BUS 864

Class Notes

February 22, 2006

Anton Theunissen

Contents

I plan to update this document from time to time. Please check back regularly for the latest version.

1 A Simple Default Model

1.1 Constant Default Intensity

Assume that defaults (or 'credit events') 'arrive'/occur according to a Poisson process with constant per unit time intensity, $\lambda > 0$ (λ is the average number of occurences per unit time). We refer to a Poisson variable as $X \sim \text{Pois}(\lambda)$. X indicates the number of default events that occur in some interval $[0, t]$. For $t = 1$:

$$p(x) = e^{-\lambda}\frac{\lambda^x}{x!} \tag{1}$$

$$P(x) = \sum_{k=0}^{x} e^{-\lambda}\frac{\lambda^x}{x!} \tag{2}$$

$$\text{E}(X) = \lambda \tag{3}$$

$$\text{Var}(X) = \lambda \tag{4}$$

$$\tag{5}$$

For most of our purposes we will only be concerned with the first occurrence of a credit event. Let τ denote the time of the (first) credit event. Consider the probability that τ does not occur in the interval $[0, t]$ ($t > 0$):

$$\begin{aligned} \mathbf{p}(\tau > t) &= e^{-\lambda t}\frac{(\lambda t)^0}{0!} \\ &= e^{-\lambda t} \end{aligned} \tag{6}$$

The probability that $\tau \in [0, t]$ is:

$$\begin{aligned} \mathbf{p}(\tau \leq t) &= 1 - Pr(\tau > t) \\ &= 1 - e^{-\lambda t} \end{aligned} \tag{7}$$

So default times are exponential random variables, $\tau \sim \text{Exp}(\lambda)$. For $t \geq 0$:

$$
\begin{aligned}
f(t) &= \lambda\, e^{-\lambda t} &(8)\\
F(t) &= 1 - e^{-\lambda t} &(9)\\
\mathrm{E}(\tau) &= 1/\lambda &(10)\\
\mathrm{Var}(\tau) &= 1/\lambda^2 &(11)
\end{aligned}
$$

The exponential distribution is 'memoryless'. For $s, t > 0$:

$$
\begin{aligned}
\mathbf{p}(\tau > s + t \mid \tau > t) &= \frac{1 - F(s+t)}{1 - F(t)}\\
&= \frac{e^{-\lambda(s+t)}}{e^{-\lambda t}}\\
&= e^{-\lambda s}\\
&= \mathbf{p}(\tau > s) &(12)
\end{aligned}
$$

The hazard rate function for the exponential distribution is:

$$
\begin{aligned}
h(t) &= \frac{f(t)}{1 - F(t)}\\
&= \frac{\lambda e^{-\lambda(t)}}{e^{-\lambda t}}\\
&= \lambda &(13)
\end{aligned}
$$

This follows from the 'memoryless' property.

1.1.1 Calibration

We can 'extract' λ from bond prices, default swap spreads, asset swap spreads, etc.

Notation:

$b(0,t)$	spot price of a risk-free zero couponn bond maturing at t
$V(0,T)$	spot price of a default risky bond maturing at T
c	annual coupon rate
R	recovery rate (fraction of par)
$S(0,T)$	spot spread for default swap with term T

Bond Prices:

Continuous coupons, default at any $t \in [0, T]$:

$$
\begin{aligned}
V(0,T) &= \int_0^T c\,[1 - F(t)]\, b(0,t)\,\mathrm{d}t + [1 - F(T)]\, b(0,T) + \int_0^T R\, f(t)\, b(0,t)\,\mathrm{d}t\\
&= c \int_0^T e^{-(r+\lambda)t}\mathrm{d}t + e^{-(r+\lambda)T} + R\lambda \int_0^T e^{-(r+\lambda)t}\mathrm{d}t\\
&= \frac{c + R\lambda}{r + \lambda} + \left[1 - \frac{c + R\lambda}{r + \lambda}\right] e^{-(r+\lambda)T} &(14)
\end{aligned}
$$

Discrete coupons (n per year), default on coupon dates only:

$$\begin{aligned} V(0,T) &= \frac{c}{n}\sum_{i=1}^{nT}[(1-F(t_i)]\,b(0,t_i) + [1-F(T)]\,b(0,T) \\ &\quad +R\sum_{i=1}^{nT}[F(t_i)-F(t_{i-1})]\,b(0,t_i) \end{aligned} \tag{15}$$

Default Swaps:

$$\begin{aligned} S(0,T)\int_0^T [1-F(t)]\,b(0,t)\,\mathrm{d}t &= \int_0^T (1-R)f(t)\,b(0,t)\,\mathrm{d}t \\ S(0,T)\int_0^T e^{-(r+\lambda)t}\,\mathrm{d}t &= (1-R)\lambda\int_0^T e^{-(r+\lambda)t}\,\mathrm{d}t \\ \lambda &= \frac{S(0,T)}{1-R} \end{aligned} \tag{16}$$

1.2 Time-varying Default Intensity

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March 1, 2006

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1 Reduced-form Default Models

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This implies:

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Bond Prices:

Continuous coupons, default at any $t \in [0, T]$:

$$
V(0,T) = \int_0^T c\,[1 - F(t)]\,b(0,t)\,\mathrm{d}t + [1 - F(T)]\,b(0,T) + \int_0^T R\,f(t)\,b(0,t)\,\mathrm{d}t \tag{15}
$$

If we assume that the risk-free rate is constant:

$$
\begin{aligned}
V(0,T) &= c\int_0^T e^{-(r+\lambda)t}\mathrm{d}t + e^{-(r+\lambda)T} + R\lambda\int_0^T e^{-(r+\lambda)t}\mathrm{d}t \\
&= \frac{c+R\lambda}{r+\lambda} + \left[1-\frac{c+R\lambda}{r+\lambda}\right]e^{-(r+\lambda)T} \qquad (16)
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Discrete coupons (n per year), default 'revealed' on coupon dates only:

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\begin{aligned}
V(0,T) &= \frac{c}{n}\sum_{i=1}^{nT}\left[(1-F(t_i)\right]b(0,t_i) + \left[1-F(T)\right]b(0,T) \\
&\quad + R\sum_{i=1}^{nT}\left[F(t_i)-F(t_{i-1})\right]b(0,t_i) \qquad (17)
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Default Swaps:

For deterministic recovery rates and continuously paid default swap spreads:

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\begin{aligned}
S(0,T)\int_0^T \left[1-F(t)\right]b(0,t)\,\mathrm{d}t &= \int_0^T (1-R)f(t)\,b(0,t)\,\mathrm{d}t \\
S(0,T)\int_0^T e^{-\lambda t}b(0,t)\,\mathrm{d}t &= (1-R)\lambda\int_0^T e^{-\lambda t}b(0,t)\,\mathrm{d}t \\
\lambda &= \frac{S(0,T)}{1-R} \qquad (18)
\end{aligned}
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1.1.2 Implications of constant default intensities

Consider a firm that has issued a number of zero coupon bonds, $V(0,t_1), V(0,t_2), \ldots V(0,t_n)$. Assume $R=0$ to keep things simple. For each of the n bonds, equation 17 becomes:

$$
V(0,t_i) = \left[1-F(t_i)\right]b(0,t_i) \quad i\in\{1,2,\ldots,n\} \qquad (19)
$$

since there is a single default intensity for the firm:

$$
\lambda = -\ln\left[\frac{V(0,t_i)}{b(0,t_i)}\right]\frac{1}{t_i} \quad \forall i \qquad (20)
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This implies a (very) restrictive relationship across the term structure of bond prices for any given issuer. We see the same thing if we consider this firm as the reference entity to a number of default swaps with varying terms to maturity. It is obvious from 18 that constant default intensity implies that the term structure of default swap spreads is 'flat'. In general, constant default intensity models imply constant credit spreads, or 'flat' credit spread term structures. There is no reason for this to be the case in the 'real' world.

1.2 Time-varying intensity

We can do a little better if we allow default intensities to be deterministic functions of time, $\lambda(t)$. Recall the definition of the hazard rate:

$$
\begin{aligned}
\lambda(t) &= \frac{f(t)}{1-F(t)} \\
&= -\frac{\mathrm{d}}{\mathrm{d}t}\ln\left[1-F(t)\right] \qquad (21)
\end{aligned}
$$

(22)

So the hazard rate (uniquely) determines $F(t)$:

$$\begin{aligned} \int_0^t \lambda(s)\,\mathrm{d}s &= -\ln\left[1 - F(t)\right] \\ F(t) &= 1 - \exp\left[\int_0^t \lambda(s)\,\mathrm{d}s\right] \end{aligned} \tag{23}$$

1.2.1 Calibration

Bond Prices:

Consider again a firm that has issued a number of zero coupon bonds, $V(0,t_1), V(0,t_2), \ldots V(0,t_n)$. Assume $R = 0$. For each of the n bonds:

$$V(0,t_i) = \left[1 - F(t_i)\right] b(0,t_i) \quad i \in \{1, 2, \ldots, n\} \tag{24}$$

Since default intensity is no longer required to be constant, we can extract n default intensities:

$$\lambda(0,t_i) = -\ln\left[\frac{V(0,t_i)}{b(0,t_i)}\right]\frac{1}{t_i} \tag{25}$$

We can rewrite equation 23 for discrete time intervals:

$$F(t_k) = 1 - \exp\left[\sum_{i=1}^{k} \lambda(t_{i-1}, t_i)(t_i - t_{i-1})\right] \tag{26}$$

We can now write survival probabilities as:

$$\begin{aligned} e^{-\lambda(0,t_3)t_3} &= e^{-\lambda(0,t_1)t_1}\, e^{-\lambda(t_1,t_3)(t_3-t_1)} \\ &= e^{-\lambda(0,t_2)t_2}\, e^{-\lambda(t_2,t_3)(t_3-t_2)} \\ &= e^{-\lambda(0,t_1)t_1}\, e^{-\lambda(t_1,t_2)(t_2-t_1)}\, e^{-\lambda(t_2,t_3)(t_3-t_2)} \quad \text{etc.} \end{aligned} \tag{27}$$

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For a term structure of default swap spreads, $S(0,t_1), S(0,t_2), \ldots, S(0,t_n)$, default intensities are:

$$\lambda(0,t_i) = \frac{S(0,t_i)}{1 - R} \tag{28}$$

The 'spot' hazard rates (intensities), $\lambda(0,t_i)$ are constant over the time intervals $(0,t_i)$. Hence we are specifying a 'step-function' or 'piece-wise' constant function for the 'forward' hazard rates, $\lambda(t_i, t_{i+1})$.

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March 8, 2006
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1.3 Stochastic Intensity

2 Dependence or Correlation

2.1 Default Event Dependence

(Read: Lucas (2003), Schönbucher (2003), pp. 289 - 301).

2.2 Default Time Dependence

2.2.1 Copula Functions

(Read: Li (2000), Schönbucher (2003), pp. 326 - 331).

3 Credit Instruments and Derivatives

Read:

3.1 "Single Name" Products

3.1.1 Asset Swaps

3.1.2 Total Return Swaps

3.1.3 Credit Default Swaps

Exotics

3.2 Correlation Instruments

3.2.1 Basket Default Swaps

3.2.2 Index Tranches

4 Reading

Giesecke, Kay, *Credit Risk Modeling and Valuation: An Introduction*, 2002. (Web site).

Li, David, *On Default Correlation: A Copula Function Approach*, RiskMetrics, 2000. (Web site).

Lucas, Douglas, N^{th} *to Default Swaps and Notes: All about Default Correlation*, UBS, 2003. (Web site).

Schönbucher, Phillip, *Credit Derivatives Pricing Models*, Wiley, 2003. (Reserve).

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O'Kane and Schlögl (2001), pp.13 - 30.

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Bond Prices:

Continuous coupons, default at any $t \in [0,T]$:

$$V(0,T) = \int_0^T c\,[1-F(t)]\,b(0,t)\,\mathrm{d}t + [1-F(T)]\,b(0,T) + \int_0^T R\,f(t)\,b(0,t)\,\mathrm{d}t \tag{15}$$

If we assume that the risk-free rate is constant:

$$\begin{aligned} V(0,T) &= c\int_0^T e^{-(r+\lambda)t}\mathrm{d}t + e^{-(r+\lambda)T} + R\lambda\int_0^T e^{-(r+\lambda)t}\mathrm{d}t \\ &= \frac{c+R\lambda}{r+\lambda} + \left[1-\frac{c+R\lambda}{r+\lambda}\right]e^{-(r+\lambda)T} \end{aligned} \tag{16}$$

Discrete coupons (n per year), default ‘revealed’ on coupon dates only:

$$\begin{aligned} V(0,T) &= \frac{c}{n}\sum_{i=1}^{nT}[(1-F(t_i)]\,b(0,t_i) + [1-F(T)]\,b(0,T) \\ &\quad +R\sum_{i=1}^{nT}[F(t_i)-F(t_{i-1})]\,b(0,t_i) \end{aligned} \tag{17}$$

Default Swaps:

For deterministic recovery rates and continuously paid default swap spreads:

$$\begin{aligned} S(0,T)\int_0^T[1-F(t)]\,b(0,t)\,\mathrm{d}t &= \int_0^T(1-R)f(t)\,b(0,t)\,\mathrm{d}t \\ S(0,T)\int_0^T e^{-\lambda t}b(0,t)\,\mathrm{d}t &= (1-R)\lambda\int_0^T e^{-\lambda t}b(0,t)\,\mathrm{d}t \\ \lambda &= \frac{S(0,T)}{1-R} \end{aligned} \tag{18}$$

1.1.2 Implications of constant default intensities

Consider a firm that has issued a number of zero coupon bonds, $V(0,t_1), V(0,t_2), \dots V(0,t_n)$. Assume $R=0$ to keep things simple. For each of the n bonds, equation 17 becomes:

$$V(0,t_i) = [1-F(t_i)]\,b(0,t_i) \quad i \in \{1,2,\dots,n\} \tag{19}$$

since there is a single default intensity for the firm:

$$\lambda = -\ln\left[\frac{V(0,t_i)}{b(0,t_i)}\right]\frac{1}{t_i} \quad \forall i \tag{20}$$

This implies a (very) restrictive relationship across the term structure of bond prices for any given issuer. We see the same thing if we consider this firm as the reference entity to a number of default swaps with varying terms to maturity. It is obvious from 18 that constant default intensity implies that the term structure of default swap spreads is 'flat'. In general, constant default intensity models imply constant credit spreads, or 'flat' credit spread term structures. There is no reason for this to be the case in the 'real' world.

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We can do a little better if we allow default intensities to be deterministic functions of time, $\lambda(t)$. Recall the definition of the hazard rate:

$$\begin{aligned} \lambda(t) &= \frac{f(t)}{1-F(t)} \\ &= -\frac{\mathrm{d}}{\mathrm{d}t}\ln\left[1-F(t)\right] \qquad (21) \\ & \qquad (22) \end{aligned}$$

So the hazard rate (uniquely) determines $F(t)$:

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1.2.1 Calibration

Bond Prices:

Consider again a firm that has issued a number of zero coupon bonds, $V(0,t_1), V(0,t_2), \ldots V(0,t_n)$. Assume $R = 0$. For each of the n bonds:

$$V(0,t_i) = \left[1-F(t_i)\right]b(0,t_i) \quad i \in \{1,2,\ldots,n\} \tag{24}$$

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$$\lambda(0,t_i) = -\ln\left[\frac{V(0,t_i)}{b(0,t_i)}\right]\frac{1}{t_i} \tag{25}$$

We can rewrite equation 23 for discrete time intervals:

$$F(t_k) = 1-\exp\left[\sum_{i=1}^{k}\lambda(t_{i-1},t_i)(t_i-t_{i-1})\right] \tag{26}$$

We can now write survival probabilities as:

$$\begin{aligned} e^{-\lambda(0,t_3)t_3} &= e^{-\lambda(0,t_1)t_1}\,e^{-\lambda(t_1,t_3)(t_3-t_1)} \\ &= e^{-\lambda(0,t_2)t_2}\,e^{-\lambda(t_2,t_3)(t_3-t_2)} \\ &= e^{-\lambda(0,t_1)t_1}\,e^{-\lambda(t_1,t_2)(t_2-t_1)}\,e^{-\lambda(t_2,t_3)(t_3-t_2)} \quad \text{etc.} \qquad (27) \end{aligned}$$

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For a term structure of default swap spreads, $S(0,t_1), S(0,t_2), \ldots, S(0,t_n)$, default intensities are:

$$\lambda(0,t_i) = \frac{S(0,t_i)}{1-R} \tag{28}$$

The 'spot' hazard rates (intensities), $\lambda(0,t_i)$ are constant over the time intervals $(0,t_i)$. Hence we are specifying a 'step-function' or 'piece-wise' constant function for the 'forward' hazard rates, $\lambda(t_i, t_{i+1})$.

1.3 Stochastic Intensity

2 Dependence or Correlation

2.1 Default Event Dependence

Reading

Goodman (2003).
Schönbucher (2003), pp. 289 - 301.

2.2 Default Time Dependence

2.2.1 Copula Functions

Reading

Li (2000).
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$C(u_1, \ldots, u_n)$ is a copula function if:

$C(u_1, \ldots, u_n) = \mathbf{p}(U_1 \leq u_1, \ldots U_n \leq u_n)$ ($C(.)$ is a distribution function)

$C(1, \ldots, 1, u_i, 1, \ldots, 1) = u_i$ ($C(.)$ has uniform marginals)

Sklar's Theorem (1959)

$$\begin{aligned} F(x_1, \ldots, x_n) &= C\left(F_1(x_1), \ldots, F_n(x_n)\right) \\ &= C(u_1, \ldots, u_n) \end{aligned} \tag{29}$$

If $F_1, \ldots, F_n$ are continuous, then $C(.)$ is unique.

Useful properties:

1. $C(.)$ is invariant to (strictly) increasing transformations.

2. $\max(u_1 + u_2 \ldots + u_n - n + 1, 0) \leq C(u_1, \ldots, u_n) \leq \min(u_1, \ldots, u_n)$

Product Copula

$$C(u_1, \ldots, u_n) = u_1 \cdot u_2 \cdot \ldots \cdot u_n \tag{30}$$

Gaussian Copula

$$\begin{aligned} C_{\mathbf{\Sigma}}(u_1, \ldots, u_n) &= C_{\mathbf{\Sigma}}\left(F_1(x_1), \ldots, F_n(x_n)\right) \\ &= \Phi^n_{\mathbf{\Sigma}}\left(\Phi^{-1}\left(F_1(x_1)\right), \ldots, \Phi^{-1}\left(F_n(x_n)\right)\right) \end{aligned} \tag{31}$$

Σ is the covariance matrix.

3 Credit Instruments and Derivatives

3.1 Single Reference Entity Products

3.1.1 Asset Swaps

3.1.2 Total Return Swaps

3.1.3 Credit Default Swaps

3.1.4 Exotics

3.2 Portfolio Products

3.2.1 Credit Indices

- Standardized portfolios of corporate and sovereign reference entities ('credits') with liquid markets for outstanding debt/CDS.
- Standardized documentation for unfunded credit default swap contracts referencing the indexes.
- Transparent rules based approach to portfolio construction and maintenance ('the roll').
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  Markit Group Ltd - index administration - data tracking and dissemination, oversees index rolls, day-to-day running of IndexCo.

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  Portfolio of 125 North American corporate reference entities, distributed among 5 industry sectors and 18 sub-sectors.

  All reference entities are investment grade (S&P and Moody's) at time of 'roll' (every March 20 and September 20).

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  Index composition identical across all maturities.

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- Dow Jones iTraxx Asia ex Japan - 30 most liquid (mostly investment grade) corporate reference entities.
- Dow Jones iTraxx Japan - 50 most liquid (mostly investment grade) corporate reference entities.
- Dow Jones iTraxx Australia - 25 most liquid (mostly investment grade) corporate reference entities.

3.3 Correlation Instruments

3.3.1 Basket Default Swaps

3.3.2 Index Tranches

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BUS 864
Class Notes
March 15, 2006

Anton Theunissen

Contents

I plan to update this document from time to time. Please check back regularly for the latest version.

1 Reduced-form Default Models

Reading

Giesecke (2002), pp. 15 - 24.
O'Kane and Schlögl (2001), pp.13 - 30.
O'Kane et al (2003), pp.31 - 52.

We assume that defaults (or 'credit events') 'arrive'/occur according to a Poisson process with some per unit time intensity, $\lambda > 0$. We can choose to have the intensity be constant, time-varying (but deterministic), or stochastic.

1.1 Constant Intensity

Defaults (or 'credit events') 'arrive'/occur according to a (homogeneous) Poisson process with constant per unit time intensity, $\lambda > 0$ (λ is the average number of occurences per unit time). We refer to a Poisson variable as $X \sim \text{Pois}(\lambda)$. X indicates the number of default events that occur in some interval $[0, t]$. For $t = 1$:

$$
\begin{aligned}
p(x) &= e^{-\lambda}\frac{\lambda^x}{x!} && (1)\\
P(x) &= \sum_{k=0}^{x} e^{-\lambda}\frac{\lambda^x}{x!} && (2)\\
\mathrm{E}(X) &= \lambda && (3)\\
\mathrm{Var}(X) &= \lambda && (4)\\
& && (5)
\end{aligned}
$$

For most of our purposes we will only be concerned with the first occurrence of a credit event. Let τ denote the time of the (first) credit event. Consider the probability that τ does not occur in the interval $[0, t]$ $(t > 0)$:

$$
\begin{aligned}
\mathbf{p}(\tau > t) &= e^{-\lambda t}\frac{(\lambda t)^0}{0!}\\
&= e^{-\lambda t} && (6)
\end{aligned}
$$

The probability that $\tau \in [0, t]$ is:

$$
\begin{aligned}
\mathbf{p}(\tau \le t) &= 1 - \mathbf{p}(\tau > t)\\
&= 1 - e^{-\lambda t} && (7)
\end{aligned}
$$

So default times are exponential random variables, $\tau \sim \mathrm{Exp}(\lambda)$. For $t \ge 0$:

$$
\begin{aligned}
f(t) &= \lambda\, e^{-\lambda t} && (8)\\
F(t) &= 1 - e^{-\lambda t} && (9)\\
\mathrm{E}(\tau) &= 1/\lambda && (10)\\
\mathrm{Var}(\tau) &= 1/\lambda^2 && (11)
\end{aligned}
$$

The exponential distribution is 'memoryless'. For $s, t > 0$:

$$
\begin{aligned}
\mathbf{p}(\tau > s + t \mid \tau > t) &= \frac{1 - F(s+t)}{1 - F(t)}\\
&= \frac{e^{-\lambda(s+t)}}{e^{-\lambda t}}\\
&= e^{-\lambda s}\\
&= \mathbf{p}(\tau > s) && (12)
\end{aligned}
$$

This implies:

$$
1 - F(s + t) = [1 - F(t)]\,[1 - F(s)] \qquad (13)
$$

The hazard rate function for the constant intensity exponential distribution is:

$$
\begin{aligned}
h(t) &= \frac{f(t)}{1 - F(t)}\\
&= \frac{\lambda e^{-\lambda(t)}}{e^{-\lambda t}}\\
&= \lambda && (14)
\end{aligned}
$$

This follows from the 'memoryless' property. In this document we will refer to $h(t)$ and $\lambda(t)$ interchangeably.

1.1.1 Calibration

If default intenstities are constant, we can 'extract' λ from a single bond price, default swap spread, asset swap spread, etc., for a given issuer or reference entity:

Notation:

$b(0,t)$	spot price of a risk-free zero coupon bond maturing at t
$V(0,T)$	spot price of a default risky bond maturing at T
c	annual coupon rate
R	constant recovery rate (fraction of par)
$S(0,T)$	spot par or 'break-even' spread for a default swap with term T

Bond Prices:

Continuous coupons, default at any $t \in [0,T]$:

$$V(0,T) = \int_0^T c\,[1-F(t)]\,b(0,t)\ \mathrm{d}t + [1-F(T)]\,b(0,T) + \int_0^T R\,f(t)\,b(0,t)\ \mathrm{d}t \tag{15}$$

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Discrete coupons (n per year), default 'revealed' on coupon dates only:

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For deterministic recovery rates and continuously paid default swap spreads:

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Consider a firm that has issued a number of zero coupon bonds, $V(0,t_1), V(0,t_2), \ldots V(0,t_n)$. Assume $R=0$ to keep things simple. For each of the n bonds, equation 17 becomes:

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Schönbucher (2003), pp. 289 - 301.

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3 Credit Instruments and Derivatives

Reading

O'Kane (2001).
O'Kane et al (2003), pp.1 -30.
Schönbucher (2003), chapter 2.

3.1 Single Reference Entity Products

3.1.1 Asset Swaps

Reading

Lando (2004), pp. 169 - 178. (Background on interest rate swaps).
O'Kane and Sen (2004), pp. 9 - 11.

Default-risky bond combined with a swap of coupons for a floating rate (risk-free rate + asset swap spread).

Par floater:

$$1 = \int_0^T r(t) b(0,t)\,\mathrm{d}t + b(0,T) \tag{32}$$

Par-par asset swap spread satisfies:

$$\begin{aligned}
1 + c\int_0^T b(0,t)\,\mathrm{d}t &= V(0,T) + \int_0^T [r(t) + S_a(0,T)]\, b(0,t)\,\mathrm{d}t \\
&= V(0,T) + S_a(0,T)\int_0^T b(0,t)\,\mathrm{d}t + \int_0^T r(t) b(0,t)\,\mathrm{d}t
\end{aligned} \tag{33}$$

Rearranging:

$$\begin{aligned}
S_a(0,T) &= \frac{1}{\int_0^T b(0,t)\,\mathrm{d}t}\left[1 - V(0,T) + c\int_0^T b(0,t)\,\mathrm{d}t - \int_0^T r(t) b(0,t)\,\mathrm{d}t\right] \\
&= \frac{1}{\int_0^T b(0,t)\,\mathrm{d}t}\left[c\int_0^T b(0,t)\,\mathrm{d}t + b(0,T) - V(0,T)\right]
\end{aligned} \tag{34}$$

3.1.2 Total Return Swaps

3.1.3 Credit Default Swaps

Reading

O'Kane and Turnbull (2003).

3.1.4 Exotics

3.2 Portfolio Products

3.2.1 Credit Indices

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- Standardized documentation for unfunded credit default swap contracts referencing the indexes.

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  14 sovereign issuers - e.g. Brazil, Russia, South Africa.

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- Dow Jones iTraxx Australia - 25 most liquid (mostly investment grade) corporate reference entities.

3.3 Correlation Instruments

3.3.1 Basket Default Swaps

3.3.2 Index Tranches

Reading

Kakodkar(2003).
O'Kane and Livesey (2004).

DJ.CDX.NA.IG

Same term structure of maturities as underlying index.

Five year and ten year maturities have greatest liquidity.

0% – 3% (‘Equity’)

3% – 7%

7% – 10%

10% – 15%

15% – 30%

4 References

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