

Homework Set 3
Constant intensity default models

1. Suppose a firm issues a 2 yr. zero-coupon bond with maturity (par) value 100. The firm has constant Poisson (objective) default intensity of $h = .15/\text{yr}$. The riskfree interest rate is constant at $r = .05/\text{yr}$. Assume the market is risk-neutral toward the default event (it is fully diversifiable), so that the risk-adjusted default intensity is the same as the objective one.
 - (a) What is the probability the firm defaults before maturity? (determine by formula)
 - (b) Suppose there are 0 recoveries in default. What is the initial market value of the bond and its contractual yield if purchased for that price?
 - (c) Repeat for maturities 1 to 5 years and plot the resulting credit spread curve for this issuer.
 - (d) Suppose recovery upon default is 60% of par, with payment *deferred to the bond maturity date*. What will be the initial market value and contractual yield of the 2 year bond?
 - (e) Suppose recovery upon default is 60% of par, with payment *made at the time of default*. What will be the initial market value and contractual yield of the 2 year bond?
 - (f) Suppose recovery upon default is 60% of *contractual balance owing* (i.e., $100e^{-c(2-t)}$ if t is the default time), with payment made at the time of default. What will be the initial market value and contractual yield of the 2 year bond?

2. Suppose the above firm issues a 5 year bond of 100 par value, with a coupon rate of 20%/year paid semiannually, and with expected recoveries in default being 40% of par value (paid at the time of default). The riskfree yield curve is flat at 5%/year (continuously compounded).
 - (a) For what price should this bond trade, and what would be the associated promised yield to maturity, if the default intensity is .15/year?
 - (b) If we observe this bond trading at a price of 95, and everyone agrees that recoveries in default will be 40%, what is the market implied default intensity? (i.e., what value of h would make 95 the market price)
 - (c) Now consider a 5 year credit default swap on this firm. The protection buyer pays a constant, continuous, default swap spread at a rate c/year on a notional debt amount of 100 until the earlier of 5 years or the time of default. If default occurs before 5 years, the protection seller pays at that time a lump sum equal to $(100 - \text{recovery rate})$. If $h = .15$, what should be the credit default swap spread c ? (i.e., what value of c would make the initial market value of the CDS equal to 0?)

3. Consider a portfolio of long-term bonds issued by three firms: 100 of firm X, 150 of firm Y, 200 of firm Z. The firms' market implied default intensities are .20, .10 and .05 respectively. Expected recovery rates in default are all 40% of par. You wish to buy a 5 year first-to-default swap reimbursing you for the loss in par value just for whichever firm defaults first. You pay for the swap with a single lump sum premium C at the start of the contract. The riskfree interest rate is constant at 5%.

Do a Monte Carlo simulation of the firms' default times (assume they are independent) to determine the fair market value of the protection and hence what C you should pay.