Fixed Income
North America

Product Note

Correlation

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May 15, 2004

The Layman's Guide to Implied Correlation

After the stellar performance of 2003, fixed income has received a tremendous amount of attention. The credit market in particular has seen increased interest based both on its recent performance and its massive product innovation. In the past two years, traditional credit trading has gone from a long-only world to a two-sided, tiered, cancelable, callable, extendable opportunity set.

With the advent of the indexation product called Dow Jones TRAC-X[SM], the ability to trade credit in a liquid, diversified manner has shaken up the way credit trades. As a result of the standard liquid product, next-generation products such as options on credit and tranched credit risk have had the opportunity to flourish. The last time technology met liquidity in fixed income the CMO market was born and mortgage option adjusted spreads (OAS) became one of the key tools in defining relative value in the mortgage market.

Like OAS, implied correlation affords one the ability to think about relative value within a leveraged framework. For those familiar with options, a better analogy to implied correlation is implied volatility. Even if one has no inclination to trade structured credit, the powerful information that comes from potential flows merits taking the time to better understand this relatively new, but exploding market.

There has been so much focus recently on "correlation" in the credit space that the traditional corporate investor has been bombarded with terms like Gaussian copula, pair wise correlation and random variable generators. We hope to shed light on this topic in the most straightforward way possible but have tried to simplify – which always comes with a risk. One has to weigh precision against simplicity, and we hope that we achieved the right balance in this paper. Those wishing to find a more technically focused approach to understanding correlation should read Peter Cotton's report titled "What Is Correlation?" dated November 7, 2003.

In this Product Note, we first define correlation and briefly discuss its applicability to credit. Second, we present an example that introduces many of the key concepts discussed in depth within the paper. Third, in the heart of the paper, we walk through the mechanics of building your own simple correlation model[1] and, in so doing, hope to provide you with a strong intuitive understanding of correlation as it applies to the credit markets. And last, we provide a glossary of important terms.

[1] *One quick caveat, the "Gaussian copula" model that we discuss is not the only model used to price implied correlation, however, it is the framework most widely used by the Street.*

What Is Correlation?

The textbook definition of correlation is “a measure of association between two variables. If two variables tend to move up or down together, they are said to be positively correlated. If the variables tend to move in opposite directions, they are said to be negatively correlated.”

When you hear “correlation” in the credit markets, chances are it refers to the correlation of default between two credits, or even more precisely the correlation of time to default. Correlation of time to default measures the propensity of a default of one credit to be associated with increased or decreased probability of default of another credit.[2]

Estimating default correlation between companies or industry sectors is extremely difficult because of the lack of observable historical default data. Some proxies for default correlation include the correlation of spread movements and the correlation of stock prices. The imperfection of these measures combined with a desire for simplicity has led the Street to use one number for all pair wise correlations. For instance, in a basket with 5 names there are 10 distinct pair wise correlation numbers that could be inputs into a model (credit 1’s correlation with credits 2 thru 5, credit 2’s correlation with credits 3 thru 5, credit 3’s correlation with credits 4 and 5; and credit 4’s correlation with credit 5). As a result, the implied correlation that one might see on a trader’s run of a synthetic CDO tranche is more a measure of relative value than an actual indication of the trader’s view of correlation of defaults.

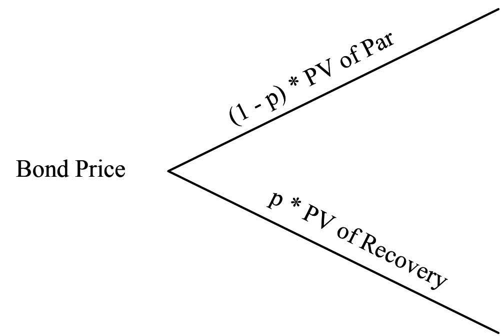
Default Correlation and a Simple CDO Example

We are now going to present a simple but powerful example highlighting correlation’s impact on a portfolio of credits, extracted from the Stanford Business School Case “TRAC-X Derivatives: Structured Credit Index Products and Default Correlation,” because it introduces many of the key concepts regarding correlation that we explore in the paper.

Assume there are two one-year zero-coupon bonds backing a collateralized debt obligation (CDO). Bond 1 has a par of \$100, a price of \$95 and a 50% recovery rate in the event of default. Bond 2 has a par of \$200, a price of \$179.5 and a 40% recovery rate in the event of default. Using a probability tree, the assumptions above, and a 2% interest rate we can solve for each bond’s implied default probability, “p,” as seen in Exhibit 1.

Exhibit 1

Probability Tree



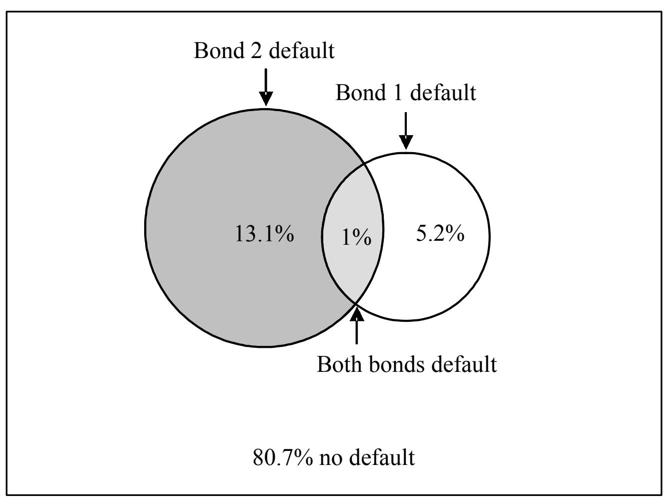
Source: Stanford Business School Case – “TRAC-X Derivatives: Structured Credit Index Products and Default Correlation”

For Bond 1: \$95 = ((1-p) * \$100/1.02) + (p * \$50/1.02), so its implied default probability is 6.2%. For Bond 2: \$179.5 = ((1-p) * \$200/1.02) + (p * \$80/1.02), so its implied default probability is 14.1%.

Using the Venn diagram shown in Exhibit 2 and assuming the probability of both notes defaulting at the same time is 1%, you can see that the probability of only Bond 1 defaulting is 5.2% (6.2% - 1%), the probability of only Bond 2 defaulting is 13.1% (14.1% - 1%) and the probability of neither note defaulting is 80.7% (100% - (13.1% + 5.2% + 1%)).

Exhibit 2

Venn Diagram



Source: Stanford Business School Case – “TRAC-X Derivatives: Structured Credit Index Products and Default Correlation”

[2] *The technical definition of default correlation is discussed in detail in CDO Insights, Understanding Mezzanine Notes (June 2002).*

Exhibit 3

Two Bond Scenario Analysis

Default Scenario:	Probability of each Default Scenario	Total CDO Payout for each Default Scenario	Expected Payout per Tranche = Σ (Payout per Tranche per Default Scenario x Probability of Default Scenario Occurring)		
			Senior Tranche ($220 principal)	*Mezzanine Tranche ($60 principal)*	*Junior Tranche ($20 principal)*
1. No default	80.7%	$300	$220 x .807 = $177.5	$60 x .807 = $48.4	$20 x .807 = $16.1
2. Bond 1 default only	5.2%	$250	$220 x .052 = $11.4	$30 x .052 = $1.6	$0 x .052 = $0.0
3. Bond 2 default only	13.1%	$180	$180 x .131 = $23.6	$0 x .131 = $0.0	$0 x .131 = $0.0
4. Both bonds default	1%	$130	$130 x .010 = $1.3	$0 x .010 = $0.0	$0 x .010 = $0.0
Expected Payout per Tranche			$213.8	$50.0	$16.1
Present Value of expected payout per tranche			$209.6	$49.0	$15.8
Yield per Tranche			220.0/209.6 – 1 = **4.9%**	60.0/49.0 – 1 = **22.4%**	20.0/15.8 – 1 = **26.6%**

Source: Stanford Business School Case – "TRAC-X Derivatives: Structured Credit Index Products and Default Correlation"

Let's now use these two bonds to create a $300 CDO divided into three tranches: a senior tranche ($220 par), a mezzanine tranche ($60 par), and a junior tranche ($20 par) and look at the probability-adjusted payoffs given our earlier assumptions (Exhibit 3).

Most of the data in Exhibit 3 are self-explanatory, but the "Total CDO Payout for each Default Scenario" is simply the payout under each of the four possible scenarios. For instance, if there are no defaults, the par of each bond is paid out, or $300. If Bond 1 defaults, the payout is the recovery (as shown in the previous example) on Bond 1 plus par of Bond 2, or $250 (($100-$50) + $200). If Bond 2 defaults, the payout is the recovery on Bond 2 plus par of Bond 1, or $180 (($200-$120) + $100). If both bonds default, the payout is the recovery on both, or $130.

Using this analysis one can now see how default correlation can impact the present value and yield of each tranche. Let's think through the intuition – what happens if the probability that both bonds default increases (analogous to an increase in default correlation)? According to our Venn diagram, the probability of only Bond 1 or Bond 2 defaulting must decrease and the probability of no defaults must increase. As a result, the value of the mezzanine and junior tranches increase and the values of the senior tranche decreases. The opposite happens if default correlation decreases.

There are a number of very important observations that we can draw from this example. First, as shown by Exhibit 1, the value of a bond is primarily a function of your view on two variables: the bond's default probability and its recovery rate. Other factors that impact the price that are not dependent on your "beliefs" are the timing of cash flows and the interest rate used to discount the cash flows. Second, the value of a portfolio of bonds is simply the sum of the value of the individual bonds. While this seems obvious, we have found that many people forget this financial truth when they begin learning about correlation – the value of an entire portfolio is independent of the default correlation between the bonds in the portfolio. Third, the only variable needed to price a tranche of a portfolio of bonds, besides the inputs required to price the individual bonds themselves, is the default correlation between the bonds. Correlation, as an input to a model, simply distributes risk among the tranches. And last, as default correlation increases, the price of junior tranches increases and the value of senior tranches decreases (price should not be confused with yield or spread, which both move in the opposite direction to price).

A Toolbox to Evaluate Structured Credit

We now transition into the main focus of the paper – a six-step process that provides the building blocks for creating your own analytics to "price" implied correlation and give you an intuitive understanding of correlation's impact on pricing behavior.

1. How to calculate the spread from implied probability of default on a single-name CDS

2. How to price an nth-to-default basket with a 0% implied correlation of default and introduce correlation's impact on pricing

3. How to price an nth-to-default basket with a 100% correlation of default, and continue our exploration of correlation's impact on pricing

4. How to use simulation to price Morgan Stanley's on-the-run High Quality Benchmark First -to-Default Basket

5. How to use simulation to price Morgan Stanley's on-the-run High Quality Benchmark First -to-Default Basket incorporating correlation between the credits into the analysis and comparing our results to actual levels

6. The final but highest quest: how to price the standard tranches on DJ TRAC-X NA Series 2, a 100-name credit default swap basket, and compare our results to actual levels

1. Calculating the Price of a Single Name CDS – Turning Probability into Spread

To understand how a tranche on a basket of credits is priced, we must first understand how a single-name credit default swap is priced.

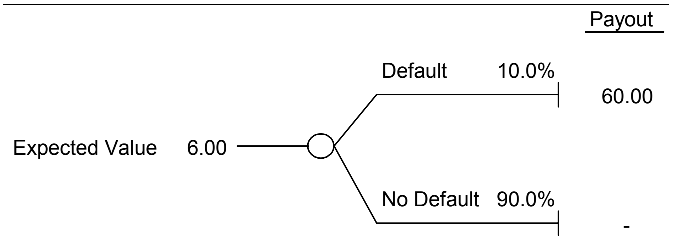
Let's begin by developing a simple three-step framework that we can use to price CDS. The CDS we price has a 5-year maturity, a 40% recovery rate, a 10% market implied probability of default, par of $100 and fair pricing.[3] Please note that the market tends to back into the implied default rate from market spread, but for ease of examples, we start with the implied default rate and work back to spread.

Step 1 – Expected Value Calculation

Let's draw a one-period "probability tree" of the two possible outcomes for a default protection buyer: 1) the underlying credit defaults, or 2) it does not default. If the credit defaults, the buyer of protection would receive the notional of the trade less the recovery ($100 less $40 = $60). If the credit does not default, the buyer of protection receives nothing. The expected value of what a buyer of protection would receive is the probability of default multiplied by the payout if a default occurs plus the probability of no default multiplied by the payout received if there is no default. In this case, the expected value is equal to $6.00 ($60 * 10% + $0 * 90%).

Exhibit 4

One-Period Probability Tree



Source: Morgan Stanley

Step 2 – Net Present Value Calculation

Let's now convert the $6.00 into a net present value (NPV). In doing so, we make two simplifying assumptions for the purposes of this paper: 1) because we are pricing the default swap using a one-period probability tree, if the credit defaults it does so halfway between years 0 and 5 (year 2.5), and 2) the annual interest rate to discount 2.5-year cash flows is 3%. With these assumptions, the net present value of the $6.00 expected value is $5.57. This is equal to (($100 – $40) * 10%) / ((1 + 3%) ^ (2.5)).

Step 3 – Annuity Calculation

We must now convert the $5.57 present value to a five-year annuity with quarterly payments. The reason this must be done is that the standard structure of a CDS contract pursuant to the ISDA 2003 definitions requires the buyer of protection to make quarterly payments as opposed to making one upfront payment. We do this by solving for S in Exhibit 5 below to get $0.317 (or 31.7 bps). Because CDS are quoted annually, we multiply the 31.7 bps by 4 to get 127 bps.

[3] *A CDS is priced fairly when the NPV of what the buyer pays is equal to the NPV of what the buyer expects to receive.*

Exhibit 5

Annuity Calculation

$$5.57 = \frac{S}{(1+.0509/4)^{0.25*4}} + \frac{S}{(1+.0509/4)^{0.50*4}} + \frac{S}{(1+.0509/4)^{0.75*4}} + ... + \frac{S}{(1+.0509/4)^{5.00*4}}$$

Source: Morgan Stanley

A very important item to note - we added a 2.09% "risk premium" to the 3% "risk free" interest rate to get to a 5.09% "risky rate." We must use a risky rate when solving for the annuity because the underlying credit may default prior to the CDS maturity, which would terminate the trade before all 20 payments were made. The risk premium in this case is equal to the one-year annual probability of default or (1 – ((1-0.1) ^ (1 / 5))). The $5.57 net present value can also be converted to an annuity in Excel by using the PMT function (-PMT(0.0509/4,20,5.57,0,0)).

It must be noted that there are a number of simplifying assumptions used in this framework. First, as opposed to the "single-stage" tree used above, most models use "multi-stage" trees that would break the 5-year period down to years, months, weeks or days (we discuss "multi-stage" trees in Section 5). It is important to note that the lower the default probability on a given credit, the closer the spread generated from a "single-stage" tree would be compared to that of a "multi-stage" tree, all else equal. Second, the implied cumulative probability of default up to a certain point in time is determined by the shape of a credit's credit curve. Given our one-stage tree, we implicitly assume that the credit curve is flat. Third, we used a flat interest rate curve for discounting, whereas most models use the swap curve.

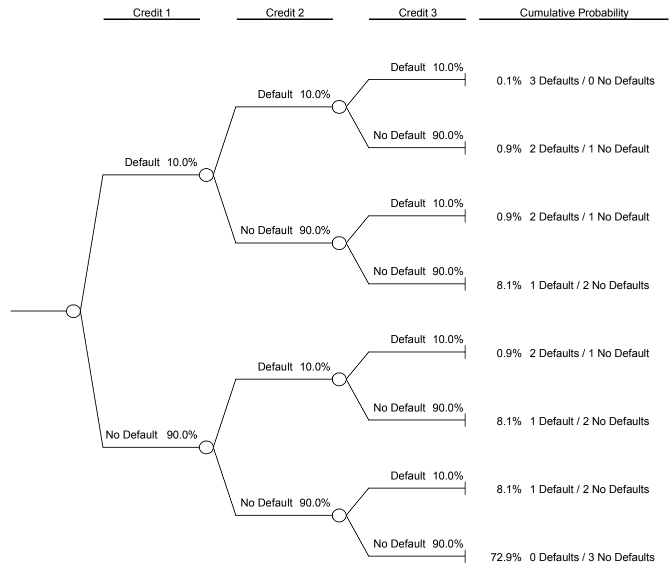
2. Nth-to-Default Pricing Intuition Using Probability Trees

The simple three-step framework that we developed above can also be used to develop intuition for pricing nth-to-default baskets. Let's assume we are pricing a homogenous basket of three credits and each credit has the same characteristics used in the example above (5-year maturity, 40% recovery rate, 10% probability of default and par of $100).

Given that the basket has three credits, there are three pair wise correlations. The lack of historical default information plus a desire to make first-to-default baskets as liquid and transparent as possible, led the Street to develop a standard whereby one correlation figure is used for all 3 pair wise correlations. Let's first price the nth-to-default tranches using a 0% correlation. The best way to think of 0% correlation between credits is that a default of one tells us nothing about the likelihood of the other's defaulting. The probability tree for this basket is shown in Exhibit 6.

Exhibit 6

3-Credit Nth-to-Default Probability Tree – 0% Correlation



Source: Morgan Stanley

First-to-Default Pricing Intuition Using Probability Trees

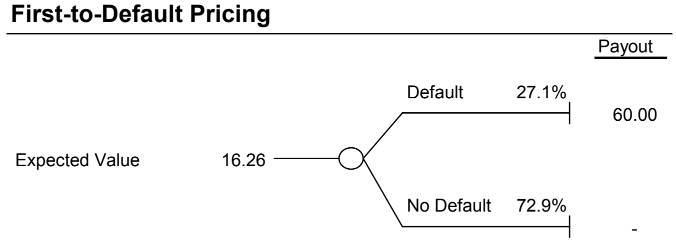
A buyer of protection in a first-to-default basket is paid if one (or more) of the credits in the basket defaults. The tree above shows the eight possible distinct outcomes, as well as the probability of each. As you can see, each of the top seven paths has at least one default; therefore, the probability of one or more defaults is 27.1% (100% - 72.9%).

Step 1 … Again

We can now use the simple framework that we developed for pricing a single-name credit default swap to price this first-to-default basket. As we determined above, a buyer of protection in this first-to-default basket has a 27.1% chance of being paid and a 72.9% chance of not being paid. As Exhibit 7 shows, the expected value of what the buyer of protection would receive is then $16.26 (27.1% * $60 + 72.9% * $0).

Exhibit 7

First-to-Default Pricing



Source: Morgan Stanley

Steps 2 and 3 … Again

We now repeat Steps 2 and 3, which we used to price a single-name CDS.

Step 2 (NPV Calculation): Assuming an interest rate of 3% and a payment of $16.26 halfway between time 0 and the maturity, the net present value of what a buyer would expect to receive is $15.10. This is equal to ((par – recovery) * probability of at least 1 default) / ((1 + interest rate) ^ (expected time of default)), or ((100 – 40) * .271) / ((1 + .03) ^ (2.5)).

Step 3 (Annuity Calculation): We now must convert the $15.10 so that it is equivalent to a standard CDS contract – quarterly payments quoted in basis points paid annually. Once again, we can do this in Excel using the PMT function: (-PMT(0.0913/4,20,15.10,0,0)*400) = a premium of 380 basis points per annum. The 0.0913 is equal to 0.03 plus the annual probability of default of 0.0613, or (1 - (1 - 0.271) ^ (1 / 5)).

2nd- and 3rd-to-Default Pricing Intuition Using Probability Trees

A buyer of protection in a second-to-default basket is paid if two or more of the credits in the basket default. As shown in Exhibit 6, four paths lead to this result and this occurs with a 2.8% probability (0.1% + 0.9%+ 0.9% + 0.9%). Using the same pricing method as before, a 2.8% probability of being paid results in an expected value of $1.68, an NPV of $1.56 and a premium of 34 basis points per annum.

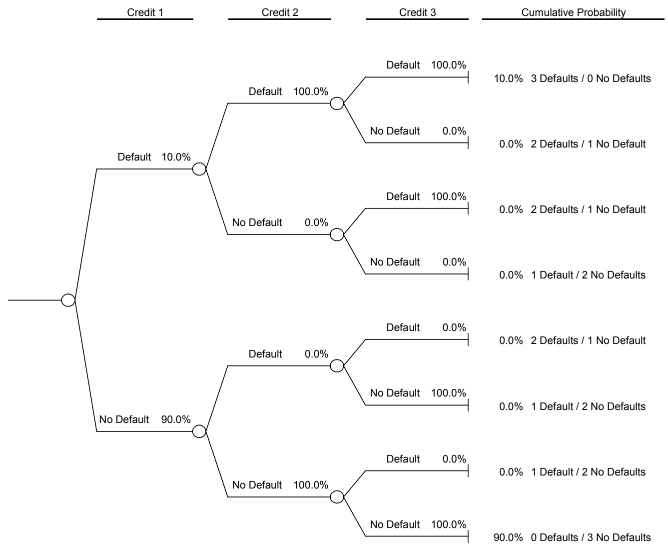
A buyer of protection in a third-to-default basket is paid if three of the credits in the basket default. As shown in Exhibit 6, one path leads to this result and occurs with a 0.1% probability. As before, a 0.1% probability of being paid results in an expected value of $0.06, an NPV of $0.06 and a premium of 1 basis point per annum.

3. Nth-to-Default and 100% Correlation

What happens to our framework if we assume that the three credits are 100% correlated? Correlation of 100% between the credits means that, if one credit defaults, the other two credits *MUST* default. The probability tree shown in Exhibit 6 now becomes Exhibit 8.

Exhibit 8

3-Credit Nth-to-Default Probability Tree – 100% Correlation



Source: Morgan Stanley

The key takeaway from this diagram is that the probability of at least 1 default is equal to the probability of at least 2 defaults is equal to the probability of at least 3 defaults, which is 10.0%. Going through steps 1, 2 and 3 of the framework we have established, the expected value, net present value and premium on a first-to-default, second-to-default or a third-to-default basket is $6.00, $5.57 and 127 basis points, respectively.

In the two prior examples we have used the extreme correlation assumptions of 0% and 100%, and assumed identical pricing for each credit. In Morgan Stanley's benchmark first-to-default baskets, implied correlations generally range between 30% and 80% and comprise CDS with a wide range of premiums. Calculating the premiums of nth-to-default baskets in which the correlation is somewhere between 0% and 100% using an analytical method is very challenging. As a result, Monte Carlo simulation is the method most frequently used to price such baskets. In Section 4, we will use Monte Carlo simulation to price Morgan Stanley's High Quality First-to-Default Basket using a 0% implied correlation and in Section 5 we use Monte Carlo simulation to price the same basket using a market-based implied correlation.

Intuition for Pricing Behavior

To begin developing an understanding of how correlation affects pricing behavior let's take a minute to compare the premiums from the 0% correlation basket and the 100% correlation basket.

Exhibit 9

Correlation's Impact on Nth-to-Default Pricing

	Spread		
	First-to-Default	Second-to-Default	Third-to-Default
0% Correlation	380	34	1
100% Correlation	127	127	127

Source: Morgan Stanley

In the first-to-default tranche, spread decreases as implied correlation increases. In the second- and third-to-default tranches, the opposite happens, the spread increases as implied correlation rises. From this we can make a generalization that applies to most nth-to-default and loss baskets: in subordinate tranches, the spread/risk decreases as implied correlation increases, and in senior tranches the spread/risk increases as implied correlation rises.

It must also be noted that the implied correlation from our model effectively serves to allocate the total spread from the 3 credits among the various tranches. The total spread from the three credits is 381. In a basket with 0% correlation, the first-to-default tranche gets the vast majority of the spread, the second-to-default tranche the next most and so on. In a basket with 100% correlation, all equally-sized tranches get an equal share of the total spread. The total spread under different correlation scenarios (e.g., 415 and 381 in the 0% and 100% correlation scenarios in Exhibit 9 above, respectively) need not sum to the same number because the simulated timing of the defaults occur at different times in the two different scenarios.

4. Pricing Morgan Stanley's Benchmark High Quality First-to-Default Basket Using Simulation

Morgan Stanley's High Quality First-to-Default Basket includes AIG, Fannie Mae ("FNM"), GE Capital Corp ("GECC"), Pfizer ("PFE") and Wal-Mart ("WMT"). Each week, Morgan Stanley research shows various standard FTD baskets; we will use pricing from Sivan Mahadevan's strategy piece, Tailoring Baskets – One Size Doesn't Fit All, published on April 16, 2004 (page 13). The basket has an effective date of April 15, 2004, and a maturity date of June 20, 2009. A buyer of protection pays the premium on a quarterly basis on standard ISDA payment dates (March 20, June 20, September 20 and December 20). We assume that each CDS has a notional of $100 and a recovery of 40% (Exhibit 10), and we discount cash flows using an interest rate of 3.84% (the five-year swap rate on April 20, 2004).

Exhibit 10

High Quality First-to-Default Basket Characteristics – April 15, 2004

Credit	5-Year Spread (bps)	Notional	Recovery (%)
Fannie Mae	24	100.00	40.00
AIG	25	100.00	40.00
GE Capital Corp.	33	100.00	40.00
Pfizer	15	100.00	40.00
Wal-Mart	19	100.00	40.00
Totals	116	500.00	

Source: Morgan Stanley

Pair Wise Correlation

Given that the basket has 5 credits, there are, in theory, 10 pair wise correlations. As previously discussed, the Street uses one correlation number for all pair wise correlations. Let's first price the first-to-default basket using a 0% correlation. We will see exactly where correlation enters into the pricing calculation when we price a "correlated" basket in Section 5.

Random Variable Generation

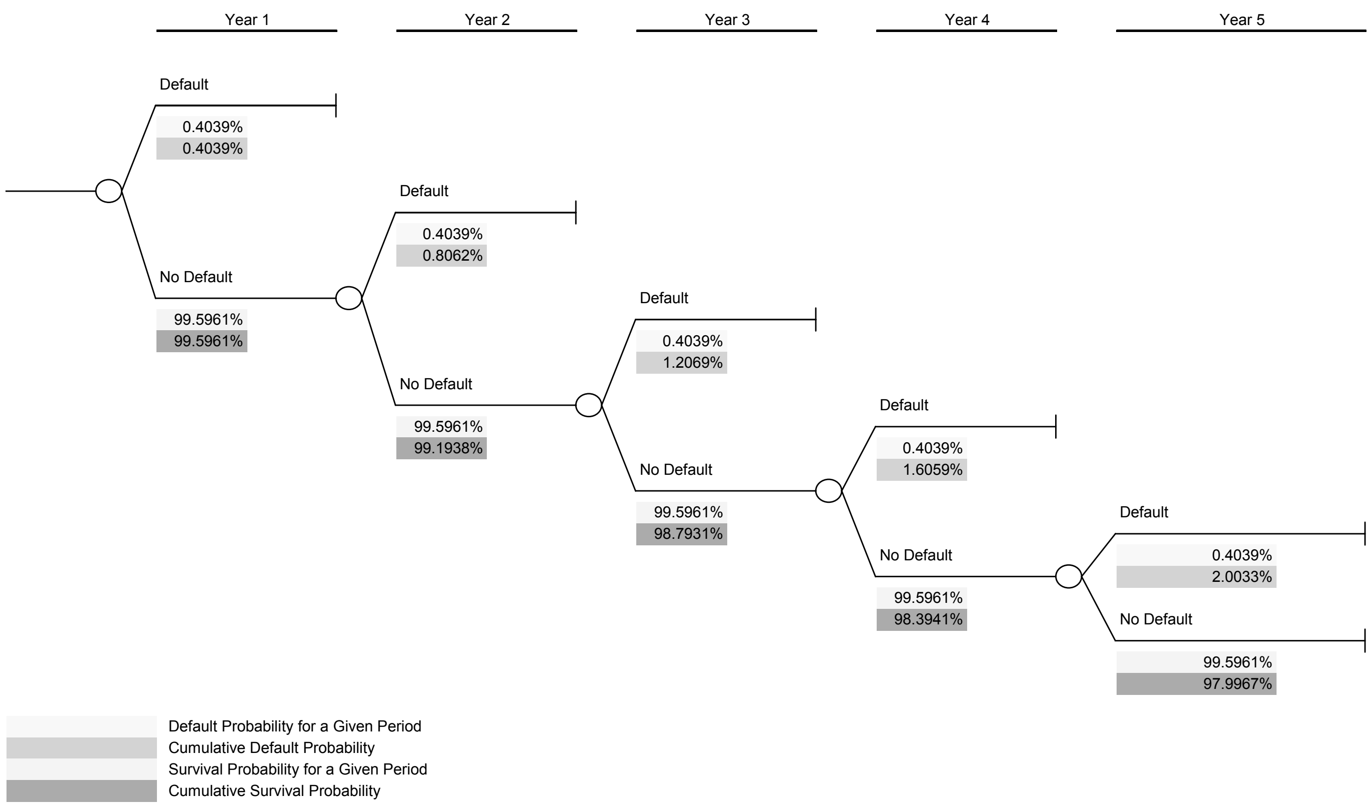
How does one "simulate" one possible future outcome? This can be done in Excel by using the random number generator function, =rand(), which generates a uniform random number between 0 and 1. Uniform means that there is an equal probability of generating any number between 0 and 1 – this implies that if we were to generate a million random numbers, we would expect to see 100,000 of the numbers fall between 0 and 0.1, 200,000 between 0.1 and 0.3, 300,000 between 0.4 and 0.7, and so on.

Cumulative Default Probability

For each credit in the basket, we generate a separate random number. Let's assume the random number generated for FNM is 0.012069. What does the 0.012069 tell us? We will define the random number to represent the cumulative default probability of FNM at a certain point in time. We use Exhibit 11 to help explain what cumulative default probability means.

Exhibit 11

Default Probabilities by Year Assuming a Flat Credit Curve



Source: Morgan Stanley

The numbers in the first row of each column are FNM's probabilities of default for a given year – the 0.4039% indicates that FNM has a 0.4039% chance of defaulting in any given year and the fact that these probabilities are the same in each year implies that FNM has a flat credit curve. The numbers in the second row of each column are FNM's cumulative default probabilities – these must increase in each successive period because FNM's probability of default for each period is greater than 0. The numbers in the third row of each column are the survival probabilities for a given period and are simply 1 less the default probabilities for a given period. The numbers in the fourth row of each column are the cumulative survival probabilities and are simply 1 less the cumulative default probabilities. A random number of 0.012069 tells us that, in this one simulated scenario, FNM defaults at the end of year 3, because at year 3 the cumulative default probability is 0.012069.

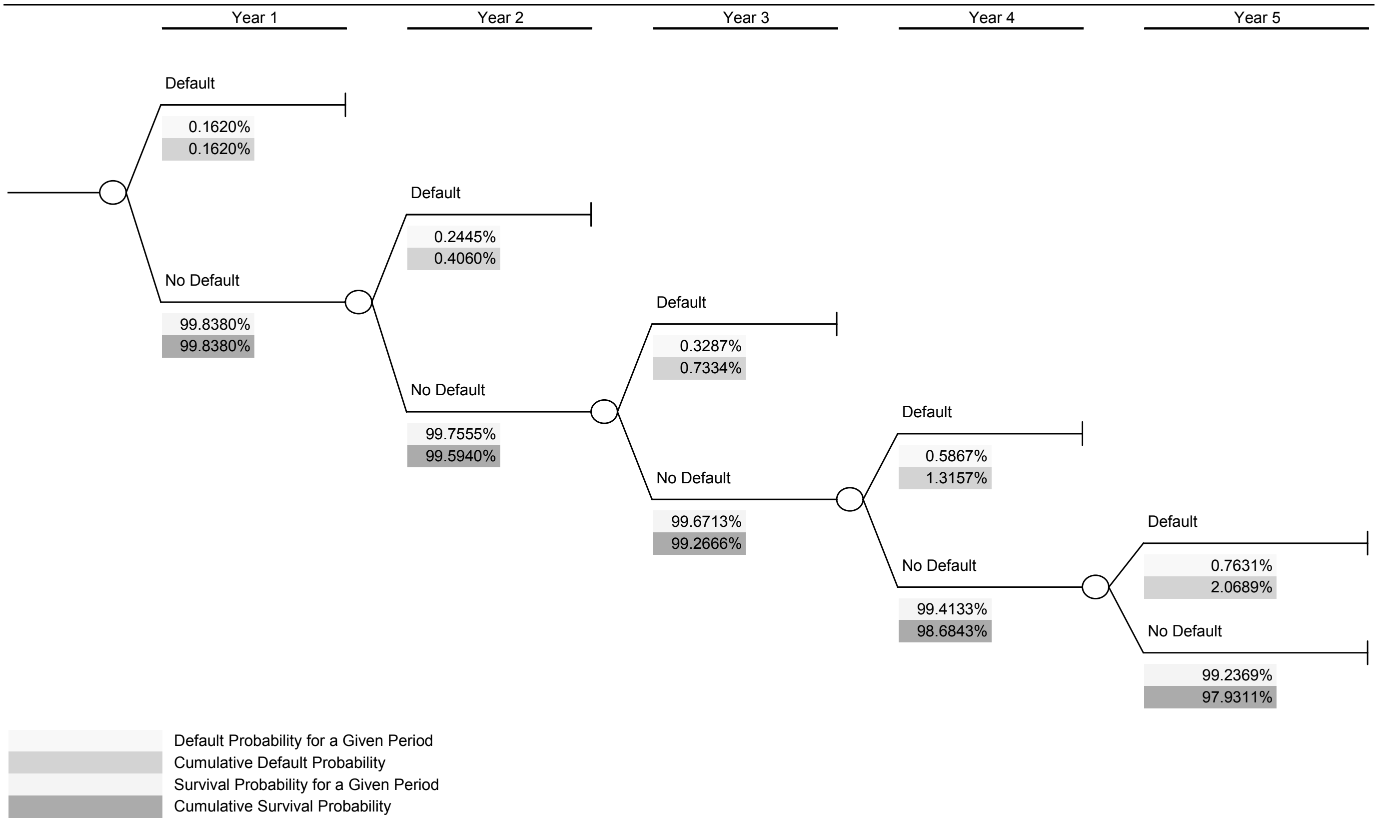
It is important to note that we use a more precise "multi-stage" tree process for calculating the default probabilities by year in Exhibits 11 and 12 than the 3-step framework established in Section 1. This process, known as "bootstrapping a credit curve," is discussed on the next two pages. We use a more precise process because credit curve shape materially affects default probabilities and the Street typically uses non-flat curves in pricing nth-to-default and other tranched products.

Probability and a "Non-Flat" Credit Curve

Morgan Stanley's benchmark first-to-default baskets are not priced with flat credit curves. How would the chart in Exhibit 11 change if we assumed that the 1-year, 2-year, 3-year and 4-year spreads for Fannie Mae were 40%, 50%, 60%, and 80% of the 5-year spread, respectively? Incorporating this non-flat credit curve into a new version of Exhibit 11 results in Exhibit 12 below.

Exhibit 12

Default Probabilities by Year Assuming a Non-Flat Credit Curve



Source: Morgan Stanley

So what is the impact of this non-flat credit curve on our analysis of FNM's time to default? In our flat curve example, a random number of 0.012069 meant that FNM defaulted at year 3.00. In Exhibit 12, we see that 0.012069 falls between the year 3 and year 4 cumulative default probabilities. Using simple linear interpolation, a random number of 0.012069 indicates that FNM defaults at time 3.81 (3 + ((0.012069 - 0.007334)/(.013157-.007334))).

The process for calculating default probabilities by year as done in Exhibits 11 and 12 is discussed next.

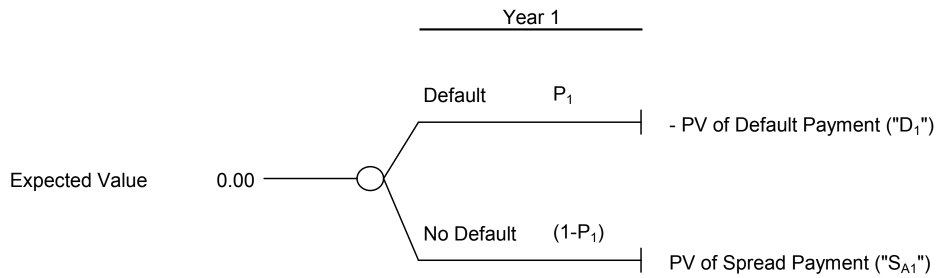
Bootstrapping a Credit Curve

The probabilities in Exhibits 11 and 12 were calculated by bootstrapping FNM's credit curve - let's walk through the calculations underlying Exhibit 12 in order to understand the process by which these probabilities were calculated.

Bootstrapping a credit curve is a "multi-year" (or "multi-stage" – can be more granular) process, by which we start with year 1 and then move to year 2, and so on. Exhibit 13 below shows the process for calculating the year 1 probabilities.

Exhibit 13

Bootstrapping the Credit Curve – Calculation of Year 1 Probabilities



Equations Used to Solve for P_1

Equation 1:	$0 = -P_1D_1 + (1 - P_1)S_{A1}$
Equation 2:	$P_1 = S_{A1} / (D_1 + S_{A1})$

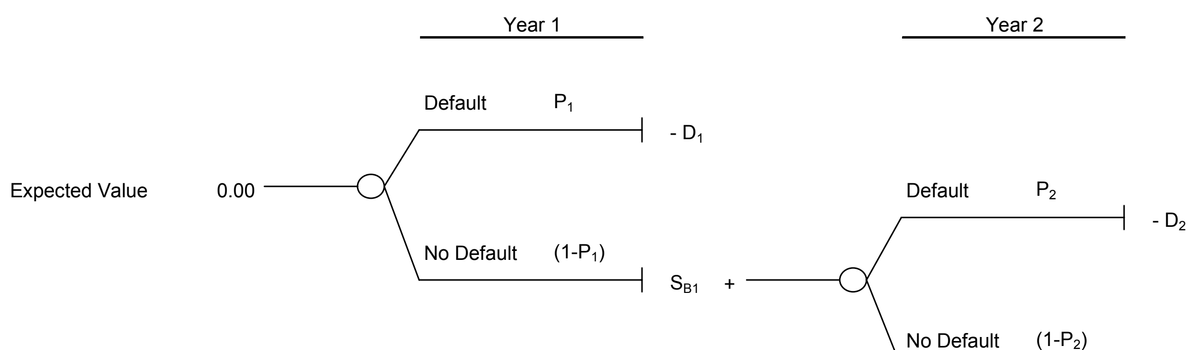
Source: Morgan Stanley

"P_1" is the year 1 probability of default, "D_1" is the present value of the net payment in the event of a default, and "S_{A1}" is the premium payment for a one year FNM CDS (the one year FNM spread is 9.6 bps). As can be seen from Equation 1, the opposite of the present value of a default payment in year 1 multiplied by the probability of a default in year 1 plus the premium payment for a 1-year CDS on FNM multiplied by 1 less the probability of default in year 1 must equal $0. Equation 1 simplifies to Equation 2. Continuing with our 40% recovery rate assumption and an interest rate of 3.84%, the 1 year probability and cumulative probability of default for FNM are both 0.1620%. The year 1 probability and cumulative probability of survival are simply 1 less the year 1 probabilities of default or 99.838%.

Now that we have calculated the year 1 probabilities we can move on to the year 2 probabilities.

Exhibit 14

Bootstrapping the Credit Curve – Calculation of Year 2 Probabilities



Equations Used to Solve for P_2

Equation 1:	$0 = ((-P_2D_2 + (1 - P_2)S_{B2} + S_{B1}) * (1-P_1)) - P_1D_1$
Equation 2:	$P_2 = ((P_1D_1 / (1-P_1)) - S_{B1} - S_{B2}) / (-D_2-S_{B2})$

Source: Morgan Stanley

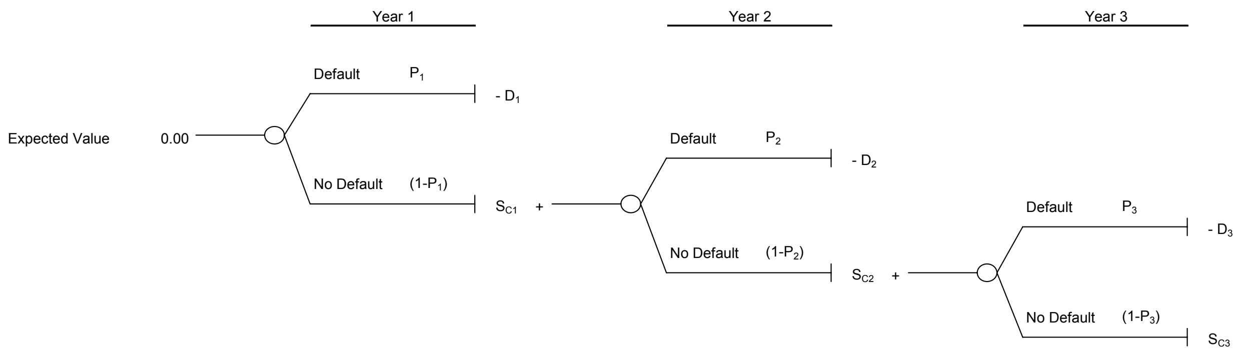
"D" continues to represent the present value of the net payment in the event of an FNM default (D_1 is for year 1 and D_2 is for year 2), and "S_{B1}" and "S_{B2}" are the present values of the premium payment for a two year FNM CDS in years 1 and 2, respectively (the two year FNM spread is 12 bps).

We know every variable in Exhibit 14 except for the default probability for year 2 ("P_2") and we know that the expected value must equal $0. As such, we simply need to solve for P_2 in Equation 1. While this Equation may look intimidating, it easily simplifies into equation 2. From the simplified equation we determine that P_2 is 0.2445%. The survival probability for year 2 is 1 less 0.2445% or 99.7555%. The cumulative survival probability is the probability of survival in year 1 multiplied by the probability of survival in year 2, or 99.594%. The cumulative default probability is 1 less the cumulative survival probability or 0.4060%.

We won't continue this process out to year 5, but let's do one more year to make sure we fully understand the bootstrapping concept. Below is the diagram for solving the year 3 probability of default.

Exhibit 15

Bootstrapping the Credit Curve – Calculation of Year 3 Probabilities



Equations Used to Solve for P_3

Equation 1:	$0 = (((((-P_3D_3 + (1 - P_3)S_{C3} + S_{C2}) * (1-P_2)) - P_2D_2) + S_{C1}) * (1-P_1)) - P_1D_1$
Equation 2:	$P_3 = ((((P_1D_1 / (1-P_1)) - S_{C1} + P_2D_2) / (1-P_2)) - S_{C2} - S_{C3}) / (-D_3 - S_{C3})$

Source: Morgan Stanley

The notation is the same as in the prior two Exhibits, we simply added another year. Also like Exhibits 13 and 14, we know every variable except for the final year probability of default, in this case "P_3", and we know that the expected value must equal $0. As such, we simply need to solve for P_3 in Equation 1, which is easily done by isolating P_3 as shown in Equation 2. From the simplified equation we determine that P_3 is equal to 0.3287%. The survival probability for year 3 is 1 less 0.3287% or 99.6713%. The cumulative survival probability is the probability of survival in year 3 multiplied by the cumulative probability of survival in year 2, or 99.2666%. The cumulative default probability is 1 less the cumulative survival probability or 0.7334%.

Quick Calculation for Simulated Time to Default

How do we calculate time to default without having to put together a chart similar to the one in Exhibits 11 or 12. Let's answer with an example. Continuing with FNM and assuming a flat credit curve, what is the time to default implied by a random number of 0.3825?

Step 1: The 5-year cumulative survival probability for FNM is 97.9967%. The flat credit curve assumption implies a *daily* survival probability of 99.998891% (97.9967% ^ (1 / (365 * 5).

Step 2: Remembering that the random number represents the cumulative default probability, let's convert the random number to a cumulative survival probability. We do this by simply subtracting 1 from it to get 0.6175.

Step 3: Let's now convert the cumulative survival probability to the simulated time of default. 99.998891% ^ x days must equal 0.6175. Therefore, in Excel, log (0.6175,.99998891) = x = 43,475 days or 119.1 years, which means FNM does not default in the 5-year period in this simulation scenario.

A One-Scenario Simulation

We are now ready to run through a simulation of one scenario in order to price the various nth-to-default tranches of the High Quality Basket.

As we can see from Exhibit 16, each credit in the first-to-default basket is assigned a uniform random variable – the formula in each of these cells is the same ("rand()") and each time you hit "F9," Excel assigns a new uniform random variable to the cell.

As we walk through above, we can see that the uniform random variable assigned to each credit indicates when the credit will default. In this simulation, one credit, PFE, with a simulated default time of 2.45 years, defaults prior to the end of the term of the nth-to-default trade, which is approximately 5 years.

Exhibit 16

One Scenario of the Simulation

	General Information						Time to Default Correlation			
Credit	5-Year Spread (bps)	5-Year Default Probability	3-Year Default Probability	1-Year Default Probability	Notional	Recovery (%)	Uniform Random Variable	Time-to-Default (Years)	Defaults Before Maturity	Losses
Fannie Mae	24	2.07%	0.73%	0.16%	100.00	40.00	0.4300	134.60	No	-
AIG	25	2.15%	0.76%	0.17%	100.00	40.00	0.0510	12.03	No	-
GE Capital Corp.	33	2.83%	1.01%	0.22%	100.00	40.00	0.8770	364.79	No	-
Pfizer	15	1.30%	0.46%	0.10%	100.00	40.00	0.0036	2.45	Yes	60.00
Wal-Mart	19	1.64%	0.58%	0.13%	100.00	40.00	0.6550	321.98	No	-
Totals	**116**				**500.00**					**60.00**

Note: default probabilities are calculated assuming that 4-year, 3-year, 2-year and 1-year spreads are 80%,60%, 50% and 40% of 5-year spreads.
Source: Morgan Stanley

Because there was only one default in this scenario, only one of the nth-to-default tranches experiences a loss – the first-to-default tranche. It records a loss of $60 because the notional of Pfizer was $100 and we assumed a 40% recovery rate. The net present value of the $60 loss is $54.72 (60 / (1+.0384) ^ 2.45). The losses and the corresponding present values for each nth-to-default tranche for this one simulation are listed in Exhibit 17.

Exhibit 17

Nth-to-Default Losses in One Scenario of Simulation

	Nth-to-Default Size, Losses and PV of Losses				
Nth-to-Default Tranche	1	2	3	4	5
Tranche Size	100	100	100	100	100
Tranche Losses	60	–	–	–	–
PV of Losses	54.72	–	–	–	–

Source: Morgan Stanley

Calculating Nth-to-Default Premiums for the Scenario

If nth-to-default baskets were structured so that the buyer of protection made one upfront payment, we could simply run thousands of simulations in which we recorded the present value of losses for each tranche. We would then average the present value of losses for each tranche to calculate the amount that a buyer would pay at the inception of the trade. However, nth-to-default baskets, like CDS, are not structured this way. Instead, the buyer of protection makes quarterly payments until the earlier of a credit event that affects the tranche or the expiration of the trade. The implication of this structure is that the timing of defaults affects nth-to-default pricing.

The net present value of what a buyer of protection expects to pay is equal to the net present value of what the buyer expects to receive (the "PV of Losses" in Exhibit 17). Therefore, the sum of the present values of the quarterly payments that the buyer of protection makes must equal the present value of the tranche losses, or $54.72 for the first-to-default tranche in this case.

We express this mathematically for the first-to-default tranche in the equation in Exhibit 18. "S" is the quarterly premium paid by the buyer of protection, "AB" is the average notional balance of the tranche for the quarter and 0.0384 is the interest rate (the 5-year swap rate on April 20, 2004), used for discounting. We do not add a risk premium to this interest rate as we did in the 3 step framework created in Section 1 because our simulation takes into account the fact that the trade may terminate before all payments are made (the average balance can go to $0).

Because the maturity date of the basket is June 20, 2009, the term of the trade is 5.18 years. Although payment dates are quarterly, they are not "on quarters" (.25, .50, .75, etc.) because we enter into the trade on April 15 and the first payment date is June 20 (standard ISDA payment dates). Therefore, the last payment date is 5.18 years and the first payment date is 0.18 years.

Like a bondholder, the buyer of protection receives accrued interest at the inception of the trade. This is because the buyer pays for protection at year 0.18 as if he received protection for the entire quarter (0.25 years) when in fact he received protection for only 0.18 years. The first term in the equation shown in Exhibit 18 reflects the accrued interest payment.

Exhibit 18

Calculation of Quarterly Premiums and Accrued Interest

$$54.72=\frac{S\times AB_{0.00}\times(.18-.25)}{.25}+\frac{S\times AB_{0.18}}{(1+.0384/4)^{0.18*4}}+\frac{S\times AB_{0.43}}{(1+.0384/4)^{0.43*4}}+....+\frac{S\times AB_{5.18}}{(1+.0384/4)^{5.18*4}}$$

Source: Morgan Stanley

By isolating the premium, "S," in the equation in Exhibit 18 we get the new equation in Exhibit 19:

Exhibit 19

Isolating the Premium

$$S = 54.72 \div \left(\frac{AB_{0.00} \times (0.18 - 0.25)}{0.25} + \frac{AB_{0.18}}{(1+.0384/4)^{0.18*4}} + \frac{AB_{0.43}}{(1+.0384/4)^{0.43*4}} + \ldots + \frac{AB_{5.18}}{(1+.0384/4)^{5.18*4}} \right)$$

Source: Morgan Stanley

The next step is to determine the average balance ("AB") at each payment date. Because there are no defaults prior to year 2.43, the average balance for the first-to-default tranche through year 2.43 is $100. At year 2.45, PFE defaults resulting in an average balance of $6.37 for the quarter ending at year 2.68. The calculation is (((2.45 – 2.43)/(2.68 – 2.43)) * $100). Please note that due to rounding, you will not get this calculation to match exactly. Time periods 2.93 through 5.18 have an average balance of $0 because the first-to-default trade has ended due to the occurrence of the first default. The second- through fifth-to-default tranches in this one scenario have average balances of $100 for the term of the trade because there was only one default. Exhibit 20 summarizes the average balances for all tranches and their respective present values for this one scenario.

Exhibit 20

Average Tranche Balance and Present Value of Average Balance

	Nth-to-Default Average Balance Information									
	Average Tranche Balance					Discounted Average Tranche Balance				
Time	1	2	3	4	5	1	2	3	4	5
-	100.00	100.00	100.00	100.00	100.00	(27.67)	(27.67)	(27.67)	(27.67)	(27.67)
0.18	100.00	100.00	100.00	100.00	100.00	99.31	99.31	99.31	99.31	99.31
0.43	100.00	100.00	100.00	100.00	100.00	98.37	98.37	98.37	98.37	98.37
0.68	100.00	100.00	100.00	100.00	100.00	97.43	97.43	97.43	97.43	97.43
0.93	100.00	100.00	100.00	100.00	100.00	96.51	96.51	96.51	96.51	96.51
1.18	100.00	100.00	100.00	100.00	100.00	95.59	95.59	95.59	95.59	95.59
1.43	100.00	100.00	100.00	100.00	100.00	94.68	94.68	94.68	94.68	94.68
1.68	100.00	100.00	100.00	100.00	100.00	93.78	93.78	93.78	93.78	93.78
1.93	100.00	100.00	100.00	100.00	100.00	92.89	92.89	92.89	92.89	92.89
2.18	100.00	100.00	100.00	100.00	100.00	92.00	92.00	92.00	92.00	92.00
2.43	100.00	100.00	100.00	100.00	100.00	91.13	91.13	91.13	91.13	91.13
2.68	6.37	100.00	100.00	100.00	100.00	5.75	90.26	90.26	90.26	90.26
2.93	-	100.00	100.00	100.00	100.00	-	89.40	89.40	89.40	89.40
3.18	-	100.00	100.00	100.00	100.00	-	88.55	88.55	88.55	88.55
3.43	-	100.00	100.00	100.00	100.00	-	87.71	87.71	87.71	87.71
3.68	-	100.00	100.00	100.00	100.00	-	86.88	86.88	86.88	86.88
3.93	-	100.00	100.00	100.00	100.00	-	86.05	86.05	86.05	86.05
4.18	-	100.00	100.00	100.00	100.00	-	85.23	85.23	85.23	85.23
4.43	-	100.00	100.00	100.00	100.00	-	84.42	84.42	84.42	84.42
4.68	-	100.00	100.00	100.00	100.00	-	83.62	83.62	83.62	83.62
4.93	-	100.00	100.00	100.00	100.00	-	82.82	82.82	82.82	82.82
5.18	-	100.00	100.00	100.00	100.00	-	82.04	82.04	82.04	82.04
Total						**929.75**	**1,871.01**	**1,871.01**	**1,871.01**	**1,871.01**

Source: Morgan Stanley

We now have all the information we need to calculate the annual fair value premium for each nth-to-default tranche in this one scenario. Remembering from Exhibit 19 that the quarterly fair value premium is equal to the expected loss divided by the sum of the discounted average balances, we show the annual fair value premium for each tranche in Exhibit 21.

Exhibit 21

Fair Value Premium for One Scenario

	Tranche				
	1	2	3	4	5
Annual Premium – One Scenario	2,354	–	–	–	–

Source: Morgan Stanley

Please note that the annual premium is quoted in basis points and is simply the quarterly premium multiplied by 4.

Simulating Multiple Scenarios

The purpose of simulation is to generate multiple scenarios. For each one, we record the present value of losses for each tranche and the present value of the average balance for each tranche (the two items required to calculate the annual premium). For the purposes of this analysis, we simulated 10,000 scenarios (there are add-ins for Excel that can simulate multiple scenarios). It is important to note that the more simulations that are run, the lower the standard error of the premium calculated; the downside is increased computing time. Results from a few of the scenarios appear in Exhibit 22, as well as the averages for all of the results.

Exhibit 22

Simulating Multiple Scenarios

	Present Value of Losses by Tranche					Present Value of Average Quarterly Balance by Tranche				
Simulation	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	-	-	-	-	-	1,871	1,871	1,871	1,871	1,871
2	-	-	-	-	-	1,871	1,871	1,871	1,871	1,871
3	-	-	-	-	-	1,871	1,871	1,871	1,871	1,871
4	-	-	-	-	-	1,871	1,871	1,871	1,871	1,871
9,997	-	-	-	-	-	1,871	1,871	1,871	1,871	1,871
9,998	58	-	-	-	-	401	1,871	1,871	1,871	1,871
9,999	53	-	-	-	-	1,871	1,871	1,871	1,871	1,871
10,000	-	-	-	-	-	1,304	1,871	1,871	1,871	1,871
Average	**5.13**	**0.37**	-	-	-	**1,809.67**	**1,868.34**	**1,871.01**	**1,871.01**	**1,871.01**
Annual Premium (bps)	**113.37**	**7.87**	-	-	-					

Source: Morgan Stanley

As you can see, the fair value premium for the first-to-default tranche according to this simulation is 113 bps (5.13 / 1,809.67 * 4 * 10,000). The third, fourth and fifth-to-default tranches have fair value spreads of 0 in our simulation because no more than 2 credits ever defaulted over the 5.18 year term of the trade in any simulated scenario.

5. Pricing Morgan Stanley's Benchmark High Quality First-to-Default Basket Using Simulation – Incorporating Correlation into the Model

In pricing the nth-to-default tranches of the High Quality Basket we used five uniform random variables to simulate the time to default of each of the five companies. By not correlating these random variables, we implicitly assumed that the correlation of the time to default for each of the five credits was 0. By correlating the random variables assigned to each credit, we effectively correlate the time-to-default of the credits.

Generating Correlated Uniform Random Variables

We incorporate "correlation" into our model by generating correlated uniform random variables, which requires a five-step process.

Step 1: As we did above, we generate a uniform random variable for each credit. In Excel we use the formula = rand().

Step 2: We convert the uniform random variables to normal random variables, also known as Gaussian random variables. A normal random variable is one with a mean of 0 and a standard deviation of 1. We do this in Excel by using the formula = normsinv(x) where x is the uniform random variable from Step 1. We do this because the process used to correlate random variables (Step 4 below) requires normal random variables as opposed to uniform random variables.

Step 3: We generate one normal random variable (using the same process as Steps 1 and 2 above) that we will use as a "community" normal random variable – in other words, we will use this normal random variable for all credits in our basket (see Exhibit 23 – this explanation should make sense when you see it in context).

Step 4: We generate correlated normal random variables. Because we are using one correlation number for all pair wise correlations, the process is relatively simple.[4] To generate a correlated normal random variable for FNM, we multiply the normal random variable generated for FNM by the square root of 1 less the implied correlation for the basket. We add to this the "community" normal random variable multiplied by the square root of the implied correlation on the basket. We repeat this process for each credit in the basket.

Step 5: We generate correlated uniform random variables from the correlated normal random variables. In Excel, we use the formula = normsdist(x), where x is the correlated normal random variable.

Now that we have calculated correlated uniform random variables, we are able to calculate correlated times to default by simply plugging the correlated uniform random variables into the formula that we used for calculating time to defaults. The output from the five steps above and the time to default calculation for the High Quality Basket are shown below in the highlighted portions of Exhibit 23 for one single scenario. We assume a correlation of 55% in the calculations.

4 *The mathematical process to generate correlated normal random variables when pair wise correlations are not the same uses the Cholesky Decomposition.*

Exhibit 23

Generating Correlated Uniform Random Variables

General Information	
Correlation	55.0%
Interest Rate	3.84%
Notional Per Name	100
Recovery Rate	40%
3-Year Spread / 5-Year Spread	60%
1-Year Spread / 5-Year Spread	40%
Effective Date	4/15/04
First Coupon Date	6/20/04
Maturity Date	6/20/09
Years to Maturity	5.18
"Community" Normal Random Variable	(1.67)

								Step 1	Step 2	Step 3	Step 4	Step 5			
Credit Specific Information								Time-to-Default correlation							
Credit	5-Year Spread (bps)	5-Year Default Probability	3-Year Default Probability	1-Year Default Probability	0-Year Default Probability	Notional	Recovery	Uniform Random Variable	Normal Random Variable	"Community" Normal Random Variable	Correlated Normal Random Variable	Correlated Uniform Random Variable	Time to Default (Years)	Defaults Before Maturity	Losses
Fannie Mae	24	2.07%	0.73%	0.16%	0.00%	100.00	40.00	0.55	0.12	(1.67)	(1.16)	12.4%	31.62	No	-
AIG	25	2.15%	0.76%	0.17%	0.00%	100.00	40.00	0.09	(1.35)	(1.67)	(2.14)	1.6%	4.21	Yes	60.00
GE Capital Corp.	33	2.83%	1.01%	0.22%	0.00%	100.00	40.00	0.83	0.97	(1.67)	(0.59)	27.9%	56.89	No	-
Pfizer	15	1.30%	0.46%	0.10%	0.00%	100.00	40.00	0.72	0.59	(1.67)	(0.84)	20.0%	85.54	No	-
Wal-Mart	19	1.64%	0.58%	0.13%	0.00%	100.00	40.00	0.56	0.15	(1.67)	(1.14)	12.7%	41.23	No	-
Totals	**116**					**500.00**									**60.00**

Source: Morgan Stanley

Pulling the Pieces Together

Now that we've walked through all the elements of a simple correlation model, let's price Morgan Stanley's High Quality First-to-Default Basket and compare the results from our model with those published in Tailoring Baskets – One Size Doesn't Fit All, published on April 16, 2004 (page 13).

The High Quality Benchmark basket had a bid/offer of 87 bps/101 bps and corresponding implied correlations of 55%/35%. Using an implied correlation of 55% and running 10,000 simulations, our model results in a spread of 87.77 bps with a standard error of 3.09 bps (Please see Exhibit 24).

Exhibit 24

Pricing Morgan Stanley's High Quality First-to-Default Basket

General Information	
Correlation	55.0%
Interest Rate	3.84%
Notional Per Name	100
Recovery Rate	40%
3-Year Spread / 5-Year Spread	60%
1-Year Spread / 5-Year Spread	40%
Effective Date	4/15/04
First Coupon Date	6/20/04
Maturity Date	6/20/09
Years to Maturity	5.18
"Community" Normal Random Variable	(1.67)

	Credit-Specific Information							Time-to-Default Correlation							
								Step 1	Step 2	Step 3	Step 4	Step 5			
Credit	5-Year Spread (bps)	5-Year Default Probability	3-Year Default Probability	1-Year Default Probability	0-Year Default Probability	Notional	Recovery	Uniform Random Variable	Normal Random Variable	"Community" Normal Random Variable	Correlated Normal Random Variable	Correlated Uniform Random Variable	Time to Default (Years)	Defaults Before Maturity	Losses
Fannie Mae	24	2.07%	0.73%	0.16%	0.00%	100.00	40.00	0.55	0.12	(1.67)	(1.16)	12.4%	31.62	No	-
AIG	25	2.15%	0.76%	0.17%	0.00%	100.00	40.00	0.09	(1.35)	(1.67)	(2.14)	1.6%	4.21	Yes	60.00
GE Capital Corp.	33	2.83%	1.01%	0.22%	0.00%	100.00	40.00	0.83	0.97	(1.67)	(0.59)	27.9%	56.89	No	-
Pfizer	15	1.30%	0.46%	0.10%	0.00%	100.00	40.00	0.72	0.59	(1.67)	(0.84)	20.0%	85.54	No	-
Wal-Mart	19	1.64%	0.58%	0.13%	0.00%	100.00	40.00	0.56	0.15	(1.67)	(1.14)	12.7%	41.23	No	-
Totals	**116**					**500.00**									**60.00**

Tranche Number, Attachment/Detachment Points, Size, Losses & PV of Losses					
Tranche Number	1	2	3	4	5
Tranche Size	100	100	100	100	100
Tranche Losses	60	-	-	-	-
PV of Losses	51.20	-	-	-	-

Tranche Ending Balance Information

	Tranche Number and Ending Tranche Balance					Tranche Number & Discounted Ending Tranche Balance (T0 Reflects Accrued Interest Payment)				
Time	1	2	3	4	5	1	2	3	4	5
-	100.00	100.00	100.00	100.00	100.00	(27.67)	(27.67)	(27.67)	(27.67)	(27.67)
0.18	100.00	100.00	100.00	100.00	100.00	99.31	99.31	99.31	99.31	99.31
0.43	100.00	100.00	100.00	100.00	100.00	98.37	98.37	98.37	98.37	98.37
0.68	100.00	100.00	100.00	100.00	100.00	97.43	97.43	97.43	97.43	97.43
0.93	100.00	100.00	100.00	100.00	100.00	96.51	96.51	96.51	96.51	96.51
1.18	100.00	100.00	100.00	100.00	100.00	95.59	95.59	95.59	95.59	95.59
1.43	100.00	100.00	100.00	100.00	100.00	94.68	94.68	94.68	94.68	94.68
1.68	100.00	100.00	100.00	100.00	100.00	93.78	93.78	93.78	93.78	93.78
1.93	100.00	100.00	100.00	100.00	100.00	92.89	92.89	92.89	92.89	92.89
2.18	100.00	100.00	100.00	100.00	100.00	92.00	92.00	92.00	92.00	92.00
2.43	100.00	100.00	100.00	100.00	100.00	91.13	91.13	91.13	91.13	91.13
2.68	100.00	100.00	100.00	100.00	100.00	90.26	90.26	90.26	90.26	90.26
2.93	100.00	100.00	100.00	100.00	100.00	89.40	89.40	89.40	89.40	89.40
3.18	100.00	100.00	100.00	100.00	100.00	88.55	88.55	88.55	88.55	88.55
3.43	100.00	100.00	100.00	100.00	100.00	87.71	87.71	87.71	87.71	87.71
3.68	100.00	100.00	100.00	100.00	100.00	86.88	86.88	86.88	86.88	86.88
3.93	100.00	100.00	100.00	100.00	100.00	86.05	86.05	86.05	86.05	86.05
4.18	100.00	100.00	100.00	100.00	100.00	85.23	85.23	85.23	85.23	85.23
4.43	10.81	100.00	100.00	100.00	100.00	9.12	84.42	84.42	84.42	84.42
4.68	-	100.00	100.00	100.00	100.00	-	83.62	83.62	83.62	83.62
4.93	-	100.00	100.00	100.00	100.00	-	82.82	82.82	82.82	82.82
5.18	-	100.00	100.00	100.00	100.00	-	82.04	82.04	82.04	82.04
Total						**1,547.23**	**1,871.01**	**1,871.01**	**1,871.01**	**1,871.01**
Spread - One Scenario						**1,323.73**	**-**	**-**	**-**	**-**

	Present Value of Losses by Tranche					Present Value of Average Quarterly Balance by Tranche				
Simulation	1	2	3	4	5	1	2	3	4	5
1	53	-	-	-	-	1,261	1,871	1,871	1,871	1,871
2	-	-	-	-	-	1,871	1,871	1,871	1,871	1,871
3	52	-	-	-	-	1,471	1,871	1,871	1,871	1,871
4	-	-	-	-	-	1,871	1,871	1,871	1,871	1,871
9,997	-	-	-	-	-	1,871	1,871	1,871	1,871	1,871
9,998	-	-	-	-	-	1,871	1,871	1,871	1,871	1,871
9,999	-	-	-	-	-	1,871	1,871	1,871	1,871	1,871
10,000	-	-	-	-	-	1,871	1,871	1,871	1,871	1,871
Average	**3.99**	**1.06**	**0.33**	**0.12**	**0.02**	**1,818.27**	**1,859.49**	**1,867.05**	**1,869.44**	**1,870.60**
Standard Error	**0.14**	**0.07**	**0.04**	**0.03**	**0.01**					
Annual Premium	**87.77 bps**	**22.84 bps**	**7.03 bps**	**2.62 bps**	**0.47 bps**					
Standard Error of Premium	**3.09 bps**	**1.59 bps**	**0.89 bps**	**0.55 bps**	**0.24 bps**					

Note: Shading = input to model, box = fair value of first-to-default premium.
Source: Morgan Stanley

6. Pricing the Tranches of Tranched DJ TRAC-X

With slight modifications, the correlation model that we developed to price nth-to-default baskets (as shown in Exhibit 24) can also be used to price the loss tranches of large baskets such as Tranched DJ TRAC-X. Below we discuss the modifications to the model as well as the structure of loss baskets versus nth-to-default baskets and price the various tranches of Tranched DJ TRAC-X.

"Loss" vs. Nth-to-Default Tranches

The primary modifications that must be made to the model are due to the structure of a "loss basket" (Tranched DJ TRAC-X) versus the structure of an nth-to-default basket (also called a "notional basket"). In an nth-to-default tranche, the size of a first-to-default tranche, second-to-default tranche, and so on, is equal to the notional of one of the credits in the portfolio (assuming the same notional for each credit). For instance, in Exhibit 24 the notional of each name in the portfolio is $100, and the size of each tranche is $100. This means that the size of an nth-to-default tranche is larger than the amount that a buyer of protection can receive (assuming a recovery rate greater than 0%). In contrast, Tranched DJ TRAC-X and most large baskets are structured as "loss" baskets. In a loss tranche, the total amount that a buyer of protection can receive from defaults in the portfolio is equal to the tranche size, irrespective of the assumed recovery rates.

Let's walk through an example. The standard tranches on Tranched DJ TRAC-X are 0%-3%, 3%-7%, 7%-10%, 10%-15%, and 15%-30%. What do we mean by 3%-7%, for instance? In a 100-name portfolio with $10MM notional per name, the total potential amount that a buyer of protection could receive, also known as the size of the tranche, is $40MM. This is the "width" of the tranche multiplied by the notional size of the portfolio, or ((7% - 3%) * (100 * $10MM)).

The tranche size may decline with losses experienced by the portfolio. For example, if the overall portfolio has experienced $30MM of losses, any additional losses would reduce the size of the 3%-7% tranche dollar for dollar, up to $40MM in additional losses. So if another credit defaults and its recovery rate is 30%, then the size of the 3%-7% tranche declines from $40MM to $33MM (($40MM – ($10MM * (100%-30%))). A buyer of protection on this tranche would now pay for protection on a notional balance of $33MM as opposed to $40MM.

We incorporate the loss tranche structure of Tranched DJ TRAC-X into our model by modifying the table "Tranche Number, Attachment / Detachment Points, Size, Losses, and PV of Losses" (Exhibit 25). We assume that the notional of each credit in the basket is $10MM; therefore, the total size of the basket is $1B.

Exhibit 25

Various Tranches of Tranched DJ TRAC-X ($MM)

	Tranche Number, Attachment/ Detachment Points, Size, Losses & PV of Losses						
Tranche	1	2	3	4	5	6	7
Attachment	-	30	70	100	150	-	20
Detachment	30	70	100	150	300	1,000	50
Initial Size	30	40	30	50	150	1,000	30

Source: Morgan Stanley

The attachment and detachment points are the lower and upper bounds of each tranche and can be defined in percentages or dollar terms. In our model and in Exhibit 25 above we define the points in dollar terms. Continuing with our 3%-7% tranche example, the dollar attachment point is the 3% percentage attachment point multiplied by the notional of the portfolio ($10MM per name * 100 names in the portfolio), or $30MM. The dollar detachment point is the 7% percent detachment point multiplied by $1B or $70MM.

The attachment and detachment points not only define the "width" of the tranche but also the seniority of the tranche. For instance, the 3%-7% tranche is more senior than the 0%-3% tranche, because losses in the portfolio affect the 0%-3% tranche before the 3%-7%. Conversely, the 3%-7% tranche is subordinate to the 7%-10% tranche, because losses in the portfolio affect the 3%-7% tranche before the 7%-10% tranche. In Tranched DJ TRAC-X the 0%-3% tranche is considered "equity," the 3%-7% tranche is "junior mezzanine," the 7%-10% tranche is "AAA," the 10%-15% tranche is "AAA+" (the latter two tranches are not actually rated) and the 15%-30% tranche is "junior super senior."

Pulling the Pieces Together

Now that we've discussed important modifications to the model, let's price the various tranches of Tranched DJ TRAC-X using data as of April 20, 2004, and compare the results with Morgan Stanley's pricing runs.

Exhibit 26

Morgan Stanley's April 20, 2004 Tranched DJ TRAC-X Pricing Run on Bloomberg

1 Govt MSGS
2 <GO> to REPLY. 3 <GO> to FORWARD.

4/20 9:26 From: MSCREDIT MSCREDIT,MORGAN STANLEY
fwd by MSCREDIT MSCREDIT of MORGAN STANLEY NEW YORK
Phone #: 1-212-762-5847

***TRANCHED DOW JONES TRAC-X NA SERIES 2 ("Fresh")
VS 65.5

TRANCHE	MARKET	CHG	DELTA	CORR(MID)
0% - 3% EQUITY	39.7/43.7+500PA	-0.1%	12.1	21.0%
3% - 7% JR MEZZ	363/403	unch	8.6	5.5%
7% -10% "AAA"	127/147	unch	4.0	17.5%
10%-15% "AAA+"	57/67	unch	2.1	23.0%
15%-30% JR S-S	11/16	unch	.5	28.0%
2% - 5%	645/720	unch	11.9	32.0%

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Source: Morgan Stanley pricing run sent via Bloomberg

Exhibit 26 is a Bloomberg screenshot on April 20 showing indicative prices for the various tranches of Tranched DJ TRAC-X. We will compare the pricing from our model with the pricing on this run. There are a few things to note: 1) Morgan Stanley prices six tranches of Tranched DJ TRAC-X on a daily basis, 2) each tranche is priced at a different implied correlation (so we must run our model six times), 3) the dispersion of the implied correlations is a function of technicals in the structured credit market place, and 4) the "delta" of a tranche is its spread "beta" (to borrow a term from the equity markets) relative to the spread movements of the DJ TRAC-X index. Our complete "loss" model for pricing Tranched DJ TRAC-X at a 17.5% correlation (which corresponds to the implied mid-market correlation on the 7%-10% tranche) follows as Exhibit 27 on the next two pages.

Exhibit 27

Pricing DJ Tranched DJ TRAC-X 7%-10% at a 17.5% Correlation

General Information	
Correlation	17.5%
Interest Rate	3.84%
Notional Per Name	10
Recovery Rate	40%
3-Year Spread / 5-Year Spread	60%
1-Year Spread / 5-Year Spread	40%
Effective Date	4/20/04
First Coupon Date	6/20/04
Maturity Date	3/20/09
Years to Maturity	4.92
"Community" Normal Random Variable	0.41

	Tranche Number, Attachment/Detachment Points, Size, Losses & PV of Losses						
Tranche Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Attachment Point	-	30	70	100	150	-	20
Detachment Point	30	70	100	150	300	1,000	50
Initial Tranche Size	30	40	30	50	150	1,000	30
Tranche Losses	6	-	-	-	-	6	-
PV of Losses	5.07	-	-	-	-	5.07	-

	Tranche Ending Balance Information													
	Tranche Number and Ending Tranche Balance							Tranche Number & Discounted Ending Tranche Balance (T0 Reflects Accrued Interest Payment)						
Time	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
-	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	(9.95)	(13.26)	(9.95)	(16.58)	(49.73)	(331.51)	(9.95)
0.17	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	29.81	39.75	29.81	49.68	149.05	993.63	29.81
0.42	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	29.53	39.37	29.53	49.21	147.63	984.19	29.53
0.67	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	29.24	38.99	29.24	48.74	146.22	974.83	29.24
0.92	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	28.97	38.62	28.97	48.28	144.83	965.56	28.97
1.17	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	28.69	38.26	28.69	47.82	143.46	956.38	28.69
1.42	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	28.42	37.89	28.42	47.36	142.09	947.28	28.42
1.67	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	28.15	37.53	28.15	46.91	140.74	938.27	28.15
1.92	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	27.88	37.17	27.88	46.47	139.40	929.35	27.88
2.17	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	27.62	36.82	27.62	46.03	138.08	920.52	27.62
2.42	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	27.35	36.47	27.35	45.59	136.76	911.76	27.35
2.67	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	27.09	36.12	27.09	45.15	135.46	903.09	27.09
2.92	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	26.84	35.78	26.84	44.73	134.18	894.51	26.84
3.17	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	26.58	35.44	26.58	44.30	132.90	886.00	26.58
3.42	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	26.33	35.10	26.33	43.88	131.64	877.58	26.33
3.67	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	26.08	34.77	26.08	43.46	130.38	869.23	26.08
3.92	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	25.83	34.44	25.83	43.05	129.14	860.97	25.83
4.17	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	25.58	34.11	25.58	42.64	127.92	852.78	25.58
4.42	30.00	40.00	30.00	50.00	150.00	1,000.00	30.00	25.34	33.79	25.34	42.23	126.70	844.67	25.34
4.67	24.00	40.00	30.00	50.00	150.00	990.00	30.00	20.08	33.47	25.10	41.83	125.50	828.27	25.10
4.92	24.00	40.00	30.00	50.00	150.00	990.00	30.00	19.89	33.15	24.86	41.43	124.30	820.40	24.86
Total								**525.34**	**713.78**	**535.33**	**892.22**	**2,676.66**	**17,827.76**	**535.33**
Spread - One Scenario								**385.93**	**-**	**-**	**-**	**-**	**11.37**	**-**

	Present Value of Losses by Tranche							Present Value of Average Quarterly Balance by Tranche						
Simulation	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	28	35	12	-	-	75	27	140	546	510	892	2,677	16,865	354
2	27	10	-	-	-	37	19	310	689	535	892	2,677	17,428	476
3	26	5	-	-	-	31	13	386	709	535	892	2,677	17,587	520
4	-	-	-	-	-	-	-	535	714	535	892	2,677	17,844	535
9,997	27	26	-	-	-	53	26	263	628	535	892	2,677	17,247	422
9,998	27	10	-	-	-	37	19	332	694	535	892	2,677	17,473	492
9,999	27	10	-	-	-	37	19	318	679	535	892	2,677	17,423	465
10,000	-	-	-	-	-	-	-	535	714	535	892	2,677	17,844	535
Average	**17.79**	**7.80**	**1.81**	**0.90**	**0.23**	**28.53**	**9.35**	**373.56**	**671.56**	**527.52**	**888.86**	**2,675.89**	**17,484.52**	**482.40**
Standard Error	**0.10**	**0.13**	**0.06**	**0.05**	**0.03**	**0.28**	**0.11**							
Annual Premium	**43.73 pts + 500 bps running**	**464.57 bps**	**137.33 bps**	**40.39 bps**	**3.49 bps**	**65.27 bps**	**775.16 bps**							
Standard Error of Premium	**0.34 pts**	**7.48 bps**	**4.58 bps**	**2.38 bps**	**0.51 bps**	**0.65 bps**	**9.14 bps**							

Note: Inputs are highlighted. Annual fair value premium for 7%-10% Tranche is "boxed."
Source: Morgan Stanley

Exhibit 27 (Continued)

Credit Information and Time-to-Default Calculations for One Scenario of Simulation

Credit Specific Information								Time to Default Correlation							
Credit	5 Year Spread (bps)	5 Year Default Probability	3 Year Default Probability	1 Year Default Probability	0 Year Default Probability	Notional	Recovery	Uniform Random Variable	Normal Random Variable	"Community" Normal Random Variable	Correlated Normal Random Variable	Correlated Uniform Random Variable	Time to Default (Years)	Defaults Before Maturity	Losses
McDonald's Corporation	25	2.19%	0.78%	0.17%	0.00%	10.00	4.00	0.48	(0.05)	0.41	0.12	54.9%	179.77	No	-
Caterpillar Inc.	23	1.98%	0.70%	0.16%	0.00%	10.00	4.00	0.17	(0.94)	0.41	(0.68)	24.8%	71.32	No	-
American Express Company	24	2.11%	0.75%	0.17%	0.00%	10.00	4.00	0.74	0.66	0.41	0.77	77.9%	354.30	No	-
Bristol-Myers Squibb Company	38	3.27%	1.16%	0.26%	0.00%	10.00	4.00	0.30	(0.53)	0.41	(0.31)	37.8%	71.43	No	-
Cargill, Incorporated	27	2.35%	0.83%	0.18%	0.00%	10.00	4.00	0.86	1.06	0.41	1.14	87.2%	433.35	No	-
Deere & Company	32	2.73%	0.97%	0.21%	0.00%	10.00	4.00	0.38	(0.29)	0.41	(0.10)	46.2%	111.78	No	-
Danaher Corporation	23	1.96%	0.69%	0.15%	0.00%	10.00	4.00	0.91	1.36	0.41	1.41	92.1%	639.98	No	-
CVS Corporation	44	3.73%	1.33%	0.29%	0.00%	10.00	4.00	0.15	(1.03)	0.41	(0.76)	22.3%	33.16	No	-
Viacom Inc.	38	3.27%	1.16%	0.26%	0.00%	10.00	4.00	0.30	(0.53)	0.41	(0.31)	37.8%	71.35	No	-
ALLTEL Corporation	44	3.73%	1.33%	0.29%	0.00%	10.00	4.00	0.80	0.83	0.41	0.92	82.2%	226.59	No	-
Target Corporation	24	2.04%	0.72%	0.16%	0.00%	10.00	4.00	0.57	0.18	0.41	0.34	63.2%	242.63	No	-
Ingersoll-Rand Company	46	3.97%	1.41%	0.31%	0.00%	10.00	4.00	0.07	(1.44)	0.41	(1.14)	12.7%	16.84	No	-
BellSouth Corporation	35	2.96%	1.05%	0.23%	0.00%	10.00	4.00	0.39	(0.28)	0.41	(0.09)	46.6%	104.19	No	-
Wells Fargo & Company	23	1.98%	0.70%	0.16%	0.00%	10.00	4.00	0.85	1.02	0.41	1.10	86.4%	499.10	No	-
Whirlpool Corporation	28	2.42%	0.86%	0.19%	0.00%	10.00	4.00	0.86	1.09	0.41	1.16	87.8%	427.85	No	-
Ryder System, Inc.	38	3.28%	1.17%	0.26%	0.00%	10.00	4.00	0.51	0.04	0.41	0.20	58.1%	130.42	No	-
International Business Machines Corporation	26	2.22%	0.79%	0.17%	0.00%	10.00	4.00	0.03	(1.87)	0.41	(1.52)	6.4%	14.67	No	-
Alcoa Inc.	30	2.58%	0.92%	0.20%	0.00%	10.00	4.00	0.06	(1.55)	0.41	(1.24)	10.8%	21.92	No	-
ConAgra Foods, Inc.	41	3.50%	1.25%	0.28%	0.00%	10.00	4.00	0.60	0.25	0.41	0.40	65.5%	149.42	No	-
Boeing Company (The)	37	3.20%	1.14%	0.25%	0.00%	10.00	4.00	0.79	0.82	0.41	0.92	82.0%	264.29	No	-
Nordstrom, Inc.	29	2.50%	0.89%	0.20%	0.00%	10.00	4.00	0.85	1.02	0.41	1.10	86.4%	393.10	No	-
Computer Sciences Corporation	43	3.66%	1.30%	0.29%	0.00%	10.00	4.00	0.36	(0.36)	0.41	(0.15)	44.0%	77.76	No	-
Wyeth	64	5.40%	1.93%	0.43%	0.00%	10.00	4.00	0.43	(0.17)	0.41	0.02	50.9%	63.99	No	-
General Electric Capital Corporation	31	2.66%	0.94%	0.21%	0.00%	10.00	4.00	0.68	0.47	0.41	0.60	72.5%	239.38	No	-
Baxter International Inc.	45	3.88%	1.38%	0.31%	0.00%	10.00	4.00	0.56	0.15	0.41	0.30	62.0%	121.98	No	-
Cingular Wireless LLC	47	4.04%	1.44%	0.32%	0.00%	10.00	4.00	0.12	(1.17)	0.41	(0.89)	18.6%	24.96	No	-
Nabors Industries, Inc.	38	3.28%	1.17%	0.26%	0.00%	10.00	4.00	0.59	0.23	0.41	0.38	64.7%	156.29	No	-
Simon Property Group, L.P.	55	4.65%	1.66%	0.37%	0.00%	10.00	4.00	0.56	0.14	0.41	0.30	61.9%	101.38	No	-
Marriott International, Inc.	48	4.11%	1.47%	0.32%	0.00%	10.00	4.00	0.30	(0.52)	0.41	(0.30)	38.2%	57.27	No	-
Maytag Corporation	48	4.08%	1.45%	0.32%	0.00%	10.00	4.00	0.88	1.17	0.41	1.24	89.2%	267.45	No	-
Federated Department Stores, Inc.	45	3.81%	1.36%	0.30%	0.00%	10.00	4.00	0.83	0.97	0.41	1.05	85.4%	247.85	No	-
General Mills, Inc.	39	3.35%	1.19%	0.26%	0.00%	10.00	4.00	0.65	0.40	0.41	0.53	70.3%	178.32	No	-
May Department Stores Company (The)	45	3.88%	1.38%	0.31%	0.00%	10.00	4.00	0.74	0.64	0.41	0.75	77.4%	187.95	No	-
SBC Communications Inc.	42	3.58%	1.27%	0.28%	0.00%	10.00	4.00	0.09	(1.34)	0.41	(1.05)	14.8%	21.96	No	-
Verizon Global Funding Corp.	41	3.50%	1.25%	0.28%	0.00%	10.00	4.00	0.33	(0.43)	0.41	(0.22)	41.2%	74.45	No	-
Centex Corporation	61	5.17%	1.85%	0.41%	0.00%	10.00	4.00	0.22	(0.76)	0.41	(0.52)	30.1%	33.69	No	-
Constellation Energy Group, Inc.	47	4.04%	1.44%	0.32%	0.00%	10.00	4.00	0.63	0.32	0.41	0.46	67.8%	137.49	No	-
EOP Operating Limited Partnership	57	4.88%	1.74%	0.39%	0.00%	10.00	4.00	0.95	1.66	0.41	1.68	95.4%	306.61	No	-
Monsanto Company	35	3.04%	1.08%	0.24%	0.00%	10.00	4.00	0.17	(0.94)	0.41	(0.68)	24.9%	46.34	No	-
Southwest Airlines Co.	67	5.70%	2.04%	0.45%	0.00%	10.00	4.00	0.75	0.68	0.41	0.79	78.5%	130.98	No	-
Aetna Inc.	32	2.73%	0.97%	0.21%	0.00%	10.00	4.00	0.90	1.29	0.41	1.34	91.0%	434.29	No	-
Sears Roebuck Acceptance Corp.	46	3.96%	1.41%	0.31%	0.00%	10.00	4.00	0.33	(0.44)	0.41	(0.23)	41.1%	65.44	No	-
Cox Communications, Inc.	59	5.02%	1.80%	0.40%	0.00%	10.00	4.00	0.70	0.51	0.41	0.64	73.8%	129.80	No	-
Sempra Energy	39	3.35%	1.19%	0.26%	0.00%	10.00	4.00	0.50	0.01	0.41	0.18	57.2%	124.58	No	-
Transocean Inc.	38	3.27%	1.16%	0.26%	0.00%	10.00	4.00	0.02	(2.17)	0.41	(1.80)	3.6%	5.52	No	-
SLM Corporation	40	3.44%	1.23%	0.27%	0.00%	10.00	4.00	0.01	(2.28)	0.41	(1.90)	2.9%	4.48	Yes	6.00
Textron Financial Corporation	41	3.50%	1.25%	0.28%	0.00%	10.00	4.00	0.55	0.12	0.41	0.28	61.0%	132.22	No	-
Dominion Resources, Inc.	48	4.11%	1.47%	0.32%	0.00%	10.00	4.00	0.44	(0.15)	0.41	0.03	51.3%	85.57	No	-
Carnival Corporation	52	4.42%	1.58%	0.35%	0.00%	10.00	4.00	0.46	(0.10)	0.41	0.08	53.1%	83.79	No	-
CIT Group Inc.	47	4.04%	1.44%	0.32%	0.00%	10.00	4.00	0.05	(1.67)	0.41	(1.35)	8.9%	11.25	No	-
Kroger Co. (The)	56	4.74%	1.69%	0.38%	0.00%	10.00	4.00	0.54	0.09	0.41	0.26	60.1%	94.61	No	-
MeadWestvaco Corporation	70	5.93%	2.12%	0.47%	0.00%	10.00	4.00	0.63	0.34	0.41	0.48	68.4%	94.40	No	-
Walt Disney Company (The)	52	4.46%	1.59%	0.35%	0.00%	10.00	4.00	0.95	1.68	0.41	1.70	95.6%	341.09	No	-
Weyerhaeuser Company	55	4.65%	1.66%	0.37%	0.00%	10.00	4.00	0.40	(0.24)	0.41	(0.05)	48.0%	68.79	No	-
Clear Channel Communications, Inc.	59	5.02%	1.80%	0.40%	0.00%	10.00	4.00	0.37	(0.33)	0.41	(0.12)	45.0%	58.08	No	-
International Paper Company	56	4.80%	1.71%	0.38%	0.00%	10.00	4.00	0.10	(1.31)	0.41	(1.02)	15.5%	17.09	No	-
Countrywide Home Loans, Inc.	46	3.92%	1.40%	0.31%	0.00%	10.00	4.00	0.28	(0.58)	0.41	(0.36)	36.0%	55.67	No	-
Dow Chemical Company (The)	45	3.81%	1.36%	0.30%	0.00%	10.00	4.00	0.64	0.37	0.41	0.50	69.3%	151.92	No	-
AOL Time Warner Inc.	62	5.25%	1.88%	0.42%	0.00%	10.00	4.00	0.60	0.27	0.41	0.41	66.0%	100.07	No	-
Cendant Corporation	45	3.88%	1.38%	0.31%	0.00%	10.00	4.00	0.11	(1.22)	0.41	(0.94)	17.4%	24.06	No	-
Comcast Cable Communications, Inc.	60	5.10%	1.82%	0.40%	0.00%	10.00	4.00	0.52	0.06	0.41	0.23	59.0%	85.18	No	-
Citizens Communications Company	291	22.68%	8.48%	1.93%	0.00%	10.00	4.00	0.24	(0.69)	0.41	(0.46)	32.3%	7.59	No	-
Albertson's, Inc.	66	5.63%	2.01%	0.45%	0.00%	10.00	4.00	0.31	(0.49)	0.41	(0.27)	39.3%	43.09	No	-
Raytheon Company	55	4.65%	1.66%	0.37%	0.00%	10.00	4.00	0.43	(0.18)	0.41	0.01	50.3%	73.54	No	-
Safeway Inc.	66	5.63%	2.01%	0.45%	0.00%	10.00	4.00	0.43	(0.17)	0.41	0.02	50.8%	61.15	No	-
Eastman Chemical Company	57	4.87%	1.74%	0.39%	0.00%	10.00	4.00	0.82	0.91	0.41	0.99	84.0%	183.49	No	-
Progress Energy, Inc.	50	4.27%	1.52%	0.34%	0.00%	10.00	4.00	0.32	(0.48)	0.41	(0.26)	39.6%	57.87	No	-
American Electric Power Company, Inc.	52	4.42%	1.58%	0.35%	0.00%	10.00	4.00	0.69	0.50	0.41	0.63	73.5%	146.93	No	-
Radian Group Inc.	57	4.87%	1.74%	0.39%	0.00%	10.00	4.00	0.87	1.11	0.41	1.18	88.1%	213.13	No	-
Valero Energy Corporation	53	4.49%	1.60%	0.36%	0.00%	10.00	4.00	0.56	0.16	0.41	0.31	62.3%	106.05	No	-
Computer Associates International, Inc.	114	9.46%	3.42%	0.76%	0.00%	10.00	4.00	0.56	0.15	0.41	0.31	62.1%	48.80	No	-
Amerada Hess Corporation	85	7.12%	2.56%	0.57%	0.00%	10.00	4.00	0.80	0.83	0.41	0.93	82.3%	117.34	No	-
MBNA Corporation	57	4.80%	1.72%	0.38%	0.00%	10.00	4.00	0.07	(1.49)	0.41	(1.18)	11.9%	12.83	No	-
Tyson Foods, Inc.	86	7.27%	2.61%	0.58%	0.00%	10.00	4.00	0.43	(0.19)	0.41	0.00	50.1%	46.08	No	-
News America Incorporated	56	4.80%	1.71%	0.38%	0.00%	10.00	4.00	0.15	(1.06)	0.41	(0.79)	21.6%	24.69	No	-
Goodrich Corporation	68	5.78%	2.07%	0.46%	0.00%	10.00	4.00	0.08	(1.40)	0.41	(1.10)	13.6%	12.28	No	-
AT&T Corp.	122	10.11%	3.66%	0.82%	0.00%	10.00	4.00	0.75	0.66	0.41	0.78	78.1%	71.19	No	-
AT&T Wireless Services, Inc.	56	4.72%	1.69%	0.37%	0.00%	10.00	4.00	0.41	(0.23)	0.41	(0.03)	48.6%	68.82	No	-
Motorola, Inc.	76	6.38%	2.29%	0.51%	0.00%	10.00	4.00	0.60	0.25	0.41	0.40	65.5%	80.80	No	-
Sprint Corporation	76	6.38%	2.29%	0.51%	0.00%	10.00	4.00	0.05	(1.63)	0.41	(1.31)	9.5%	7.56	No	-
Sun Microsystems, Inc.	118	9.82%	3.55%	0.79%	0.00%	10.00	4.00	0.24	(0.70)	0.41	(0.46)	32.2%	18.78	No	-
Halliburton Company	68	5.78%	2.07%	0.46%	0.00%	10.00	4.00	0.54	0.09	0.41	0.25	60.0%	76.90	No	-
Loews Corporation	50	4.27%	1.52%	0.34%	0.00%	10.00	4.00	0.08	(1.43)	0.41	(1.13)	13.0%	15.92	No	-
Duke Capital Corporation	77	6.52%	2.34%	0.52%	0.00%	10.00	4.00	0.59	0.24	0.41	0.39	65.1%	78.09	No	-
Liberty Media Corporation	82	6.90%	2.48%	0.55%	0.00%	10.00	4.00	0.61	0.27	0.41	0.41	66.1%	75.63	No	-
FirstEnergy Corp.	85	7.12%	2.56%	0.57%	0.00%	10.00	4.00	0.67	0.43	0.41	0.56	71.2%	84.36	No	-
Delphi Corporation	100	8.35%	3.01%	0.67%	0.00%	10.00	4.00	0.31	(0.51)	0.41	(0.29)	38.6%	27.98	No	-
UnumProvident Corporation	250	19.78%	7.34%	1.66%	0.00%	10.00	4.00	0.45	(0.13)	0.41	0.05	52.0%	16.67	No	-
Toys "R" Us, Inc.	232	18.47%	6.83%	1.54%	0.00%	10.00	4.00	0.57	0.18	0.41	0.33	63.0%	24.36	No	-
Bombardier Inc.	138	11.37%	4.13%	0.92%	0.00%	10.00	4.00	0.74	0.64	0.41	0.75	77.4%	61.59	No	-
Altria Group, Inc.	118	9.82%	3.55%	0.79%	0.00%	10.00	4.00	0.33	(0.44)	0.41	(0.23)	40.9%	25.44	No	-
Arrow Electronics, Inc.	114	9.46%	3.42%	0.76%	0.00%	10.00	4.00	0.47	(0.08)	0.41	0.10	54.0%	39.02	No	-
Electronic Data Systems Corporation	218	17.47%	6.44%	1.45%	0.00%	10.00	4.00	0.39	(0.28)	0.41	(0.09)	46.5%	16.31	No	-
General Motors Acceptance Corporation	140	11.52%	4.18%	0.94%	0.00%	10.00	4.00	0.39	(0.28)	0.41	(0.08)	46.7%	25.72	No	-
Eastman Kodak Company	150	12.32%	4.48%	1.00%	0.00%	10.00	4.00	0.85	1.04	0.41	1.11	86.7%	76.76	No	-
Ford Motor Credit Company	171	13.96%	5.10%	1.15%	0.00%	10.00	4.00	0.14	(1.09)	0.41	(0.82)	20.6%	7.67	No	-
Visteon Corporation	225	17.96%	6.63%	1.50%	0.00%	10.00	4.00	0.53	0.07	0.41	0.24	59.4%	22.76	No	-
Hewlett-Packard Company	27	2.35%	0.83%	0.18%	0.00%	10.00	4.00	0.62	0.32	0.41	0.46	67.7%	237.94	No	-
FleetBoston Financial Corporation	23	1.98%	0.70%	0.16%	0.00%	10.00	4.00	0.16	(1.00)	0.41	(0.74)	23.0%	65.36	No	-
SUPERVALU INC.	64	5.40%	1.93%	0.43%	0.00%	10.00	4.00	0.03	(1.87)	0.41	(1.52)	6.4%	5.94	No	-
Totals	6,619					1,000.00									6.00

Note: Inputs are highlighted.

Source: Morgan Stanley

The simulation run in the model above generated a spread of 137.33 bps for the 7%-10% tranche. This compares very favorably with the mid-market spread on Morgan Stanley's run of 137 bps.

Let's now run our model using the mid-market correlations for the other tranches. A summary of the outputs from each of the runs (10,0000 simulations in each) follows in Exhibit 28. As you can see for tranche 1, the equity tranche, our model generated a fair value premium of 41.44 pts upfront + 500 bps running, compared with the mid-market premium that Morgan Stanley showed on its April 20, 2004 run of 41.70 pts upfront + 500 bps running. The difference in premium, 0.26 pts upfront, is well within the 0.35 pts standard error generated in the simulation. We don't walk through the Exhibit for every tranche, but as you can see, the results are relatively close to the actual mid-market premiums on Morgan Stanley's run.

Exhibit 28

Summary Model Pricing for the Tranches of DJ TRAC-X on April 20, 2004

	Present Value of Losses by Tranche						
Tranche Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Attachment Point	-	30	70	100	150	-	20
Detachment Point	30	70	100	150	300	1,000	50
MS Mid Market Correlation	21.0%	5.5%	17.5%	23.0%	28.0%	NA	32.0%
MS Mid Market Premium	41.70 pts	383.00 bps	137.00 bps	62.00 bps	13.50 bps	65.50 bps	682.50 bps
Model Premium	41.44 pts	379.64 bps	137.33 bps	60.67 bps	14.02 bps	64.96 bps	680.70 bps
Difference (Absolute Value)	0.26 pts	3.36 bps	0.33 bps	1.33 bps	0.52 bps	0.54 bps	1.80 bps
Model Std. Error of Premium	0.35 pts	5.73 bps	4.58 bps	3.04 bps	1.26 bps	0.70 bps	9.27 bps

Source: Morgan Stanley

In summary, we've learned how to price a single-name CDS using a simple probability tree. We then built on the three-step framework by discussing how to price tranches on nth-to-default baskets assuming implied correlations of 0% and 100%, again using probability trees. From the analysis, we concluded that a higher implied correlation translates into a lower spread on junior tranches and a higher spread on senior tranches. Next, we further developed our intuition by pricing Morgan Stanley's High Quality Benchmark First-to-Default Basket using simulation. In doing so, we generated correlated uniform normal random variables, bootstrapped the probabilities in a credit curve and learned the significance of a non-flat credit curve in pricing. We then reached our final goal – pricing Tranched DJ TRAC-X. Interestingly enough, it was not too difficult to adapt our nth-to-default model to this task. The primary modification involved going from a "notional" basket to a "loss" basket.

As you can imagine, the surface has only been scratched in regard to using the results of this study. We hope to introduce additional product notes, in which we focus on how to use implied correlation to define relative value and various trading strategies.

Glossary of Terms

attachment point: the amount of losses in the underlying portfolio that need to occur before a particular tranche starts to lose value. Expressed as either a percentage or an absolute value.

asset correlation: a measure of how asset prices move in relation to one another. Equity prices are commonly used as a proxy for asset correlation.

collateralized debt obligation (CDO): an investment collateralized by or referenced to a diverse portfolio of debt, in which the investor is exposed to losses above and below certain thresholds.

Gaussian copula: a statistical tool describing how the distribution of single risks join to form joint risk distribution. In the case of portfolio credit risk, copulas describe how patterns of individual default risk join to form the distribution of loss on a portfolio.

correlation: see *asset correlation, spread correlation, default correlation*. Correlation is usually measured as a percentage, with 100% representing perfect correlation, 0% no relationship and -100% perfectly negative correlation.

credit default swap (CDS): an over-the-counter contract to transfer credit risk, in which one counterparty (the buyer of protection) pays a premium and the other (the seller of protection) makes a payment in the event of default.

default correlation: a measure of how the default probabilities of different credits move in relation to one another.

first-loss tranche: the most junior tranche in a portfolio.

Monte Carlo simulation: generation of thousands of random scenarios to establish the probability of an event occurring.

loss basket: a credit derivative in which the payout is linked to losses within a portfolio that are defined by an attachment and detachment point.

nth-to-default basket: a credit derivative in which the payout is linked to one in a series of defaults (such as first-, second- or third-to-default), with the contract terminating at that point.

pair wise correlation: In any given portfolio, each credit has a correlation with every other credit in the portfolio. In a portfolio consisting of credits A, B and C, credit A has a correlation with B and a separate correlation with C. Credits B and C also have a correlation. Therefore, in a portfolio of 3 names, there are 3 pair wise correlations. In a portfolio of n names there are $n! \div (2 \times (n-2)!)$ pair wise correlations. "!" is the "factorial" function. n!= 1 x 2 x 3 x 4 x 5 x ... x (n-2) x (n-1) x n. e.g.,: 5! = 1 x 2 x 3 x 4 x 5.

spread correlation: a measure of how the tradable spreads of different credits move in relation to one another.

Dow Jones TRAC-X: Morgan Stanley and JP Morgan 's proprietary structured credit product that is a tradable portfolio of equally weighted single name credit default swaps representing broad exposure to credit markets.

Tranched Dow Jones TRAC-X: Morgan Stanley and JP Morgan 's proprietary structured credit product that offers prioritized cash flows from an underlying pool of equally weighted single name credit default swaps. Like CDOs, this product allows the investor to be exposed to losses above and below certain thresholds.

Source: Morgan Stanley and the CreditFlux Inside Guide to Portfolio Credit Swaps

Morgan Stanley

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