Homework Set 4

Constant intensity default models

1. Consider two firms, Shady Deals Inc and Best Co. Suppose we know that Shady's (constant) hazard rate of default (λ) is exactly double that of Best. Assume the market is risk-neutral toward the default events (fully diversifiable), so that risk-adjusted hazard rates are the same as objective hazard rates.

   (a) Is it correct to conclude that Best's 'survival' probability to any time t, is double that of Shady? Explain.

   (b) Suppose there are 0 recoveries in default for both firms, and that the (constant) risk-free interest rate is zero. Best issues a two-year bond at par with annual coupon rate c_B (proportion of par). If Shady were to issue a similar bond, what annual coupon rate would it have to offer (expressed as a function of c_B) such that this bond also traded at par?

2. A constant hazard rate of default implies that the probability of default over any interval of time is constant. True or False? Explain.

Time-varying intensity default models

3. A firm issues the following zero-coupon bonds:

t	$V(0, t)$
1	0.90
2	0.78
3	0.62
4	0.55
5	0.50

   Assume the following:

   $$r = 0.045$$
   $$R = 0$$

   Recover the following information from the bond prices:

   (a) The term structure of contractual yields to maturity (continuously compounded).
   (b) The term structure of credit spreads.
   (c) The term structure of 1-year 'forward' hazard rates or default intensities.
   (d) The continuous par default swap spreads for each maturity.

   Do a Monte Carlo simulation (minimum of 1,000 traces) to determine the following:

   (a) The risk-neutral expected default time.
   (b) The risk-neutral probability of default occurring in the interval $(4, 5]$.
   (c) The risk-neutral probability of no defaults on the bonds.

   See if you can do this analytically as well to check the accuracy of your simulation.