Econ. 811
Summer 2010

Homework Set 2

1. **Vasicek interest rate model:** In the Vasicek (1977) one factor interest rate model, the instantaneous risk-free rate is assumed to follow the Ito process

$$dr = \kappa(\theta - r)\,dt + \sigma\,dz$$

with market price of r-risk being $-\lambda$. Assume $r = .10$ now, $\sigma = .2$, $\theta = .12$, $\kappa = .2$, $\lambda = -.02$ (i.e., risk-neutral drift is $\kappa(\theta - r) - \lambda$).

   (a) What would be the current prices of 2 and 3 year pure discount bonds of \$100 maturity value?

   (b) What is the risk-adjusted expected *instantaneous* return on holding the two year bond? What is the objective expected *instantaneous* return on the bond?

   (c) What would be the current *forward price* of a 1 year bond to be delivered in 2 years?

   (d) What would be the the current *futures price* of a 1 year pure discount bond to be delivered in 2 years? Why does this differ from (c)?

   (e) If you held the 2 year bond, what size position in the above futures contract would initially be required to hedge against changes in rates?

   (f) What is the objective probability that 2 year bond yields exceed 10% 2 years from now?

   Also express your answers to (a,c,d) as continuously compounded yields to maturity.

2. **Convertible bonds:** XXX Breweries stock follows the assumptions of the Black-Scholes model, pays no dividend, currently trades for \$10 per share, and has constant instantaneous proportional volatility of 25%/year. The risk-free interest rate is currently 5%/year and follows the Cox- Ingersoll-Ross process of Homework Assignment 1. The company has issued 5 year bonds with fixed coupon rate of 6%/year, paid annually. Suppose the bonds have no chance of default, but are convertible anytime, at the option of the holder, into stock at a conversion price of \$12.50 per share.

   What is the fair market value of each \$100 par value bond?