Econ. 482
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Homework Set 1

1. **Ito's Lemma:** Let the price $s(t)$ of a security follow the Ito process $ds = \alpha s\,dt + \sigma s\,dz$ in which α and σ are constants.

   (a) Use Ito's Lemma to determine the process followed by $y(t) \equiv \ln s(t)$.

   (b) What is the probability distribution of $y(2)$ in terms of $y(0)$, α and σ? (i.e., what is the type of distribution, its mean and its variance)

   (c) If you were given $s_0 = s(0)$ and a random draw X from the type of distribution in (b), but it was standardized to have mean 0 and variance 1, how would you convert it into a random draw of $s(2)$? (i.e., of the security price at time 2)

2. **Black-Scholes-Merton default model:** Consider a firm whose assets have value $A(t)$ at time t. Initial asset value $A(0) = 150$. Asset value evolves as a lognormal diffusion described by:

$$dA = \alpha A\,dt + \sigma A\,dz$$

   Time is measured in years. Parameters α and σ are constant at .08 and .30 respectively. The riskfree interest rate r is constant at .04/year (continuously compounded). The firm's liabilities consist of a zero coupon bond with maturity value 100 at maturity date T, and common shares. No coupons are paid on the bond and no dividends paid on the shares prior to T. Let $B(t)$ denote the value of the firm's bonds and $S(t)$ the value of the firm's shares at time t.

   (a) Let $T = 2$ years. Use the Black-Scholes formula to determine the fair market value of the firm's bonds $B(0)$ and shares $S(0)$ at time 0.

   (b) If you bought the bonds for price $B(0)$, what would be the promised or contractual yield to maturity c (continuously compounded) on the bonds? What would thus be their credit spread $c - r$ relative to default-free bonds? Repeat this for bonds with maturities T of 0.5, 1, 3 and 10 years to see what the firm's term structure of credit spreads looks like.

   (c) Construct a spreadsheet to re-answer question (a) using a Monte Carlo method. I.e., for each simulation make a random draw of the asset value $A(T)$ and determine the payoffs to the bondholders and shareholders. Then average the discounted (at riskfree rate) payoffs to obtain estimated fair market value. [use about 500 simulations]

   (d) **Optional bonus question:**

   The simulation above was done using the *risk-neutral drift* (expected growth rate) in asset value. Now substitute the *objective drift* and re-run to determine the following:

      i. objective expected discounted cash flows on the bond and the stock

      ii. expected yield to maturity on the bond if bought for price $B(0)$

      iii. objective probability that the bond defaults

      iv. expected recovery rate on the bond conditional on default occurring