Bus. 864
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Homework Set 2
Crank-Nicolson finite difference method

1. Use the Crank-Nicolson method to solve question (1) of Homework Set 1 for the Black-Scholes-Merton model 2 year bond. Start from the template program stored under the code section of the course website as CN_012606.xls (copy and save under a different name). State variable $S(t)$ will now refer to the firm's asset value.[1]

   (a) Adapt the spreadsheet as indicated above to produce a vector of *intial fair market bond values* for each possible intial asset value.

   (b) Add a column to your spreadsheet giving, for each initial S, the bond's *contractual yield to maturity* if purchased for the fair initial asset value.

   (c) Add a further column determining for each asset level (except the top and bottom) the approximate first derivative of the bond's price (it's 'delta') with respect to S. Also, for each level of S, determine the standard deviation of S changes over the next one week. Put the two together to determine the standard deviation of the *mark-to-market* risk on holding this bond over that first week. (Comment: This is one possible measure of the short term riskiness of this bond.)

2. Next consider the bond cash flows under the *objective* process for S. I.e., the proportional drift in S is $\alpha = .10$ rather than r. It is probably best to do this in a new spreadsheet. Add α as a model parameter and incorporate the necessary changes in coefficient functions of subroutine CNPRICE. Do the following for the 2 year bond.

   (a) Produce a vector of objective expected discounted cash flows $\mathbf{E}[e^{-rT}B(T)]$ from the bond for the various initial asset values. Discounting is at the riskfree interest rate $r = .05$ (Note: This occurs automatically if you leave FNC equal to $-r$).

   (b) Produce a vector of the objective probability of default on this bond for various initial asset values.[2]

3. Black-Cox diffusion to barrier model: Suppose default occurs at the time τ that firm assets first hit a trigger level of 100. Assume that recoveries in default are just 60 because bankruptcy costs consume the other 40.

   (a) Adapt the Crank-Nicolson spreadsheet to determine the intial market value of the bond as function of intial firm assets.[3]

   (b) In an additional column, display the contractual yield to maturity on the bond implied by the corresponding prices for each initial asset level.

4. **Optional bonus question:** For question (2), produce a vector of the objective *standard deviations* of the *undiscounted* cash flows from the bond.[4] (Comment: This is one possible measure of the long term riskiness of this bond.)

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[1]Hint: Reset SMAX to 500 since our intial asset level of interest is $S(0) = 150$. Since our asset value process is the same as the stock price process in the template program, the only change you will have to make, other than input parameter values, is in the loop of subroutine CNPRICE that sets the boundary condition at maturity.

[2]Hint: First, probability of default is the expected value of a variable taking value 1 at time T if default occurs and 0 otherwise. Modify your maturity boundary condition accordingly. Second, since there is no 'discounting by the riskfree rate' involved in answering this question, that must be 'turned off' by setting r equal to 0 (or by setting IFUT = 1) before calling CNSET again.

[3]Hint: Set SMIN to the trigger level, SMAX to 500 above that, and IMIN = 1 to indicate known lower boundary value. Modify the function FMIN appropriately to reflect the recovery rate.

[4]Hint: Proceed in stages using the fact that variance of a random variable X can be computed as $\mathbf{E}[X^2] - (\mathbf{E}[X])^2$. Your answer to (a) gives $e^{-rT}\mathbf{E}[X]$ where X is the terminal payoff. To get $e^{-rT}\mathbf{E}[X^2]$, you do this again with maturity value set to $B(T)^2$. This should give you the pieces necessary to answer the question.