

Homework Set 4

1. **Diffusion to barrier default model:** Consider a firm with current assets valued at \$100 million. The fair market value of these assets follow a diffusion with constant proportional volatility of 25%/year. The firm's liabilities consist of common shares plus \$80 million par value of 5 year bonds with coupon rate of 7%/year paid continuously. The shares pay no dividends. The risk-free interest rate is constant at 5%/year (continuously compounded). Suppose that the bond indenture forces bankruptcy whenever the asset value falls below \$50 million. Suppose there are no costs of liquidating the assets in bankruptcy.
 - (a) What will be the current fair market value of the bonds per \$100 of par value? What is the corresponding yield to maturity?
 - (b) What is the fair market value of the shares?
 - (c) How do your answers to (a) and (b) change if the debt maturity is reduced to 2 years?
 - (d) How do your answers to (a) and (b) change if the bankruptcy barrier is raised to \$80 million?
 - (e) How do your answers to (a) and (b) change if liquidation costs are raised to 20% of asset value?
 - (f) How do your answers to (a) and (b) change if the firm pays continuous dividends of \$2 million per year?

2. **Poisson default model:** Suppose you enter a 5 year fixed-for-floating interest rate swap with some counterparty. You receive quarterly payments equal to the then prevailing 3 mo. Libor on the swap notional amount, and pay quarterly payments at a rate c on the same notional. The interest rate environment is the CIR model of the first homework assignment.
 - (a) Assuming that you and counterparty have no default risk, what is the equilibrium fixed rate c you should pay in the swap?
 - (b) Now assume that the counterparty defaults as a Poisson process with constant intensity of 10%/year, and that there are no recoveries in the event of default. What fixed rate c should you be willing to pay?

3. **Portfolio default swap:** Consider a portfolio with 10 reference firms that might default on their debt. Suppose arrival of default to each is a Poisson process with intensity .05/year. The notional debt amount associated with each of the first 5 firms is \$5 million, and of the second 5 firms is \$15 million. Let the default-free interest rate be constant at 5%/year. Use a Monte Carlo method in which you simulate the default times of the reference names.
 - (a) Consider a 3 year first-to-default swap on this basket of firms, with default swap spread c paid continuously on the total notional up to the time for first default. Assume defaults by the various firms are statistically independent. What is the arbitrage-free level of c ?
 - (b) Consider a senior tranche 5 year synthetic CDO on this portfolio, where (for simplicity) you pay a counterparty at the 5 year point the notional amount of defaults in the portfolio exceeding \$40 million. Again assume statistical independence. What up-front one time premium as a percent of the \$60 million maximum payout should you ask for providing this protection?
 - (c) Now assume that the default times of the reference names are not independent. Specifically, assume that their dependence is generated by a Gaussian copula with equal pairwise correlation of .2, but that the marginal distributions of default times are unaltered. Recompute your answers to (a) and (b).