Overnight borrowing, interest rates and extreme value theory

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Abstract

We examine the dynamics of extreme values of overnight borrowing rates in an inter-bank money market before a financial crisis during which overnight borrowing rates rocketed up to (simple annual) 4000 percent. It is shown that the generalized Pareto distribution fits well to the extreme values of the interest rate distribution. We also provide predictions of extreme overnight borrowing rates before the crisis. The examination of tails (extreme values) provides answers to such issues as what are the extreme movements expected in financial markets; have we already seen the largest moves; is there a possibility for even larger movements and, are there theoretical processes that can model the type of fat tails in the observed data? The answers to such questions are essential for proper management of financial exposures and laying ground for regulations.

Key Words: Financial crises, risk management, extreme value theory, overnight rate, federal funds rate.
JEL No: G0, G1, C1

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1. Introduction

The Turkish government started implementing a far reaching restructuring and reform program after the general elections in April 1999.\(^1\) The aim of the program was to reduce inflation from its 60-70 percent level per year to single digits by the end of the year 2002. The program gained further momentum after the country made a stand-by arrangement with the International Monetary Fund (IMF) in December 1999 and announced the technical aspects of the disinflation program. The main tool of the disinflation program was the adoption of a *tablita* with an exit, that is, the percent change in the value of the Turkish Lira (TRL) against a basket of foreign currencies (1 U.S. Dollar (USD) plus 0.70 Euro) was fixed (crawling peg) for the one and a half year period beginning January 2000. The government announced that a band around the crawling peg would start in July 2001, and would continue widening towards the end of 2002. Meanwhile, the stand-by arrangement determined a ceiling for the net domestic assets of the Central Bank.\(^2\) Accordingly, the Central Bank was able to create TRL liquidity only through net foreign capital inflows.

The structure of the stabilization program implied that the interest rate would be market determined in line with the exchange rate depreciation and capital flows, and the volatility of interest rates would be higher than the volatility before the program.\(^3\) A close inspection of Figure 1 reveals that this was the case. After the launch of the program on January 1, 2000, the average level of daily overnight interest rates dropped immediately in accord with the slowed down, fixed depreciation rate, and the volatility of the overnight interest rate increased.\(^4\)

During the second half of the year 2000, market participants and foreign investors were uneasy about developments in the economy. There were several reasons for these concerns. The government was slow in taking action to solve the chronic financial problems of the state banks and to implement other structural reforms; privatization efforts were not taking place as planned; some of the ministers in the government raised their voice against the privatization of Turkish Telecom; the current account deficit was increasing to historically high levels; and the volatility of the overnight interest rate increased.

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\(^1\) See Selçuk (1998) and Ertuğrul and Selçuk (2001) for the developments in the Turkish economy in recent years.

\(^2\) The net domestic assets of the Central Bank of Turkey are defined as the base money less the net foreign assets of the Central Bank valued at actual exchange rates. The base money is defined as currency issued by the Central Bank plus the banking sector’s deposits in TRL with the Central Bank.

\(^3\) In macroeconomics literature, this situation is known as “an impossible trinity” (a fixed exchange rate, free capital flows and an active monetary policy), i.e., a policy maker cannot fully control both interest rates and exchange rates if capital flows are free in the economy. Since the stabilization program fixed the exchange rate depreciation and the capital flows were free, there was little room for the Central Bank to affect interest rates.

\(^4\) The sample coefficient of variation of the overnight interest rates increased to 0.36 during January 3 - November 17, 2000, from 0.10 which is based on the previous four year sample. The sample coefficient of variation is defined as the sample standard deviation divided by the sample mean.
levels as a result of the appreciation of the Turkish Lira and negative domestic real interest rates; the relations between Turkey and the European Union started to become very tense; the possibility of closing a major opposition party in the parliament by the constitutional court implied general elections and, finally, Turkey had a history of unsuccessful stand-by arrangements with the IMF. As in many exchange rate-based stabilization programs, credibility appeared to increase in the first phase of the program and, as time passed, inability to deal with fundamental problems and an unsound banking system started to erode it.\(^5\)

It was well-known by the market participants that one of the commercial banks (Demirbank) had an extremely risky position during the year 2000. The bank (with a paid capital of USD 300 million) was funding its estimated USD 7.5 billion government securities portfolio mostly from the money market with short term obligations. On Monday, November 20, 2000, Demirbank was not able to borrow from the money market and the Central Bank stepped in to cover Demirbank’s position. In the following days, market makers in the government securities market stopped posting prices and Demirbank was not able to liquidate its positions. As a result, overnight interest rates started to increase. Meanwhile, a significant portion of foreign creditors withdrew their credit lines to Turkish banks and began to liquidate their positions out of devaluation fears. Major international investment houses and banks started to recommend “fire sale” to their clients in their bulletins and reports. Suddenly, there was a rapid capital outflow, starting on Wednesday, November 22.\(^6\)

As a result of the heavy capital outflow and decrease in the Central Bank reserves, liquidity pressure rocketed the interest rates. The Central Bank started to provide liquidity to the market violating the rule set by the stand-by agreement for the net domestic assets. However, the injected liquidity bounced back to the Central Bank in the form of additional demand for foreign currency. Therefore, the Central Bank stopped providing liquidity after six business days, on Thursday, November 30, 2000. Immediately, the overnight interest rate reached its peak at (simple annual) 873 percent on Friday, December 1, 2000. Total capital outflow during this period reached an estimated USD 6 billion, eroding approximately 25 percent of the foreign exchange reserves of the Central Bank. Over the weekend, the IMF rushed in an “emergency team” consisting of two delegations to discuss an emergency loan. On Tuesday, December 5, Turkish authorities announced a USD 7.5 billion rescue package with the IMF. The following morning before the markets opened, Demirbank became the 11th Turkish bank to be taken under control of the Saving Deposits Insurance Fund. The owner of Demirbank went on record stating that:

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\(^5\)See Guidotti and Végh (1999) for a political-economy model that focuses on the evolution of credibility over time under exchange rate-based stabilization programs.

\(^6\)Dornbusch (2001) discusses the makings of emerging market crises in general and claims that a large number of poorly managed banks and the banking system’s short term funding caused the Turkish financial crisis in 2000. Stanley Fischer, the first deputy managing director of IMF, relates the crisis in Turkey to banking sector problems and the failure to undertake corrective fiscal actions against the widening current account deficit. See Fischer (2001).
“Naturally, while buying the treasury bond Demirbank thought interest rates might rise and accounted for that. But the rise we are talking about was above and beyond logic and reason. No institution could foresee [annualized compound] interest rates in thousands, ten thousands or even billions. [R]ates suddenly became crazy and rose to the levels unseen in the history of the Republic. This situation ripped [us] apart.” Reuters News Service. December 12, 2000.

However, other bankers had a different perspective:

“Around mid-November, Demirbank with a paid capital of USD 300 million was carrying a T-bill stock of USD 4-5 billion and it was extremely squeezed. [These] fellows were taking an extremely high risk and this risk cost them their lives.” Reuters News Service. December 6, 2000.

After the IMF’s backing of the country in December 2000, there was some short-term capital inflow to the economy, especially in the beginning of the year 2001, and the Central Bank reserves returned to its pre-crisis level. Interest rates decreased, albeit stabilizing at a higher level than the pre-crisis average. Nevertheless, the market participants were not comfortable about developments in the economy and there were concerns about the Treasury’s ability to borrow from the domestic market at favorable terms. A scheduled domestic debt auction of the Treasury on February 20, 2001, the day before the maturing USD 7 billion domestic debt, was aimed at borrowing approximately USD 5 billion (around ten percent of the total domestic debt).

Suddenly, on February 19, 2001, the day before the auction, Turkish Prime Minister Bulent Ecevit stormed out of a key meeting of top political and military leaders stating that a “dispute” had arisen between himself and the country’s president. He further emphasized that “of course, this is a serious political crisis” without elaborating the future of the government or the economic program. The news hit the market and the stock market dived 18 percent in one day. The same day, the Central Bank sold USD 7.5 billion (approximately one-third of the total official reserves) for the next day delivery to the banks which had yet to recover from the November crisis. The next day, two state banks (Ziraat and Halkbank) were not able to meet their obligations in the markets and the Central Bank refused to provide TRL liquidity to the banks. Therefore, banks were forced to sell USD 6 billion back to the Central Bank. The daily average overnight interest rates rocketed up to (simple annual) 2000 percent on February 20, and 4000 percent on February 21. The government responded by dropping its exchange-rate controls early February 22 to take the pressure off on rates, and the TRL/USD exchange rate went up 40 percent in one week. One week after this second crisis, another bank (Ulusalbank) became the 12th Turkish bank to be

\[\text{Market sources estimate that the total obligations of two state banks were approximately USD 7 billion on that particular day.}\]
taken under control of the Saving Deposits Insurance Fund. Incidentally, Ulusalbank was controlled by the same group who used to own Demirbank.

The Turkish financial crisis in February 2001 is a case study for extreme risks and risk management practices. In recent years, the problem of extreme risks in financial markets has become topical following the recent turmoils in the Asian and Russian markets, and the unexpected big losses of investment banks such as Barings and Daiwa. The Basel Committee has set rules to be followed by banks to control their risks, but most of the well-studied models for assessing risks are based on the assumption that financial assets are distributed according to a normal distribution. In particular, the Value-at-Risk (VaR) measures with Gaussian-type innovations failed to cope with the recent turmoils within the Asian and Russian markets in market and credit risk computations. In the Gaussian model the evaluation of extreme risks is directly related to the variance, but in the case of fat-tailed distributions this is not the case since the underlying distribution may not even have a finite variance.

In this paper, we investigate the dynamics of the extreme values of overnight borrowing rates in the inter-bank money market for the TRL before the financial crisis of February 2001. It is shown that the generalized Pareto distribution model fits well to the tail of the interest rate distribution. We also provide estimates of overnight borrowing rates at 0.999 percentile, again, before the crisis. Our findings indicate that the extremely high overnight interest rates observed during the February 2001 crisis (up to daily average of 4000 percent) emerge as a possible outcome from the pre-crisis data, although it has not been observed before.

This paper is structured as follows. In Section 2, the extreme value theory with reference to the Fisher-Tippett framework, the Generalized Pareto Distribution (GPD), the tail estimation, and the tools used in the preliminary data analysis for the extreme value theory applications are presented. Section 3 reports the descriptive statistics, the maximum likelihood estimation of the GPD parameters for the overnight interest rates and other empirical results. Since the Turkish daily overnight rates has not been studied widely in the literature and is not well known, we also study the daily U.S. Effective Federal Funds Rate as a comparison. We conclude afterwards.

2. Extreme Value Theory

From the practitioners’ point of view, one of the most interesting questions that tail studies can answer is what are the extreme movements that can be expected in financial markets? Have we already seen the largest ones or are we going to experience even larger movements? Are there theoretical processes that can model the type of fat tails that come out of our empirical analysis? Answers to such questions are essential for sound risk management of financial exposures. It turns out that we can answer these questions within the framework
of the extreme value theory.

After the seminal work of Mandelbrot (1963) on cotton prices, evidence of heavy tails in financial asset returns is plentiful: See, for example, Koedijk et al. (1990), Hols and de Vries (1991), Loretan and Phillips (1994), Ghose and Kroner (1995), Danielsson and de Vries (1997), Müller et al. (1998), Pictet et al. (1998), Hauksson et al. (2001), Dacorogna et al. 2001a,b). Mandelbrot advanced the hypothesis of a stable distribution on the basis of an observed invariance of the return distribution across different frequencies and apparent heavy tails in return distributions. A continuing controversy has long been in the financial research as to whether the second moment of the returns converges. This question is central to many models in finance, which rely heavily on the finiteness of the variance of returns.

Extreme value theory is a powerful and yet fairly robust framework to study the tail behavior of a distribution. Embrechts et al. (1997) is a comprehensive source of the extreme value theory to the finance and insurance literature. Reiss and Thomas (1997) and Beirlant et al. (1996) also have extensive coverage on the extreme value theory.

Although the extreme value theory has found large applicability in climatology and hydrology, there have been a limited number of extreme value studies in finance literature in recent years. De Haan et al. (1994) study the quantile estimation using the extreme value theory. Reiss and Thomas (1997) is an early comprehensive collection of statistical analysis of extreme values with applications to insurance and finance, among other fields. McNeil (1997, 1998) study the estimation of the tails of loss severity distributions and the estimation of the quantile risk measures for financial time series using extreme value theory. Embrechts et al. (1999) overviews the extreme value theory as a risk management tool. Müller et al. (1998) and Pictet et al. (1998) study the probability of exceedances for the foreign exchange rates and compare them with the GARCH and HARCH models. Embrechts (1999, 2000) study the potentials and limitations of the extreme value theory. McNeil (1999) provides an extensive overview of the extreme value theory for risk managers. McNeil and Frey (2000) studies the estimation of tail-related risk measures for heteroskedastic financial time series. Gençay et al. (2003) and Gençay and Selçuk (2003) study the extreme value theory within the context of Value-at-Risk (VaR) calculations. Their results indicate that VaR estimates, based on extreme value theory, are more accurate at higher quantiles. In addition, the daily return distributions have different moment properties at their right and left tails. Therefore, risk and reward are not equally likely in emerging market economies. In the following section, we present the parametric framework for our study.

\footnote{Embrechts et al. (1997) has a large collection of literature in applications of the extreme value theory in other fields.}
2.1. Fisher-Tippett Theorem

The normal distribution is an important limiting distribution for sample sums or averages as summarized in the central limit theorem. Similarly, the family of extreme value distributions are the ones to study the limiting distributions of sample maxima. This family can be presented under a single parameterization known as the Generalized Extreme Value distribution (GEV). The theorem of Fisher and Tippett (1928) is in the core of the extreme value theory. The theory deals with the convergence of maxima. Suppose that $X_1, X_2, \ldots, X_n$ is a sequence of independently and identically distributed\(^9\) random variables from an unknown distribution function $F(x)$. Denote the maximum of the first $m < n$ observations of $X$ by $M_m = \text{max}(X_1, X_2, \ldots, X_n)$. Given a sequence of $a_m > 0$ and $b_m$ such that $(M_m - b_m)/a_m$, the sequence of normalized maxima converges in distribution to the following so-called Generalized Extreme Value (GEV) distribution

$$H_\xi(x) = \begin{cases} 
  e^{-(1+\xi x)^{-1/\xi}} & \text{if } \xi \neq 0 \\
  e^{-e^{-x}} & \text{if } \xi = 0,
\end{cases}$$

where $X$ is such that $1 + \xi X > 0$ and $\xi$ is the shape parameter.\(^{10}\) When $\xi > 0$, the distribution is known as the Fréchet distribution and it has a fat-tail. The larger the shape parameter, the more fat-tailed the distribution. If $\xi < 0$, the distribution is known as the Weibull distribution. Finally, if $\xi = 0$, it is the Gumbel distribution.\(^{11}\) The Fisher-Tippett theorem suggests that the asymptotic distribution of the normalized maxima belongs to one of the three distributions above and the tail behavior of the data series can be estimated from one of these three distributions.

The class of distributions of $F(x)$ where the Fisher-Tippett theorem holds is quite large.\(^{12}\) One of the conditions is that $F(x)$ has to be in the domain of attraction for the Fréchet distribution\(^{13}\) ($\xi > 0$) which in general holds for financial time series. Gnedenko (1943) shows that if the tail of $F(x)$ decays like a power function, then it is in the domain of attraction for the Fréchet distribution. The class of distributions whose tails decay like a power function are large and include the Pareto, Cauchy, Student-$t$ and several mixture distributions. These distributions are the well-known heavy tailed distributions.

The distributions in the domain of attraction of the Weibull distribution ($\xi < 0$) are the short tailed distributions such as uniform and beta distributions. The distributions in the

\(^9\)The assumption of independence can be easily dropped and the theoretical results follow through (see McNeil 1997). The assumption of identical distribution is for convenience and can also be relaxed.

\(^{10}\)The tail index is defined as $\alpha = \xi^{-1}$.

\(^{11}\)An extensive coverage can be found in Gumbel (1958).

\(^{12}\)Embrechts et al. (1997, 1999), Embrechts (1999), Reiss and Thomas (1997), Longin (2000) and McNeil (1997, 1999) have excellent discussions of the theory behind the extreme value distributions from the risk management perspective.

\(^{13}\)See Falk et al. (1994).
domain of attraction of the Gumbel distribution ($\xi = 0$) include the normal, exponential, gamma and lognormal distributions where only the lognormal distribution has a moderately heavy tail.

### 2.2. Generalized Pareto Distribution

In general, we are not only interested in the maxima of observations, but also in the behavior of large observations which exceed a high threshold. Given a high threshold $u$, the distribution of excess values of $X$ over threshold $u$ is defined by

$$F_u(y) = P\{X - u \leq y | X > u\} = \frac{F(y + u) - F(u)}{1 - F(u)}$$

which represents the probability that the value of $X$ exceeds the threshold $u$ by at most an amount $y \geq 0$ given that $X$ exceeds the threshold $u$. A theorem by Balkema and de Haan (1974) and Pickands (1975) shows that for sufficiently high threshold $u$, the distribution function of the excess may be approximated by the generalized Pareto distribution (GPD) such that, as the threshold gets large, the excess distribution $F_u(y)$ converges to the GPD which is

$$G_{\xi, \beta}(x) = \begin{cases} 
1 - \left(1 + \frac{x}{\beta}\right)^{-1/\xi} & \text{if } \xi \neq 0 \\
1 - e^{-x/\beta} & \text{if } \xi = 0,
\end{cases} \quad (3)$$

where $\xi$ is the shape parameter. The GPD embeds a number of other distributions. When $\xi > 0$, it takes the form of the ordinary Pareto distribution. This particular case is the most relevant for financial time series analysis since it is a heavy tailed one. For $\xi > 0$, $E[X^k]$ is infinite for $k \geq 1/\xi$. For instance, the GPD has an infinite variance for $\xi = 0.5$ and, when $\xi = 0.25$, it has an infinite fourth moment. For the security returns or high frequency foreign exchange returns, the estimates of $\xi$ are usually less than 0.5, implying that the returns have finite variance. See, for instance, Jansen and deVries (1991), Longin (1996), Müller et al. (1998), and Dacorogna et al. (2001b). When $\xi = 0$, the GPD corresponds to exponential distribution and it is known as a Pareto II type distribution for $\xi < 0$.

The importance of the Balkema and de Haan (1974) and Pickands (1975) results is that the distribution of excesses may be approximated by the GPD by estimating $\xi$ and $\beta$ as a function of a (high) threshold $u$. The parameters of the GPD can be estimated with various methods such as the method of probability weighted moments or the maximum likelihood method.\(^\text{14}\) For $\xi > -0.5$ which corresponds to heavy tails, Hosking and Wallis (1987) presents evidence that maximum likelihood regularity conditions are fulfilled and the

\(^{14}\)Hosking and Wallis (1987) has discussions on the comparisons between various methods of estimation.
maximum likelihood estimates are asymptotically normally distributed. Therefore, the approximate standard errors for the estimators of $\beta$ and $\xi$ can be obtained through maximum likelihood estimation.

2.3. The Tail Estimation

The conditional probability in the previous section is

$$F_u(y) = \frac{\Pr\{X - u \leq y, X > u\}}{\Pr(X > u)} = \frac{F(y + u) - F(u)}{1 - F(u)}$$

and denoting $x = y + u$, $y \geq 0$, we have the following representation

$$F(x) = [1 - F(u)] F_u(y) + F(u).$$

Notice that this representation is valid only for $x > u$. Since $F_u(y)$ converges to the GPD for sufficiently large $u$, and denoting $x = y + u$, $y \geq 0$, we have that

$$F(x) \approx [1 - F(u)] G_{\xi, \beta, u}(x - u) + F(u).$$

The last term on the right hand side can be determined by the empirical estimator $(n - N_u)/n$ where $N_u$ is the number of exceedances of $u$ and $n$ is the sample size. The tail estimator, therefore, is given by

$$\hat{F}(x) = 1 - \frac{N_u}{n} \left(1 + \frac{x - u}{\hat{\beta}}\right)^{-1/\hat{\xi}}.$$

For a given confidence level $q$, a percentile ($\hat{x}_q$) at the tail is estimated by inverting the tail estimator in (7),

$$\hat{x}_q = u + \frac{\hat{\beta}}{\hat{\xi}} \left(\left(\frac{n}{N_u}(1 - q)\right)^{-\hat{\xi}} - 1\right).$$

In statistics, this is the quantile estimation and it can be utilized in Value-at-Risk (VaR) estimations in finance applications.

2.4. Preliminary Data Analysis

In statistics, a QQ-plot (quantile-quantile plot) is a convenient visual tool to examine whether a sample comes from a specific distribution. Specifically, the quantiles of a hypothesized distribution are plotted against the quantiles of an empirical distribution. If the sample comes from the hypothesized distribution, the QQ-plot is linear. In extreme value theory and its applications, the QQ-plot is typically plotted against the exponential distribution (i.e, a distribution with a thin-sized tail) to measure the fat-tailness of a distribution.
If the data is from an exponential distribution, the points on the graph would lie along a positively sloped straight line. If there is a concave presence, this would indicate a fat-tailed distribution, whereas a convex departure is an indication of short-tailed distribution.

A second tool is the sample mean excess function (MEF) which is defined by

\[ e_n(u) = \frac{\sum_{i=1}^{n} (X_i - u)}{\sum_{i=1}^{n} I\{X_i > u\}}. \]

(9)

where \( I\{X_i > u\} \) is the indicator function yielding 1 for \( X_i > u \) and 0 otherwise. The MEF is the sum of the excesses over the threshold \( u \) divided by the number of data points which exceed the threshold \( u \). It is an estimate of the mean excess function which describes the expected overshoot of a threshold once an exceedance occurs. If the empirical MEF is a positively sloped straight line above a certain threshold \( u \), it is an indication that the data follows the GPD with a positive shape parameter \( \xi \). On the other hand, exponentially distributed data would show a horizontal MEF while short tailed data would have a negative slope.

In threshold determination, we face a trade off between bias and variance. If we choose a low threshold, the number of observations (exceedances) increases and the variance of the estimation becomes smaller. However, choosing a low threshold also introduces some observations from the center of the distribution and the estimation becomes biased. Therefore, a careful combination of several techniques, such as the QQ-plot and the MEF should be considered in threshold determination. See, Reiss and Thomas (1997, Ch. 2) and Embrechts et al. (1997, Ch 6) for these and other diagnostic tools and exploratory data analysis for extremes.

3. Empirical results

The data source for the daily overnight interest rates (simple annual) is the Central Bank of Turkey. The daily rates are calculated by the Central Bank as a weighted average of intraday transactions in the interbank money market. The descriptive statistics of daily average simple annual overnight interest rates before and after the February 2001 crisis are given in Table 1. The full sample period is January 2, 1990 - February 23, 2001 with 2806 observations. It includes all available daily interest rate data from the inter-bank money market in Turkey. In this paper, all calculations and predictions are carried out with simple annual interest rates.

In Table 1, the sample means of 73.0 and 75.7 percent correspond to compound annual

\footnote{The observed maximum \textit{intraday} interest rate is (simple annual) 6200 percent. This means borrowing Turkish Lira at an interest rate of 17.2 percent for one day \((17.2 \times 360 \approx 6200)\). We use \textit{weighted} average of intraday rates as provided by the Central Bank.}
interest rates of approximately 107 and 113 percent, respectively.\footnote{The annual compound interest rate is calculated by
\[ 1 + cr = [1 + (sr/360)]^{360} \]
where \( cr \) is the annual compound interest rate and \( sr \) is the annual simple interest rate. FX and money market days basis is 360 whereas capital markets days basis is 365 in the Turkish markets.} Although it is high by the standards of a developed market, it reflects the high inflation levels and associated risk in the economy. The annual average percent increase in consumer prices (inflation) during the sample period was 75.4 percent, implying an average annual real interest rate of 18-21 percent.\footnote{It is a common practice among researchers and practitioners in developed economies to calculate the real interest rate as “nominal rate minus inflation”. This approximation holds only for low levels of inflation. The exact formula for the real interest rate calculation is \( 1 + r = \frac{1 + i}{1 + \pi} \) where \( r \) is the real interest rate, \( i \) is the nominal rate and \( \pi \) is the corresponding inflation rate. In our calculations of the average annual real interest rate, \( i \) is the nominal annual compound interest rate and \( \pi \) is the annual inflation rate.} Both kurtosis and skewness estimates show that the interest rates are far from being normally distributed. The estimated kurtosis 82.62 before the crisis shows that the interest rate distribution has a fat tail. The estimated skewness of 7.5 before the crisis points out that the distribution is skewed. After the crisis, both skewness (26.4) and kurtosis (943.2) estimates indicate that fat-tailness and skewness of the distribution have substantially increased.

Our study should be viewed as a first attempt to investigate the extremal behavior of the Turkish interest rates. Further study on the dynamics of this data set such as stationarity, ARCH effects, dependency, etc., is required. Notice that even if the identical and independent distribution condition fails, the extreme value theory approach may still be an accurate approximation of the actual distribution function of maxima as noted by Reiss and Thomas (1997, page 10 and 172), among others. If there is a dependency in the data, the estimation result should be modified with an estimated extremal index. In this case, the quantile estimate would be different than the one estimated with weak dependency or independency assumption. We refer the reader to Longin (2000), Embrechts et al., (1997, Ch. 8) and Reiss Thomas (1997, Ch. 6) about dependency, extremal index and its implications in practice.

3.1. Tail Estimation of Excess Interest Rates

It is necessary to determine a threshold interest rate to estimate the parameters of the generalized Pareto (GPD) distribution. As presented in Section 2.4, the QQ-plot and the mean excess function are two empirical tools for this task. In a QQ-plot, the quantiles of the empirical distribution function on the \( x \)-axis are plotted against the quantiles of the exponential distribution function on the \( y \)-axis. The points should lie approximately along a straight line if the data is from an exponential distribution. Since the exponential
distribution has a medium-sized tail, a concave relationship between the quantiles of the empirical and exponential distributions indicate a heavy-tailed distribution for the time series under study. The top panel of Figure 2 indicates that the sample points start deviating from linear behavior at around 80 percent and form a concave pattern.

The sample mean excess function is another diagnostic tool to determine a threshold. If the points of the mean excess function exhibit an upward trend, this indicates heavy-tailed behavior. Short-tailed data exhibit negatively sloped behavior whereas an exponential distribution has a flat mean excess function. The bottom panel of Figure 2 demonstrates that the sample mean excess function is approximately linear and positively sloped after the 80 percent interest rate threshold. The examination of both panels in Figure 2 indicate that the approximate threshold value corresponds to 80 percent.

For a given threshold level, a tail estimation involves the estimation of the parameters of the generalized Pareto distribution. The maximum likelihood estimation is a convenient way to obtain standard errors of the parameter estimates. For the threshold \( u = 80 \), the threshold exceedances are 389 data points which constitute the upper 13.9 percent tail of the original sample of 2801 sample points. Notice that an 80 percent overnight interest rate (simple annual) is equivalent to an annualized compound interest rate of 122 percent. During the sample period, the average annual percent increase in TL/USD exchange rate is 75 percent. Therefore, the threshold implies a 23 percent annualized dollar interest rate.

The maximum likelihood estimate of \( \xi \) and \( \beta \) at threshold \( u = 80 \) are (standard errors are in parenthesis) 0.73 (0.086) and 22 (2.03), respectively. The estimated parameters are statistically significant at the 1 percent level. As discussed in Section 2.1, when \( \xi > 0 \), the distribution is known as the Fréchet distribution and it has a fat-tail with tail index \( 1/\xi \). The larger the shape parameter, the more fat-tailed the distribution. The value of \( \hat{\xi} = 0.73 \) indicates that the overnight interest rate series may come from a distribution with infinite variance.

The estimated GPD model with a threshold of 80 percent interest rate \( (u = 80) \) for the interest rate data before the February 2001 crisis is presented in the top panel of Figure 3. The estimated model is plotted in a solid curve while the interest rates above the threshold are shown in circles (on logarithmic scale). The estimated model successfully captures the underlying extreme values, and the tail behavior of excess interest rates is successfully approximated by a Fréchet-type distribution.

To examine the robustness of the results to the choice of the threshold value, the maximum likelihood estimation of the shape parameter, \( \xi \), is carried out for GPD models with a range of thresholds between 80 to 100 percent. The estimates of \( \xi \), as well as their asymp-
totical confidence intervals, are reported in the bottom panel of Figure 3. On the lower $x$-axis the number of data points exceeding the threshold is plotted and on the upper $x$-axis the threshold is located. The estimates of the shape parameter, $\xi$, are plotted on the $y$-axis. The results indicate that the shape parameter is fairly stable within the range of 80-90 percent interest rate. Of course, there are fewer observations at higher thresholds and less statistical precision. This is reflected in the wider confidence intervals.

3.2. Point Predictions at the Tails

It is possible to estimate a percentile value at the tail from the estimated parameters of the generalized Pareto distribution. In Figure 4, the tail estimates are reported with the pre-February 2001 crisis data. On a log-log plot as in Figure 4, GPD becomes linear with negative $(1/\xi)$ slope and it is easier to interpret results visually. In this figure, the left $y$-axis indicates the tail probabilities, $1 - F(x)$. The vertical dotted line starting from the $x$-axis is the estimated overnight interest rate which corresponds to the 1 percent tail probability on the $y$-axis. This value is obtained from the intersection of the 1 percent tail probability with the estimated tail and traced down to the $x$-axis. Therefore, the 99 percent quantile interest rate corresponds to 257 percent.

The dotted curve is the confidence interval calculated by the profile likelihood method.$^{19}$ The 95 percent confidence level corresponds to the intersection of the horizontal dotted line starting from the right $y$-axis. The range of the 95 percent confidence interval is 221 percent for the lower end and 313 percent for the higher end. Figure 5 has the same information as Figure 4, but it contains a different set of symmetric confidence intervals (197 and 318 percent) calculated with the Wald statistics.

Figure 4 has a straightforward message. When the interest rate corresponding to the 99 percent quantile is evaluated at the 95 percent confidence level, the maximum simple annual interest rate would go as high as 313 percent. This is a substantially high overnight interest rate but it is not in the order of ten thousands or millions percent compound interest rate as it was observed during the February crisis. Let us now go further in the tail to evaluate further extreme possibilities and investigate the 99.9 percent quantile estimates. This is presented in Figure 6 where the interest rate estimates at the 99.9 percent quantile are plotted as a function of the threshold (upper $x$-axis) or alternatively as a function of exceedances (lower $x$-axis) with the pre-February crisis data. The estimated interest rate settles above 1000 percent (simple annual) for most thresholds although variations are higher at higher thresholds. The upper confidence interval indicates that simple annual interest rates as high as 4800 percent were a possibility from the pre-crisis data.

$^{19}$The usual Wald standard errors are computed from the inversion of the Hessian of the log-likelihood function. Confidence limits may also be computed from selected methods such as bootstrapping, Bayesian, inversion of the likelihood ratio and the Lagrange multiplier statistics. Confidence limits from the inversion of the likelihood ratio statistic are also called profile likelihood confidence limits.
The results indicate that overnight interest rates observed during the financial crisis were a part of the pre-crisis interest rate distribution and the Turkish commercial banks simply ignored the possibility that such extreme tail events could be realized, considering that some banks were carrying short-term debt obligations of at least USD 4-5 billion with a paid capital of USD 300 million. Extremely high overnight interest rates observed during the February 2001 crisis (up to 4000 percent simple annual) should have been a concern before the outbreak of the crisis from a risk management perspective.

3.3. A Comparison with the U.S. Overnight Interest Rates

Although the Turkish daily overnight rate is an excellent case study with high volatility and a thick-tailed distribution, this data set has not been studied widely in the literature and is not well-known. Hence, we have repeated the extreme value analysis with the daily U.S. Effective Federal Funds Rate as a comparison. The Federal Funds Rate (FFR) is the interest rate that banks with excess reserves at a Federal Reserve District Bank charge other banks that need overnight loans. The sample period is from July 1, 1954 to December 31, 2000. The sample size is 16,986 daily observations. The data source is the Federal Reserve Board of Governors.

The GPD estimations with different numbers of exceedances in this data set indicate that the estimated shape parameter $\hat{\xi}$ is remarkably stable around -0.40, corresponding to a Pareto II type distribution which is a thin-tailed distribution with a finite tail.\textsuperscript{20} Figure 7 plots estimated interest rates at the 0.9999th quantile (one day in every ten thousand days, approximately 30 years), and at the 0.999th quantile (one day in every one thousand days, approximately 3 years) as a function of the number of exceedances. The estimated interest rate is very stable around 20.5 percent at the 0.1% tail and at around 22 percent at the 0.01% tail regardless of the number of exceedances. Moving in the tail of the distribution from the 0.1% to the 0.01% region increases the estimated interest rate by only around 150 bases points (one and a half percentage point).

4. Lessons from the Turkish Crises and Conclusions

Financial crises in emerging markets in general and the Turkish crises in November 2000 and February 2001 in particular, provide several lessons for investors both in developing and developed countries. As one prominent economist puts it, “crises are not just financial experiences but rather involve large and lasting social costs and important redistribution.

\textsuperscript{20}We do not intend to model the FFR dynamics in the US: it is far beyond the scope of this study. There is an ongoing discussion as to whether interest rates are stationary or not. Conventional economic and finance theory often assumes that interest rates are stationary and utilizes a model which is a mean-reverting process. However, some empirical studies provide evidence that interest rates are nonstationary processes. See, for example, Sarno et al. (2002) and references therein.
of income and wealth” (Dornbusch, 2001). Since a significant portion of total savings in developed economies are invested in emerging markets by hedge funds, mutual funds, and other institutions in the form of portfolio investment, the costs of financial crises are not confined to the residents of emerging market countries.21 Therefore, a careful investigation of the market dynamics and the causes of crises in these economies would benefit investors at large by increasing the investor awareness.22

Fundamental macroeconomic indicators such as growth rate, current account balance, real exchange rate, budget deficit, export-import ratio and debt-income ratio are the main sources for assessing the current and future status of an economy. Therefore, they play a significant role in the decision making process of credit rating agencies and in portfolio allocation decisions of multinational fund managers. One of the lessons from the Turkish crisis is that even if there is no deterioration in fundamental macroeconomic indicators, the balance sheet issues in the finance sector may create an environment in which even a small shock can lead to a total collapse of the system. Especially, a balance sheet mismatch situation (funding long-term illiquid assets with short-term obligations) combined with slack supervision and regulation is an invitation for a liquidity and currency crisis. Among others, Eichengreen (2001) investigates both the Argentinian and Turkish crises in detail. He points to the vulnerability of the banking sector as the main source of crisis in Turkey.23

According to a real exchange rate index recently published by the Turkish Central Bank, the lira appreciated 37 percent between January 1995 and January 2000, the month the stabilization program started. Nevertheless, the Turkish stabilization program adopted a crawling peg to reduce inflation by limiting the lira’s devaluation to 15 percent per year. Although it was expected that the lira was going to appreciate further during the program, the government could not commit to an upfront devaluation as the Turkish banks and the private sector had accumulated large unhedged foreign exchange exposures during the past 5-6 years. It was hoped by the program designers that the banking sector would strengthen before the economy would move into a floating exchange rate regime after 18 months. As Eichengreen (2001) points out, this strategy created a moral hazard in the system and an incentive to strengthen both balance sheets and supervision diminished.24

21 Although there has been a substantial decrease in recent years, the net portfolio investment in emerging markets by other countries in one year was USD 58.3 billion in 2000. The historical record is USD 109.9 billion in 1994. Source: IMF, International Financial Statistics.
22 Foreign investors may face completely different financial circumstances in emerging economies than they have in their home country. To point out this, Dornbusch (2001) refers to the catchy title of an article written in the early 1980s about the debt crisis in Latin American economies: “We are not in Kansas anymore..” (Diaz Alejandro, 1984).
23 See Dooley and Frankel (2003) for a collection of studies on currency crises in different emerging market economies.
24 According to Eichengreen (2001), the moral hazard was created “because exchange risk was socialized, that is to say, its strategy committed the government to preventing the exchange rate from moving and therefore to compensating the banks for their losses if the policy failed”.

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As a result, short foreign exchange positions of the banks doubled during the first nine months of the program. The direct consequence of this was that the fear of destabilizing the economy forced the authorities to resist any correction in the exchange rate, even if it meant extraordinary increases in overnight interest rates.\textsuperscript{25} Given the government’s strong commitment to exchange rates, the relevant question from the investors’ point of view was “what extraordinary interest rates may be observed under extreme situations?” Our results at a 99.9 percent quantile show that 2000-4800 percent interest rates (simple annual) were a possibility in the economy, although this range had not been observed before.

\textsuperscript{25}See Calvo and Reinhart (2002) on the fear of floating the exchange rate in emerging economies.
References


Table 1: Descriptive statistics of the (simple annual) overnight interest rates before and after the February 2001 crisis. n: sample size; Mean: Sample mean; Std: Standard deviation; Ku: Kurtosis; Sk: Skewness; Min: Minimum observed rate; Max: Maximum observed rate; Low: Rate corresponding to the 5th percentile; High: Rate corresponding to the 95th percentile. The full sample period is from January 2, 1990 to February 23, 2001 with 2806 observations. Pre-crisis period excludes the last five business days. Source: Central Bank of Turkey. Overnight interest rates are daily weighted average of inter-bank interest rates as calculated by the Central Bank of Turkey.

<table>
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<th>n</th>
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<th>Std</th>
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<td>99.6</td>
<td>943.2</td>
<td>26.4</td>
<td>67.7</td>
<td>13.6</td>
<td>4018.6</td>
<td>34.4</td>
<td>109.6</td>
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Figure 1: Daily average overnight interest rates in Turkey (annualized simple) including the February 2001 crisis. The daily rates are weighted average of intraday rates. The sample period is from January 2, 1990 to February 23, 2001 with 2806 observations. Source: Central Bank of Turkey.
Figure 2: **Top:** QQ-plot of daily overnight interest rates (simple annual) before the February 2001 crisis against the exponential distribution. The quantiles of the empirical distribution function on the x-axis are plotted against the quantiles of the exponential distribution function on the y-axis. The points should lie along the straight line if the data is from an exponential distribution. A concave presence indicates a fat-tailed distribution. Source: Central Bank of Turkey. **Bottom:** Sample mean excesses of daily overnight interest rates (simple annual) before the February 2001 crisis over increasing thresholds. A straight line with positive slope above a given threshold $u$ is a sign of the GPD in tail. Notice that the plot is approximately linear and positively sloped after 80 percent, indicating the Pareto behavior in the tail.
Figure 3: Top: The estimated GPD model for the excess interest rates before the February 2001 crisis. The estimated model is plotted as a curve while the actual daily overnight interest rates (simple annual) above the threshold are shown in circles (in logarithmic scale). The threshold is 80 percent interest rate, $u = 80$. The estimated parameters are $\xi = 0.73$ and $\hat{\beta} = 22$. The number of exceedances is 389. The sample period is from January 2, 1990 to February 16, 2001 which ends one week before the February 2001 crisis. Bottom: Estimates of shape parameter, $\xi$, at different thresholds (upper $x$-axis) or alternatively with different number of exceedances (lower $x$-axis) before the February 2001 crisis. The parameter estimates are from 100 GPD models with thresholds ranging between 80 percent and 100 percent interest rates (simple annual). The sample period is from January 2, 1990 to February 16, 2001. The sample period ends at the week before the February 2001 crisis.
Figure 4: Tail estimate for the excess interest rates before the February 2001 crisis. The left $y$-axis indicates the tail probabilities $1 - F(x)$. The vertical dotted line from $x$-axis is the estimated interest rate (at 257 percent) and intersects with the 1 percent tail probability on the left $y$-axis. Therefore, it is the estimated interest rate at the 99 percent quantile. The dotted curve is the confidence interval around this estimate, calculated by the profile likelihood method. It intersects with the 95 percent horizontal confidence line from the right $y$-axis and the corresponding 95% confidence interval is (221, 313) in percent. Notice that the confidence interval becomes larger when the confidence level is increased (a down movement in the level of confidence on the right $y$-axis). The sample period is from January 2, 1990 to February 16, 2001 which ends one week before the February 2001 crisis.
Figure 5: Tail estimate. This is the same tail estimate in Figure 4. The center dotted line (estimated simple annual interest rate of 257 percent) and intersects with 1 percent tail probability on $y$-axis. Symmetric confidence intervals (197 and 318 percent) are calculated by using the Wald statistic. The sample period is from January 2, 1990 to February 16, 2001 which ends one week before the February 2001 crisis.
Figure 6: Interest rate estimates of quantile 0.999 as a function of the threshold (upper x-axis) or alternatively as a function of exceedances (lower x-axis) before the February crisis. These estimates are from 100 GPD models with thresholds ranging between 80 percent and 100 percent interest rates (simple annual). The estimated interest rate at 0.999 quantile settles above 1000 percent at most thresholds although variations are higher at the higher thresholds. The Wald confidence intervals indicate that interest rates as high as 4000 percent is a possibility. The sample period is from January 2, 1990 to February 16, 2001 which ends one week before the February 2001 crisis.
Figure 7: Effective Federal Funds Rate (daily) estimates of 0.9999th (top) and 0.999 (bottom) quantiles as a function of exceedances. The estimated interest rate even at 0.9999 quantile (one day in 30 years) is less than the historical high of 22.36 percent (July 22, 1981). The sample period is from July 1, 1954 to December 31, 2000 (16,986 sample points). Data source: Federal Reserve Board of Governors, H.15 Release.