Econ 482
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Homework Set 3
Futures prices and American options

1. **Foreign exchange derivatives:** Let $s(t)$ denote the price of British £ in terms of Canadian \$. Assume it follows an Ito process with constant proportional volatility $\sigma = .20$. The current spot exchange rate is $s(0) = 2.00$. Assume the Canadian riskfree interest rate constant at $r = .04$ and the British is constant at $r_f = .07$. Recognizing a £ is a dividend paying traded security, with proportional dividend rate r_f, one can use the Black-Scholes trick to show that the risk-neutral process for the exchange rate is

$$ds = (r - r_f)s\,dt + \sigma s\,dz$$

Use the Crank-Nicolson algorithm to calculate the following:

   (a) The current futures price for £ to be delivered in 2 years.[1]

   (b) The value of a 2 year European call option to buy £100 at a price of \$2.00.

   (c) The value of the same option if it is American.

2. **Nonstorable commodity derivatives:** Suppose that the *objective* (empirical) stochastic process followed by the spot market price $s(t)$ of electricity at the California-Washington border is mean-reverting:

$$ds = \kappa(\bar{s} - s)\,dt + \sigma s\,dz$$

in which $\kappa, \bar{s}, \sigma$ are constant. Suppose the *risk-neutral* process is:

$$ds = \kappa(\bar{s} - s - \lambda)\,dt + \sigma s\,dz$$

in which λ is also constant.[2] Electricity prices are measured in \$/megawatt-hours (mwh). Time is measured in years. Values of the parameters are $\bar{s} = 600$, $\kappa = 0.5$, $\sigma = 0.4$, $r = 0.10$, $\lambda = -200$ (note the negative value). The current spot price $s(0)$ is \$900/mwh. Use the Crank-Nicolson algorithm to determine the following:

   (a) What is the *objective* forecast $E(0)$ (expected value) of the spot price 1 year from now?

   (b) What is the equilibrium futures price $F(0)$ for 1 mwh of electricity to be delivered 1 year from now?

   (c) What is the market price of a European call option $C(0)$ on 1 mwh of electricity with exercise price of \$600 deliverable in 1 year?

[1] Hint: To compute a futures price, set IFUT = 1 before calling CNSET. Remember to reset it to 0 and call CNSET again before going on to calculate market value of a security as in the next question.

[2] Since electricity is not storable, we cannot claim that the risk-adjusted proportional drift equals the riskless interest rate r, as in the Black-Scholes case.

3. **Interest rate derivatives:** Suppose that the instantaneous riskless interest rate follows the stochastic process of Cox, Ingersoll, Ross (*Econometrica* 1985)):

$$dr = \gamma(\bar{r} - r)\,dt + \sigma r^{0.5}\,dz$$

in which γ , $\bar{r}$, σ are constants. The 'price of r-risk' is assumed to be λr (i.e., the risk-adjusted drift of r is $\gamma(\bar{r} - r) + \lambda r$). Let the initial state be $r = 0.07$. The values of the fixed parameters $\gamma, \bar{r}, \lambda, \sigma$ are respectively 0.4, 0.1, 0.1 and 0.2 .

(a) Use CNSET and CNSTEP in a spreadsheet to determine the current price and yield to maturity (continuously compounded) of a $100 face value Treasury Bill with 1 year to mature.

(b) Find the value of a 6 month European Call option on such a T-bill with exercise price of $95.[3]

(c) Find the value of the above option if it is an American option.

(d) Determine the current futures price of a 1 year T-bill deliverable 3 years from now. Express that price as a yield on 1 year bills.

(e) **Optional bonus question:** Determine the objective expected value of interest rates on 1 year T-bills 3 years from now.

[3]Specifically, at option exercise, the delivered T-bill will have 1 year to mature *as of that time.*