Homework Set 6
Rating transition models

Consider a credit rating system with only three classes: A, B and D (default). Suppose the estimated probabilities of state transition over one year intervals is give by

$$
P = \begin{bmatrix}
 & A & B & D \\
A & .80 & .15 & .05 \\
B & .10 & .70 & .20 \\
D & .00 & .00 & 1.00
\end{bmatrix}
\tag{1}
$$

where P_{ij} is the probability of transitioning from state i to state j.

1. Assuming that stochastic process for ratings is Markov with constant transition probabilities, what is the transition matrix over 2 years?

2. If these are also the risk-neutral transition probabilities, the default-free interest rate is constant at 5%/year (continuously compounded), and recoveries in default are 0, what will be the prices of 1 year and 2 year zero-coupon bonds for firms currently rated A and B? What are the corresponding contractual yields to maturity? Assume the bonds have $100 maturity value.

3. What is the continuous time Markov generator matrix Q, if one exists, that corresponds to the annual transition matrix P?

4. Using your answer for Q, what would be the market contractual yield to maturity on a A-rated zero-coupon bond of maturity 0.8 years if there are 0 recoveries in default?

5. **Collateral support agreement:** Suppose that the firm in question (4) agreed to post collateral of $50 if it were downgraded to B rating before the bond either matured or defaulted. At what contractual yield to maturity would you now be willing to lend to the firm? If downgrade and default occurs, collateral is liquidated and received by you at time of default.[1] [Do this by Monte Carlo simulation of the rating/default time path]

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[1]Note: If firm is upgraded to A after downgrade to B, collateral is returned to the firm at time of the upgrade.