economy has a representative consumer with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (5)$$

where $\beta \in (0, 1)$ is a discount factor, $\{c_t\}$ is a stochastic stream of consumption of the single aggregate good and u is a bounded, twice-differentiable utility function with $u'(c) > 0$, $u''(c) < 0$ and $u''(0) = \infty$. E_t is the conditional expectations operator on information at period t . The consumer maximizes Equation 5 subject to two constraints. The first constraint is the liquidity constraint. The consumer enters period t with predetermined holdings m_t of money, learns the current state s_t , and has the opportunity to purchase goods. Consumption decisions take place in the goods market and the consumption good can only be bought with money so that

$$c_t \leq \pi_t m_t \quad (6)$$

where m_t are money balances carried from period $t - 1$, and $\pi_t = (1/p_t)$ is the purchasing power of money or the price of one dollar in terms of the consumption good. After the goods market is cleared, the consumer enters asset markets.

Let b_{t-1} be the number of one-period discount bonds held by the consumer between dates $t - 1$ and t , and r_{t-1} be the gross nominal return on such bonds which is known to the agent at date $t - 1$. In asset markets, the consumer allocates his balances among these bonds, buying and selling money in futures markets,⁵ and choosing money balances to carry to the next period. He receives a money transfer equal to $(w_t - 1)m_t^s$, the gross nominal return from his holdings of discount bonds $b_{t-1}r_{t-1}$ and the output of the representative firm y_t which the consumer is assumed to own.

Let n_{t-1} denote the number of futures contracts initiated at time $t - 1$ which the agent at time $t - 1$ considers a choice variable. At $t - 1$, the agent initiates one-period futures contracts which total $n_{t-1}r_{t-1}$ dollars. Each futures contract is initiated for the delivery of one unit of money, one period from the initiation date. When an initiation takes place at period $t - 1$, r_{t-1} is known with certainty but n_{t-1} is a choice variable to be solved. At the initiation time

the cost of $n_{t-1}r_{t-1}$, the number of futures contracts, is zero. When the agent initiates $n_{t-1}r_{t-1}$ dollars of futures contracts he also buys $n_{t-1}f_{t-1}$ dollar amount of one period discount bonds. f_{t-1} is the time $t - 1$ price of a futures contract for delivery at time t , and is in units of money. Therefore, the consumer's cash flow due to the involvement in the futures market is given as

$$\begin{array}{ll} \text{Cash Flow at } t-1 & \text{Cash Flow at } t \\ -n_{t-1}f_{t-1} & n_{t-1}f_{t-1}r_{t-1} + n_{t-1}r_{t-1}(f_t - f_{t-1}) \\ & = n_{t-1}r_{t-1}f_t \end{array}$$

where $n_{t-1}r_{t-1}(f_t - f_{t-1})$ is the futures payoff received or paid at time t and $f_{t-1}n_{t-1}r_{t-1}$ is the gross nominal return on discount bond holdings received at time t . The no-arbitrage condition is that the present value of cash flow at t is equal to the cash flow at $t - 1$, $f_{t-1} = PV(r_{t-1}f_t)$ where PV is the present value operator. This no-arbitrage condition is Proposition 2 of Cox, Ingersoll and Ross (1981). To derive a closed form solution for the present value operator we incorporate this condition into the budget constraint of the consumer. It is assumed that futures payoff $n_{t-1}r_{t-1}(f_t - f_{t-1})$ and the gross nominal return on bond holdings $f_{t-1}n_{t-1}r_{t-1}$ are received after the goods market is closed. Hence, the timing of transactions in goods and asset markets is as in Svensson (1985).

The payoffs received from the holdings of futures contracts and the holdings of discount bonds, together with the leftover money balances from the purchase of the consumption good, $(\pi_t m_t - c_t)$, are used to buy bonds, initiate futures contracts and determine next periods cash holdings. The consumer's budget constraint can then be written as

$$\begin{aligned} \pi_t m_{t+1} + \pi_t f_t n_t + \pi_t b_t \\ \leq (\pi_t m_t - c_t) + (w_t - 1)\pi_t m_t^s + \pi_t b_{t-1} r_{t-1} \\ + \pi_t r_{t-1} n_{t-1} (f_t - f_{t-1}) + \pi_t f_{t-1} n_{t-1} r_{t-1} + y_t \end{aligned} \quad (7)$$

By rearrangement we obtain

$$\begin{aligned} \pi_t m_{t+1} + \pi_t f_t n_t + \pi_t b_t \\ \leq (\pi_t m_t - c_t) + (w_t - 1)\pi_t m_t^s + \pi_t b_{t-1} r_{t-1} \\ + \pi_t r_{t-1} n_{t-1} f_t + y_t \end{aligned} \quad (8)$$

where m_{t+1} is the money holdings to be carried into period $t + 1$. Each period, the market clearing conditions for goods and asset markets are

$$c_t = y_t, \quad m_t = m_t^s, \quad m_{t+1} = m_{t+1}^s = w_t m_t^s, \quad r_t = 1, \quad n_t = 0.$$

The representative agent maximizes Equation 5 subject to Equation 6 and Equation 8. I choose to solve the problem

⁵The discussion here covers all financial futures with nominal payoffs such as treasury bills.

by forming a Lagrangean,⁶

$$J = E_0 \sum_{i=0}^{\infty} \beta^i \{u(c_i) + \lambda_i [\pi_i m_i - c_i] + (w_i - 1)\pi_i m_i^s + \pi_i b_{i-1} r_{i-1} + \pi_i r_{i-1} n_{i-1} f_i + y_i - \pi_i m_{i+1} - \pi_i f_i n_i - \pi_i b_i\} + \mu_i [\pi_i m_i - c_i] \quad (9)$$

The first-order necessary conditions are:

$$c_i: u'(c_i) = \lambda_i + \mu_i \quad (10)$$

$$b_i: \pi_i \lambda_i = \beta E_i \pi_{i+1} \lambda_{i+1} r_i \quad (11)$$

$$m_{i+1}: \pi_i \lambda_i = \beta E_i \pi_{i+1} (\lambda_{i+1} + \mu_{i+1}) \quad (12)$$

$$n_i: \pi_i \lambda_i f_i = \beta E_i \pi_{i+1} \lambda_{i+1} f_{i+1} r_i \quad (13)$$

$$c_i \leq \pi_i m_i \quad \mu_i \geq 0 \quad \mu_i (\pi_i m_i - c_i) = 0 \quad (14)$$

Equation 10 states that the marginal utility of real wealth and liquidity services of real balances are equal to the marginal utility of consumption. The equilibrium money price of a one-period discount bond is obtained from Equation 11 by rearrangement

$$r_i^{-1} = \beta E_i \left(\frac{\lambda_{i+1} \pi_{i+1}}{\lambda_i \pi_i} \right) \quad (15)$$

Equation 12 denotes that when μ is binding, the marginal utility of nominal wealth has to be equal to the expected discounted nominal value of next period's consumption. Equation 13 states that the foregone marginal utility of nominal wealth from the purchase of f_i dollars is equal to the expected marginal discounted benefit of nominal wealth of payoff $f_{i+1} r_i$. The futures pricing function obtained from⁷ Equation 13 is

$$f_i = \beta E_i \left(\frac{\lambda_{i+1} \pi_{i+1}}{\lambda_i \pi_i} f_{i+1} r_i \right) \quad (16)$$

Equation 14 denotes the complementary slackness condition. I will assume that liquidity constraint binds so that μ is always strictly positive. Under this assumption the present value operator in Equation 15 is not simply the ratio of marginal utilities of consumption in successive periods.

Define

$$k_{i+1} \equiv \beta \left(\frac{\lambda_{i+1} \pi_{i+1}}{\lambda_i \pi_i} \right) r_i, \quad l_{i+1} \equiv \frac{f_{i+1}}{f_i}$$

⁶Svensson (1985) gives sufficient conditions for the existence of a unique stochastic stationary rational expectations solution with finite value for the objective function.

⁷Equations 15 and 16 together lead to the Theorem 1 of Richard and Sundaresan (1981) which they derived in a rational expectations general equilibrium model in continuous time of a multigood, identical consumer economy with constant stochastic returns to scale production.

⁸From Equation 17 $k_{i+1} l_{i+1} = 1 + w_{i+1}$ where w_{i+1} is the forecast error and is orthogonal to the information set at time t . Then

$$e_{i+1} = \frac{1}{1 + w_{i+1}}$$

so that Equation 16 can be written as

$$1 = E_i k_{i+1} l_{i+1} \quad (17)$$

We can now state and prove our two main propositions.

Proposition 1

Assume that $k_{i+1} l_{i+1} \neq 0$ with probability one. Let

$$e_{i+1} = \frac{E_i k_{i+1} l_{i+1}}{k_{i+1} l_{i+1}}$$

be a stochastic process⁸ and l_i be the smallest σ -algebra such that $k_i, l_i, e_i, k_{i-1}, l_{i-1}, e_{i-1}, \dots$ are all measurable. Then for all integrable functions g ,

$$g(E_i k_{i+1} l_{i+1}) = E_i g(k_{i+1} l_{i+1} e_{i+1})$$

Proof:

$$E_i g(k_{i+1} l_{i+1} e_{i+1}) = E_i g(E_i k_{i+1} l_{i+1}) = g(E_i k_{i+1} l_{i+1})$$

Proposition 2

A second-order Taylor expansion of $\log(E_i k_{i+1} l_{i+1})$ around the unconditional mean of e_{i+1} , $\bar{e} = E(e_{i+1})$, is

$$\log(f_{i+1}) - \log(f_i) = \gamma_0 + \gamma_1 [\log(\lambda_{i+1} \pi_{i+1}) - \log(\lambda_i \pi_i)] + \gamma_2 \log(r_i) + \gamma_3 \sigma_i^2 + v_{i+1} \quad (18)$$

where $\gamma_0 = \frac{(\bar{e}-1)}{\bar{e}} + \frac{1}{2}(\frac{1}{\bar{e}} - 1)^2 - \log(\beta) - \log(\bar{e})$, $\gamma_1 = \gamma_2 = -1$, $\gamma_3 = \frac{1}{2}$ and $v_{i+1} \sim (0, \sigma_{v,i}^2)$.

Proof: See appendix,

Two remarks can be made from Proposition 2. First, an increase in the rate of growth of the marginal utility of real wealth and nominal gross rate of return lead to an equal decrease in the rate of growth of futures prices. Second, the higher the conditional variance of the innovations, the higher will be the increase in the rate of growth of futures prices.

III. MODELLING STRATEGY AND EMPIRICAL FINDINGS

By specifying $u(c) = \log(c)$ and assuming that the marginal utility of liquidity services for real balances, $\mu_t = \mu$ is

constant⁹ for all t , Equation 18 is written as

$$\log(f_{t+1}) - \log(f_t) = \gamma_0 + \gamma_1[\log(p_{t+1}c_{t+1}) - \log(p_t c_t)] + \gamma_2 \log(r_t) + \gamma_3 \sigma_t^2 + v_{t+1} \quad (19)$$

where $\gamma_1 = 1$; $\gamma_2 = -1$; $\gamma_3 = \frac{1}{2}$; $(p_t c_t)$ is nominal consumption expenditures at t and σ_t^2 is the time-varying risk premium. The risk premium function is represented by $E_t \phi_t^2 = \sigma_t^2$ and $\phi_t^2 = (\psi_t - E_t \psi_t)^2$ where $\psi_t = \log(f_{t+1}) - \log(f_t)$.

Notice that the change in the purchasing power of money has a direct effect on futures prices via nominal consumption expenditures. Suppose that per capita real consumption in two successive periods is the same, but the purchasing power of the dollar is increased, i.e. $\pi_{t+1} > \pi_t$. This means that the agent will spend less dollars tomorrow to buy today's consumption level. From Equation 19, this will lead to a decrease in the rate of growth of futures prices.

The estimation problem is that no direct observations on σ_t^2 exist, and the theory does not describe how σ_t^2 varies with the information at t . If it were possible to construct a series ϕ_t^2 which had the strong property,¹⁰ then σ_t^2 could be replaced by ϕ_t^2 and Equation 19 could be estimated by ordinary least squares (OLS). However, if the constructed series ϕ_t^2 has the weak property,¹¹ then OLS will be inconsistent due to the regressors σ_t^2 being observed with error. Pagan and Ullah (1988) propose a solution to this problem by estimating Equation 19 not by OLS, but by instrumental variables (IV) with instruments constructed from I_t . Of course instrument construction has to be done carefully, since the instruments must be as highly correlated with ϕ_t^2 as possible. An important aspect of the IV strategy is that it only requires $m_t = E_t \psi_t$ to be estimated consistently and some estimate of σ_t^2 .

To construct our proxy series ϕ_t^2 , we need to know how σ_t^2 varies with the information available at t , and what the mapping is between this information and σ_t^2 . Pagan and Ullah (1988) show that the consistent estimation of the parameter of the model require that I_t be known, so the choice of the information set has to be wide enough to encompass I_t . In this paper, we assume that econometrician's information set incorporates I_t .

m_t will normally be a nonlinear function of I_t . A general dependence of m_t on I_t can be carried out by non-parametric estimation methods such as the non-parametric kernel regression. With such an estimate of \hat{m}_t , $\hat{\phi}_t^2$ can be constructed by $\hat{\phi}_t^2 = (\psi_t - \hat{m}_t)^2$.

Thus the model which has to be estimated by IV is

$$\log(f_{t+1}) - \log(f_t) = \gamma_0 + \gamma_1[\log(p_{t+1}c_{t+1}) - \log(p_t c_t)] + \gamma_2 \log(r_t) + \gamma_3 \hat{\phi}_t^2 + \gamma_3(\sigma_t^2 - \phi_t^2) + \gamma_3(\phi_t^2 - \hat{\phi}_t^2) + v_{t+1} \quad (20)$$

with constant, $\log(p_{t+1}c_{t+1}) - \log(p_t c_t)$, $\log(r_t)$ and $\hat{\phi}_t^2$ as instruments.

Non-parametric kernel estimation

Consider an economic model

$$y = M(x) + u \quad (21)$$

where $M(x)$ is the conditional mean of a random variable y and x is a $q \times 1$ vector of conditioning variables. In parametric estimation $M(x)$ is known, but in non-parametric regression it is an unspecified functional form. The usual kernel estimator $\hat{M}(x) = E(y/x)$ is written as

$$\hat{M}(x) = \frac{\hat{s}(x)}{\hat{f}(x)} \quad (22)$$

where

$$\hat{s}(x) = \frac{1}{n|\Omega|} \sum_{j=1}^n y_j K_j; \quad \hat{f}(x) = \frac{1}{n|\Omega|} \sum_{j=1}^n K_j; \quad (23)$$

$K_j = K(\Omega^{-1}(x - x_j))$; $\Omega = \text{diag}(a_1, a_2, \dots, a_q)$; $a_i, i = 1, 2, \dots, q$ are the smoothing parameters or window widths; n is the sample size and j 's indicate individual observations. Since it is possible to choose the function K so that it is continuous, the resulting kernel estimator of the density function will also be continuous. In the present paper, the kernel is chosen to be the standard multivariate normal density function. Robinson (1983) derives the asymptotic distribution of the non-parametric kernel estimator of the joint density and of the conditional mean within the time series context.

An important consideration in the literature is the choice of the bandwidth parameter a . Too large a value of a induces bias and too small a value induces imprecise estimates. Robinson (1983) and Ullah (1988), among others, summarize the conditions that the kernel function and the bandwidth parameter a will have to satisfy to obtain the asymptotic properties of the regression function estimator. Ullah (1988) suggests setting the window width a_i to

$$a_i = s_i n^{-1/(4+q)} \quad (24)$$

⁹In Equation 10 if $\mu_t = 0$ then $\lambda_t = u'(c_t)$ which implies that the marginal utility of wealth is equal to the marginal utility of consumption. Since the utility function is arbitrary up to any monotonic transformation, marginal utility of wealth is also arbitrary relative to this transformation. Suppose that $u(c) = \log(c)$ and a monotonic transformation of u is $v(u) = u + \mu e^u$ so that $v'(u) > 0$. Then $v'(c) = u'(c) + \mu$. By replacing $u'(c)$ by $v'(c)$ in $\lambda_t = u'(c_t)$ we get $\lambda_t = v'(c_t) + \mu$. Therefore, we will assume that $\mu_t = \mu$ and use $\lambda_t = u'(c_t)$ instead of Equation 10.

¹⁰A series is said to possess the strong property if $\phi_t^2(N) \rightarrow \sigma_t^2$ for all t as $N \rightarrow \infty$ where N is not dependent on t .

¹¹A series is said to possess the weak property if $E_t \phi_t^2 = \sigma_t^2$.

where $i = 1, 2, \dots, q$ and s_i denotes the standard deviation of x_i . The asymptotic variance of a kernel regression estimate $\hat{M}(x)$ is given by

$$V(x) = \frac{1}{n|\Omega|} \frac{\text{Var}(y/x)}{f(x)} \int K^2 \quad (25)$$

from which it is straightforward to construct standard errors.

Data description

The daily British Pound and German Deutschemark futures exchange rate series are obtained from the Center of Research for Futures Markets of the University of Chicago covering the period July 1972 to December 1986 and Reuters for January 1987 to December 1988. The monthly series are then constructed by choosing the mid-month futures price corresponding to the shortest maturity contracts which are delivered on the third Wednesday of March, June, September and December. The mid-month refers to the 15th day of every month and 14th day of February. If the 15th day is on a weekend, the previous Friday's price is chosen. The mid-month futures are chosen to match the futures prices with the monthly average consumption. The monthly US nominal consumption expenditure series for nondurables, and the monthly US population series are compiled from the US Department of Commerce, Survey of Current Business. To obtain the per capita consumption, consumption series are normalized by the population series. The nominal gross risk-free rate of return series is constructed from the interest rate on US30-Day Commercial Paper and obtained from CANSIM matrix 2545.

Empirical findings

Choosing three lags of $\psi_t = \log(f_{t+1}) - \log(f_t)$ as the conditioning elements, the conditional mean m_t is estimated by non-parametric kernel by regressing ψ_t on ψ_{t-1} , ψ_{t-2} and ψ_{t-3} . Table 1 gives the first ten autocorrelation coefficients of the squared non-parametric residuals along with their t statistics. There is little evidence of ARCH effects observed which makes the further lags of ψ_t in the conditioning set unnecessary. The proxy $\hat{\phi}_t^2$ for σ_t^2 is generated by $\hat{\phi}_t^2 = (\psi_t - \hat{m}_t)^2$ where \hat{m}_t is the non-parametric conditional mean estimate. An estimate of σ_t^2 , $\hat{\sigma}_t^2$ is needed as an instrument for σ_t^2 . $\hat{\sigma}_t^2$ is calculated non-parametrically by calculating $\hat{\sigma}_t^2 = E_t \hat{\phi}_t^2$. IV estimation indicates the presence of a first

Table 1. Autocorrelation function of squared non-parametric residuals

Lag	British Pound		German Deutschemark	
	a.c.f.	t	a.c.f.	t
1	-0.0357	0.4703	-0.0903	1.3203
2	-0.0657	0.8655	-0.0489	0.6554
3	0.0345	0.4553	-0.0430	0.5828
4	0.0291	0.3845	-0.0029	0.0405
5	0.0629	0.8326	0.0761	1.0431
6	0.0369	0.4850	0.1388	1.8887
7	0.0931	1.2186	0.1286	1.7012
8	-0.0416	0.5426	0.0542	0.7432
9	-0.0336	0.4399	0.0567	0.7813
10	-0.0779	1.0120	0.0524	0.7236

The estimates of the autocorrelation coefficients (a.c.f.) are obtained by regressing $\hat{\phi}_t^2$ against a constant and $\hat{\phi}_{t-j}^2$ ($j = 1, 2, \dots, 10$) one at a time. $|t|$ is the absolute value of a t statistic.

order autocorrelation in British Pound futures. Accordingly, the lagged rate of growth of British Pound is included as a regressor in Equation 20.¹² Diagnostic tests applied after IV estimation give test statistic values of 0.331 and 0.572 for first order autocorrelation¹³ and 2.456 and 7.976 for heteroscedasticity.¹⁴ Thus there is some evidence of heteroscedasticity in the Deutschemark residuals. The results for the IV procedure with Equation 20 are presented in Table 2 with a White (1980) adjustment to the calculated t -statistics for the Deutschemark futures.

For both currencies it is clear that the martingale hypothesis is rejected. The risk premium for the British Pound futures and the intercept and the risk-free rate for the Deutschemark futures are statistically significant at the 5% level. This raises the issue about which model specification characterizes the data better, an *ad hoc* test equation or the test equation derived from the asset pricing model. This can easily be demonstrated by a non-nested testing methodology such as the J test of Davidson and MacKinnon (1981). Two different specifications for the *ad hoc* test equation are chosen. The first specification is

$$\log(f_{t+1}) - \log(f_t) = \alpha_0 + \alpha_2 [\log(f_{t-1}) - \log(f_{t-2})] + \varepsilon_t \quad (26)$$

The parameter estimates of this model are presented in Table 3. Equation 26 is very similar to the main test equation of McCurdy and Morgan (1988), which they estimate

¹²For British Pound the estimated model is

$$\log(f_{t+1}) - \log(f_t) = \gamma_0 + \gamma_1 [\log(p_{t+1}c_{t+1}) - \log(p_t c_t)] + \gamma_2 [\log(f_t) - \log(f_{t-1})] + \gamma_3 \log(r_t) + \gamma_4 \hat{\phi}_t^2 + \gamma_4 (\sigma_t^2 - \hat{\phi}_t^2) + \gamma_4 (\phi_t^2 - \hat{\phi}_t^2) + v_{t+1}$$

with constant, $\log(p_{t+1}c_{t+1}) - \log(p_t c_t)$, $\log(p_t c_t) - \log(p_{t-1}c_{t-1})$, $\log(r_t)$, and $\hat{\sigma}_t^2$ as instruments.

¹³The calculated test statistics are the Breusch (1978) and Godfrey (1978) procedure and should be referred to as $\chi^2(1)$.

¹⁴This is the Koenker (1981) procedure and should be referred to as $\chi^2(3)$.

Table 2. Instrumental variables estimation of

$$\log(f_{t+1}) - \log(f_t) = \gamma_0 + \gamma_1[\log(p_{t+1}c_{t+1}) - \log(p_t c_t)] + \gamma_2[\log(f_t) - \log(f_{t-1})] + \gamma_3 \log(r_t) + \gamma_4 \hat{\phi}_t^2 + \gamma_4(\sigma_t^2 - \phi_t^2) + \gamma_4(\phi_t^2 - \hat{\phi}_t^2) + v_{t+1}$$

	British Pound	German Deutschemark
$\hat{\gamma}_0$	0.0035 (0.4455)	0.0265 (3.3259) (3.1636)
$\hat{\gamma}_1$	-0.2709 (0.8642)	0.0047 (0.0130) (0.0113)
$\hat{\gamma}_2$	0.2505 (1.0633)	
$\hat{\gamma}_3$	-0.1236 (1.5463)	-0.3529 (4.1467) (3.8359)
$\hat{\gamma}_4$	8.2097 (3.0223)	6.3044 (1.7803) (1.6802)
R^2	0.0843	0.0751

The numbers in the first parentheses are the absolute values of the *t*-statistics and the numbers in the second parentheses are heteroscedastic consistent versions. For the British Pound, the constant, $\log(p_{t+1}c_{t+1}) - \log(p_t c_t)$, $\log(p_t c_t) - \log(p_{t-1}c_{t-1})$, $\log(r_t)$, and $\hat{\phi}_t^2$ are used as instruments. For the Deutschemark, the constant, $\log(p_{t+1}c_{t+1}) - \log(p_t c_t)$, $\log(r_t)$ and $\hat{\phi}_t^2$ are used as instruments.

Table 3. Ordinary least squares estimation of

$$\log(f_{t+1}) - \log(f_t) = \alpha_0 + \alpha_1(\log(f_t) - \log(f_{t-1})) + \alpha_2(\log(f_{t-1}) - \log(f_{t-2})) + \varepsilon_t$$

	British Pound		German Deutschemark	
	Model 1	Model 2	Model 1	Model 2
$\hat{\alpha}_0$	-0.0015 (0.6592)	-0.0013 (0.5728)	0.0031 (1.2406)	0.0029 (1.1618)
$\hat{\alpha}_1$		0.1527 (2.1137)		0.0623 (0.8612)
$\hat{\alpha}_2$	-0.0156 (0.2151)	-0.0378 (0.5226)	-0.0119 (0.1649)	-0.0159 (0.2189)
R^2	0.0002	0.0231	0.0001	0.0040

The absolute values of *t*-statistics are in parentheses.

by ARCH (see Engle, 1982; Bollerslev, 1986; and Engle and Bollerslev, 1986). The presence of an ARCH(1) and ARCH(2) process in the residuals of Equation 26 is tested by using monthly data. The test statistics obtained were 0.032 and 0.239 for the British Pound and 0.116 and 0.118 for the Deutschemark which are statistically insignificant.¹⁵ Assuming that Equation 26 is specified under H_0 , the artificial compound model is written by

$$\log(f_{t+1}) - \log(f_t) = \alpha_0 + \alpha_2[\log(f_{t-1}) - \log(f_{t-2})] + \alpha_3 \Psi_t + \varepsilon_t \quad (27)$$

where variable Ψ_t in Equation 27 refers to the fit obtained from the IV estimation¹⁶ of Equation 20. The logic underlying the *J* test is that, because the hypotheses are non-nested, the truth of H_0 implies that H_1 is false. If H_0 is the true model then the true value of α_3 in Equation 27 is zero. Thus, H_0 is accepted or rejected based on the significance of $\hat{\alpha}_3$: if $\hat{\alpha}_3$ is insignificant, H_0 is accepted, while if $\hat{\alpha}_3$ is significant, H_0 is rejected. Davidson and MacKinnon (1981) show that a standard *t* test can be used to determine if $\hat{\alpha}_3$ is significantly different from zero. A *t* test on the coefficient of Ψ_t is presented¹⁷ in the first row of Table 4(a). The results indicate that for both currencies the *ad hoc* model is rejected in favour of the asset pricing model. The test of the asset pricing model against the *ad hoc* model leads to retaining the asset pricing model. Therefore, the asset pricing model comes out as a better specification from the non-nested testing.

The other specification as the *ad hoc* test equation¹⁸ is

$$\log(f_{t+1}) - \log(f_t) = \alpha_0 + \alpha_1[\log(f_t) - \log(f_{t-1})] + \alpha_2[\log(f_{t-1}) - \log(f_{t-2})] + \varepsilon_t \quad (28)$$

The calculated *J* test results are presented in the second row of Table 4(a). The evidence reveals that the test equation derived from the intertemporal asset pricing theory is a better specification of the German Deutschemark and British Pound futures.

The data period covers approximately 16 years. The length of the sample horizon may raise the issue of the sensitivity of the results over shorter periods. This is done by splitting the data on three five year sub-periods. For each sub-period the *J* test is calculated and the results are presented in Table 4 (b), (c) and (d). The results here confirm the

¹⁵This is consistent with Diebold (1988) and Baillie and Bollerslev (1989) that ARCH effects tend to weaken with less frequently sampled data.

The test statistic for an ARCH(*q*) process was obtained as is nR^2 from the artificial regression $l_t^2 = \alpha_0 + \alpha_1 l_{t-1}^2 + \dots + \alpha_q l_{t-q}^2 + e_t$ where l_t^2 are the squared residuals from the AR fit. The test statistic is distributed as χ_q^2 .

¹⁶When Equation 20 is the null model then the modified version of the *J* test in MacKinnon, White and Davidson (1983) for the two-stage least squares regression is used.

¹⁷The *J* test is not the only non-nested test; others include the Cox (1961) test and the Fisher-McAleer (1981) test. In an evaluation of several non-nested tests, Godfrey and Pesaran (1983) find that in small samples the *J* test has a tendency to reject a true null model too often. This is particularly so when the true model fits the data so poorly and when the number of regressors in the false model exceeds that in the true model. The Fisher-McAleer test was also tried. The results are very comparable to the *J* test results.

¹⁸The parameter estimates of these equations are presented in Table 3.

Table 4. Nonnested comparison of the asset pricing model (AM) with an ad hoc model (HM)

	British Pound		German Deutschemark	
	(AM versus HM)	(HM versus AM)	(AM versus HM)	(HM versus AM)
(a) Sample period: July 1972–December 1988				
Model 1	0.3086 (0.2680)	5.9205 (3.7424)	0.0438 (0.0435)	4.3747 (2.7529)
Model 2	0.3082 (0.2675)	5.4533 (3.1341)	0.5756 (0.5562)	4.2941 (2.6602)
(b) Sample period: July 1972–June 1977				
Model 1	0.8562 (0.8213)	3.8452 (3.6231)	0.2145 (0.1819)	3.0631 (2.8345)
Model 2	0.8412 (0.8311)	3.5412 (3.4567)	0.3451 (0.3271)	3.0945 (3.0412)
(c) Sample period: July 1972–June 1982				
Model 1	0.3804 (0.3446)	3.2131 (3.1192)	0.6539 (0.5921)	3.5847 (3.3100)
Model 2	0.3847 (0.3404)	3.3256 (3.0123)	0.5834 (0.5380)	3.4178 (3.4178)
(d) Sample period: July 1982–June 1988				
Model 1	0.4512 (0.4449)	3.2657 (3.2145)	0.3214 (0.2903)	3.3154 (3.2234)
Model 2	0.4783 (0.4312)	3.3256 (3.1567)	0.5834 (0.5312)	3.4178 (3.2178)

AM and HM refer to the test equations derived from the asset pricing theory and the *ad hoc* testing methods, respectively. The numbers in parentheses are the heteroscedasticity consistent versions of the J tests. Models 1 and 2 are

1. $\log(f_{i+1}) - \log(f_i) = \alpha_0 + \alpha_2(\log(f_{i-1}) - \log(f_{i-2})) + \varepsilon_i$,
2. $\log(f_{i+1}) - \log(f_i) = \alpha_0 + \alpha_1(\log(f_i) - \log(f_{i-1})) + \alpha_2(\log(f_{i-1}) - \log(f_{i-2})) + \varepsilon_i$.

earlier results obtained from the full sample that the *ad hoc* test equations are rejected in favour of the test equation of the asset pricing theory.

IV. CONCLUSIONS

Some evidence in earlier literature with foreign currency futures rejected the martingale hypothesis without an explicit model of an alternative hypothesis derived from the asset pricing theory (e.g. Equation 2). One of the main reasons is that some of the arguments of function $R(z_{t+1} | I_t)$ are unobservable. The limitation of this approach is that the testing does not indicate the potential sources of rejection, as it does not utilize a specific equilibrium model as an alternative hypothesis. Without such a link, it is not possible to empirically distinguish between an inefficient market and a time-varying risk premium.

This paper has utilized the intertemporal asset pricing theory to show that the rate of growth in a futures price is a function of the rate of growth of the per capita consump-

tion, logarithm of the nominal gross risk-free rate of return and a time varying risk premium. The form of risk premium implied by the asset pricing theory is examined and its statistical significance is tested with British Pound and German Deutschemark futures. The evidence strongly rejects the *ad hoc* test equations of the martingale hypothesis in favour of the test equation derived from the asset pricing theory.

The results of this paper indicate that *ad hoc* test equations of the martingale hypothesis are misspecified. One source of the misspecification in *ad hoc* test equations is the omission of the consumption growth from the test equation. The omission of the consumption data is justified on the grounds that it is not available in higher frequencies and it is noisy. Recently, Campbell (1993) suggests a loglinear approximation to the budget constraint to substitute consumption out of a standard intertemporal asset pricing model. Campbell's results show that the innovation to log consumption equals the innovation to the log return on the market portfolio and the revisions in the discounted value of future market returns. It is possible to extend Campbell

(1993) results to the pricing of the futures exchange rates such that innovation in log consumption is expressed as a function of the innovations in the market return. Then, the test equation of the futures asset pricing theory would provide evidence to distinguish between an inefficient market and a time varying risk premium.

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APPENDIX

Proposition 2

A second-order Taylor expansion of $\log(E_t k_{t+1} l_{t+1})$ around the unconditional mean of e_{t+1} , $\bar{e} = (e_{t+1})$, is

$$\log(f_{t+1}) - \log(f_t) = \gamma_0 + \gamma_1 [\log(\lambda_{t+1} \pi_{t+1}) - \log(\lambda_t \pi_t)] + \gamma_2 \log(r_t) + \gamma_3 \sigma_t^2 + v_{t+1}$$

where $\gamma_0 = \frac{(\bar{e}-1)}{\bar{e}} + \frac{1}{2}(\frac{1}{\bar{e}} - 1)^2 - \log(\beta) - \log(\bar{e})$, $\gamma_1 = \gamma_2 = -1$, $\gamma_3 = \frac{1}{2}$ and $v_{t+1} \sim (0, \sigma_{v,t}^2)$.

Proof:

By applying Proposition 1 and choosing $g(x) = \log(x)$ Equation 17 is written as

$$0 = \log E_t k_{t+1} l_{t+1} = E_t \log(k_{t+1} l_{t+1} e_{t+1}).$$

Consider a linear expansion of $E_t \log(k_{t+1} l_{t+1} e_{t+1})$ around the unconditional mean of e_{t+1} , $\bar{e} = E(e_{t+1})$,

$$\begin{aligned} E_t \log(k_{t+1} l_{t+1} e_{t+1}) &= E_t \log(k_{t+1} l_{t+1} \bar{e}) + \frac{1}{\bar{e}} E_t (k_{t+1} l_{t+1} (e_{t+1} - \bar{e})) \\ &\quad - \frac{1}{2\bar{e}^2} E_t (k_{t+1} l_{t+1} (e_{t+1} - \bar{e})^2). \end{aligned} \tag{29}$$

From Proposition 1, $E_t k_{t+1} l_{t+1} e_{t+1} = E_t k_{t+1} l_{t+1}$ and from Equation 17, $E_t k_{t+1} l_{t+1} = 1$, hence

$$\frac{1}{\bar{e}} E_t k_{t+1} l_{t+1} (e_{t+1} - \bar{e}) = (1 - \bar{e})/\bar{e}, \tag{30}$$

and

$$E_t \log(k_{t+1} l_{t+1} \bar{e}) = E_t \log(k_{t+1} l_{t+1}) + \log(\bar{e}). \tag{31}$$

Also, notice that the third component of Equation 29 is written as,

$$\begin{aligned} & - \frac{1}{2\bar{e}^2} E_t (k_{t+1} l_{t+1} (e_{t+1} - \bar{e})^2) \\ &= - \frac{1}{2\bar{e}^2} \{E_t (1 - k_{t+1} l_{t+1} \bar{e})^2\} \end{aligned}$$

since $k_{t+1} l_{t+1} e_{t+1} = E_t k_{t+1} l_{t+1}$. By factoring out \bar{e} and adding and subtracting one, we obtain

$$= - \frac{1}{2} \left\{ E_t \left(1 - k_{t+1} l_{t+1} + \frac{1}{\bar{e}} - 1 \right)^2 \right\}.$$

By defining $w_{t+1} = k_{t+1} l_{t+1} - 1$, we get

$$\begin{aligned} &= - \frac{1}{2} \left\{ E_t \left(-w_{t+1} + \frac{1}{\bar{e}} - 1 \right)^2 \right\} \\ &= - \frac{1}{2} \left\{ E_t w_{t+1}^2 + \left(\frac{1}{\bar{e}} - 1 \right)^2 \right\} \end{aligned} \tag{32}$$

where $E_t w_{t+1} = 0$. By combining Equations 30, 31 and 32, we obtain

$$\begin{aligned} 0 &= E_t \log(k_{t+1} l_{t+1}) + \log(\bar{e}) + \frac{1 - \bar{e}}{\bar{e}} \\ &\quad - \frac{1}{2} \left\{ \sigma_t^2 + \left(\frac{1}{\bar{e}} - 1 \right)^2 \right\} \end{aligned} \tag{33}$$

where $w_{t+1} \sim (0, \sigma_t^2)$. $E_t \log(k_{t+1} l_{t+1})$ is written as

$$\begin{aligned} E_t \log(k_{t+1} l_{t+1}) &= \log(\beta) + \log(r_t) + \log(\lambda_{t+1} \pi_{t+1}) \\ &\quad - \log(\lambda_t \pi_t) + \log(f_{t+1}) \\ &\quad - \log(f_t) - v_{t+1}. \end{aligned} \tag{34}$$

By rearranging terms in Equations 33 and 34 we obtain

$$\begin{aligned} \log(f_{t+1}) - \log(f_t) &= \gamma_0 + \gamma_1 [\log(\lambda_{t+1} \pi_{t+1}) \\ &\quad - \log(\lambda_t \pi_t)] + \gamma_2 \log(r_t) + \gamma_3 \sigma_t^2 + v_{t+1} \end{aligned}$$

where $\gamma_0 = \frac{(\bar{e}-1)}{\bar{e}} + \frac{1}{2}(\frac{1}{\bar{e}} - 1)^2 - \log(\beta) - \log(\bar{e})$, $\gamma_1 = \gamma_2 = -1$, $\gamma_3 = \frac{1}{2}$ and $v_{t+1} \sim (0, \sigma_{v,t}^2)$.