# Methodology for Valuing Tier 1 Call Date Guarantee 

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This note sets out the methodology used by CFSI Research to value the proposed Tier 1 Call Guarantee transaction. The methods described are rough first cuts and do not attempt to analyze what would be rational dividend payment and call policies for issuers of the underlying securities.

## 1. Transaction description

The underlying portfolio consists of perpetual, non-cumulative, callable preferred shares issued by 15 European banks. Some are euro and some dollar denominated. Amounts of each are not identical. The shares pay fixed dividend rates up to their first call dates. The call dates are approximately 8 -10 years in the future. If not called at that time, the dividends change to floating at 3 month Libor/Euribor plus a spread of 280-390 bp, depending on the bank; they continue to be callable after that date. The shares qualify as Tier 1 capital for bank capital adequacy regulation purposes. In bankruptcy they are subordinate to all bank liabilities except common equity. Most (but not all) are currently trading above par as interest rates have fallen since date of issue.

The non-cumulative nature of the dividend and the maximum spread over Libor after first call date are imposed by regulatory authorities in order to qualify as Tier 1 capital. They may only be called with permission of the relevant regulators. Call would undoubtedly not be permitted (and dividends likely passed) if a bank did not meet the then-prevailing minimum capital adequacy requirements. No securities of this type have yet reached first call date, so there is as yet no evidence on the position regulators are likely to take.

The insurance we are asked to provide is this: In the event of non-call at first call date, we pay the protection buyer the difference between the call price (par) and the then market value of the shares. Our contract might specify physical delivery in return for payment of the call price. It has not been discussed whether this would take the form of a put option (exercisable at the discretion of the portfolio holder) or a contingent forward contract (under which we would gain if the shares are not called but are trading above par because of the substantial spread over Libor). We ignore below any possibility of gain from this source. ${ }^{1}$

[^0]The transaction currently contemplated would have fixed dollar subordination level for us. I.e., we would only pay losses once the cumulative amounts exceed, say, $25 \%$ of the total notional principal. This means that correlation among values in the non-call state must be addressed. Furthermore, and importantly, no loss would be counted for any bank that was in default on its debt prior to the first call date.

## 2. Academic finance literature

There is some - but not much - academic literature on the valuation of preferred shares. The problem for theory is that the incentive for the issuer to pay preferred dividends is unclear. Preferred shares do not typically carry voting rights. Aside from priority over common shares in liquidation, the only covenant is that common share dividends cannot be paid unless preferred share dividends (and arrears if cumulative) are paid. Since there is no consensus in the finance literature regarding what would be rational dividend policy (indeed tax considerations suggest that stock repurchase is a more efficient way to distribute earnings to shareholders than dividends), it is unclear what incentive this covenant provides. Appeal can be made to reputation and access to capital market considerations, but their force would be difficult to quantify.

The best of the existing literature takes a plausible but ad hoc approach of assuming that preferred dividends will be cancelled (or deferred) whenever the firm's financial state declines to some intermediate barrier that is below state at time of issue, but above the debt default barrier. ${ }^{2}$

## 3. Recovery rates and actuarial valuation

Each preferred share is rated by a major rating agency. Some handle on the actuarial cost of the protection is gleaned from this. Moody's 1998 study gives cumulative probabilities of dividend impairment (cancellation or deferral) during the period 1980-97 for various horizons up to 15 years. It also gives average post-impairment and post-default values for preferreds. A strong qualification is that this data is for all preferred issuers combined (not just financial institutions), is for primarily US issuers, and is for predominantly cumulative preferred shares.

We use the following results from Moody's: 10 year cumulative dividend impairment probabilities for the preferred classes in the portfolio; average recovery rate of $38 \%$ for market fixed dividend rates. They trade at a modest premium to par, suggesting market anticipation that call will not be postponed forever.
${ }^{2}$ See Emanuel (1982), Sorensen et al (1981), Ferreira et al (1992), Crabbe (1996).
preferreds which omit dividends but are not in debt default; average recovery rate of $14 \%$ for preferreds of firms in default. ${ }^{3}$ These recovery rates are treated as deterministic and also used in the market value assessment below. What differs between the actuarial and market valuations is the impairment probabilities.

We further assume, for purposes of our transaction, that impairment at any time during the 10 year period implies impairment as of the first call date (with recovery value of $38 \%$ ). This is conservative in the sense that it ignores the possibility that financial health and dividends might be restored before that point if impairment occured earlier. But it is alternatively very possible that the preferreds may trade substantially lower than the historical month-after-omission price after a prolonged interval of passed, non-cumulative dividends.

## 4. Some notation ...

Number the firms in the portfolio $i=1 \ldots n$. As of their respective preferred first call dates, each will be in one of three states: dividend impaired (denoted $m$ ), debt default (denoted $d$ ), or current on both dividends and debt. Let $N$ be the set of integers from 1 to $n$. Let $D \subset N$ be the set of firms dividend impaired at first call date. Let $\pi_{m}^{i}$ be the cumulative probability of dividend omission before first call date for firm $i$. If these events are independent across firms, the probability of impairment configuration $D$ occurring is

$$
\begin{equation*}
\mu(D)=\prod_{i \in D} \pi_{m}^{i} \prod_{i \notin D}\left(1-\pi_{m}^{i}\right) \tag{1}
\end{equation*}
$$

Let $L(D)$ denote our payout (loss) if impairment configuration $D$ occurs under the portfolio call date guarantee contract. Note that $L$ allows for possibly different recovery rates and size of holdings of the different firms' preferred shares and any subordination. Expected payout is thus

$$
\begin{equation*}
P=\sum_{D \subset N} \mu(D) L(D) \tag{2}
\end{equation*}
$$

where the sum is over all distinct subsets $D$ of $N$.
One way to evaluate $P$ is to do Monte Carlo simulation of the portfolio outcome. I.e., for each trace, draw $n$ independent uniform random variables $z_{i}$, and declare firm $i$ impaired if $z_{i}<\pi_{m}^{i}$. This would be the best way to estimate $P$ for large $n .{ }^{4}$

[^1]The actuarially fair premium rate $p$ for the deal is annuity stream for the term 0 to first call date $T$ which has the same present value as the $P$ discounted. With premiums paid at times $t=1 \ldots T$, and $z_{t}$ denoting the zero coupon riskless discount factors to those dates, fair premium rate would be ${ }^{5}$

$$
\begin{equation*}
p=\frac{z_{T} P}{\sum_{1}^{T} z_{t}} \tag{3}
\end{equation*}
$$

To obtain fair market premium rates two further considerations must be taken into account. First, dividend impairments across banks may be correlated (as a result of common regulatory changes, business cycle effects or banking crises), invalidating equation (1) as an expression for $\mu(D)$. Second, the market may price the preferreds as if the impairment probability was different from the historical value reported by Moody's. I.e., the risk-neutral $\pi_{m}$ differ from the objective values.

## 5. Correlated impairment

We have no specific evidence regarding default or impairment correlation. ${ }^{6}$ We consider first the extremes of dividend impairments being independent and of their being maximally correlated. Impairment correlation must be less than one if the $\pi_{m}$ differ across issuers. The attainable extreme is comonotonicity (in copula terminology): Under it, if a given issuer omits dividends, then all issuers with the same or higher impairment probability also omit. Thus if the highest rated preferreds are impaired, all are, and so on down the line.

Intermediate levels of dependence are handled as follows. Let $\pi_{m}^{i}$ denote the impairment probability of bank $i=1 \ldots n$. Let impairment be associated with a standard normal random variable $z_{i}$ having a realization below $F^{-1}\left(\pi_{m}^{i}\right)$, where $F$ is the standard normal cumulative distribution function. Assume the $z_{i}$ of different firms are joint normally distributed with common pairwise correlation coefficient $\rho$. Randomly draw a vector $z$ with the specified correlation matrix. Banks with $z_{i} \leq F^{-1}\left(\pi_{m}^{i}\right)$ default. Determine payoffs on the contract by adding the losses on the impaired preferreds and taking their positive excess over the subordination amount. ${ }^{7}$ Averaging of outcomes so generated give a Monte Carlo valuation indicate signify the default (1) or not (0) of the corresponding firm. This facilitates computer coding of a routine evaluating equation (2).
${ }^{5}$ Note: This assumes statistical independence of the riskless interest rate process and default processes.
${ }^{6}$ The comovement of spreads on bank preferreds during the Russian default crisis and LTCM collapse of 1998, however, anecdotally suggests the market believes it exists.
${ }^{7}$ For this calculation, losses in euros and converted to dollars using the current forward exchange rate to date of first call, implicitly assuming independence of the exchange rate and impairment processes. Losses are discounted to the present using the current Libor-swap zero coupon curve.
of the cost of the protection. Correlations $\rho$ of 0 and 1 generate the two extreme cases of the paragraph above. Note that the correlation $\rho$ ofcredit qualities $z_{i}$ (continuous random variables) is not the same as the correlation of impairment outcomes (binary 0 or 1 random variables). However the two are strongly positively related.

Using the multivariate normal distribution to generate outcomes forces a certain type of dependence on impairments that may understate the likelihood of simultaneous occurence in the less-than-maximal correlation cases. It constrains one to a Gaussian copula for the joint outcomes. An alternative proposed by Frey and McNeil (2001) is to draw the $z_{i}$ from a multivariate- $t$ distribution with pairwise correlation coefficient $\rho$ and degrees of freedom $d>1$, and letting $F$ be the cumulative univariate $t$-distribution function with $d$ degrees of freedom. The mechanics of simulating the multivariate- $t$ are given in the Appendix. Its results converge to those of the multivariate normal as $d \rightarrow \infty$. Joint 'disasters' become progressively more likely as $d$ shrinks toward 1 . We provide values for $d=2$ to contrast the $t$-copula valuation with the Gaussian copula.

## 6. Market price based valuation

A first cut at inferring risk-neutral probabilities of dividend impairment from bond and Tier 1 preferred prices is as follows. As of the first call date $T$ the issuing bank can be in one of three states: a) no default, no dividend impairment; b) no default, dividends impaired; c) in default. Default means on its debt. As above, let $\pi_{m}^{i}$ be the probability of being in the second state (dividend impaired), and $\pi_{d}^{i}$ be the probability of being in the third (default). In what follows we suppress the superscript indicating identity of the firm.

Suppose we observe the price $P_{b}$ of a bond of the firm, paying fixed annual coupon rate $c_{b}$ and maturing at $T$ with par value 100 . We also observe the price $P_{s}$ of its Tier 1 preferred shares, paying fixed annual coupon rate $c_{s}$ and callable any time after $T$ at its par value of 100. We also observe the prices $z_{t}$ of default-free zero coupon bonds (in the same currency) maturing at the associated coupon/dividend dates. Can we infer the probabilities $\pi_{m}, \pi_{d}$ from this information? We can if we add several assumptions. ${ }^{8}$

Initially assume that coupons and dividends are default-free up to $T$. Also assume that the risk-neutral expected prices of preferred shares in the bad states are the same as the actuarial average recoveries given in Moody's 1998 report. Further assume that the preferreds will be called at par if there is no dividend impairment at that time. Then the following two

[^2]relations hold:
\[

$$
\begin{align*}
P_{b} & =c_{b} \sum_{t=1}^{T} z_{t}+\left(\left(1-\pi_{d}\right) 100+\pi_{d} R^{b}\right) z_{T}  \tag{4}\\
P_{s} & =c_{s} \sum_{t=1}^{T} z_{t}+\left(\left(1-\pi_{d}-\pi_{m}\right) 100+\pi_{m} R_{m}^{s}+\pi_{d} R_{d}^{s}\right) z_{T} \tag{5}
\end{align*}
$$
\]

In the above, $R_{d}^{b}$ is the recovery rate on unsecured bank debt ( $39 \%$ for senior, $25 \%$ for subordinated from Altman and Kishore (1996)); $R_{m}^{s}$ is the average recovery rate on dividendimpaired preferreds reported by Moodys ( $38 \%$ ); $R_{d}^{s}$ is the average recovery rate on preferreds for firms in default reported by Moodys ( $14 \%$ ).

The two equations can now be solved for the two probabilities. Given that we are offrisk for banks that are in default on their senior debt at $T$, the outcome for our contract is the same as if there was no dividend impairment under the above assumptions. ${ }^{9}$ Hence the implied $\pi_{m}$ by itself (obtained for each firm separately) is what should be used in place of historical $\pi_{m}$ in the valuation of the preceeding section. They may also be used in the valuation of the perfectly correlated case.

With $T=10$ years, a large proportion of the value of both securities comes from the coupons along the way. This results in quite high implied time $T$ default and impairment probabilities. To crudely capture the possibility of default on the coupons prior to $T$, we consider a variation of the above in which default could occur either at time 0 or at time $T$. A proportion $a=.25$ of the total probability was allocated to time 0 default (uniformly for all types of events). The modified relations become

$$
\begin{aligned}
& P_{b}=(a \pi) c_{b} \sum_{t=1}^{T} z_{t}+\left(1-\pi_{d}\right) 100 z_{T}+(1-a z) \pi_{d} R^{b} \\
& P_{s}=\left(1-\left(\pi_{d}+\pi_{m}\right)\right) c_{s} \sum_{t=1}^{T} z_{t}+\left(\pi \pi 10 z\left(z \pi R^{s} \pi R^{s}\right)\right.
\end{aligned}
$$

I.e., conditional on impairment or default occurring at $T$ or earlier, the expected present value of coupons is just $1-a$ of what was promised to time $T$, and recoveries are discounted slightly less. The implied probabilities and pricing are reported as being 'with factors', with $a=0$ being the case 'without factors'. The effect of this was to reduce implied impairment probabilities and pricing of the deal.

There are many caveats to this bit of analysis. First, it ignores the timing of default/impairment and loss of dividends prior to $T$. Second, it neglects the fact that tax treatment to the investor of preferred dividends may differ from interest on debt. Third, Moody's recovery rates were for general corporate preferreds, mainly issued in the US and

[^3]mainly cumulative, rather than the non-cumulative, bank-issued, European preferreds of our contract. And fourth, it assumes the preferreds have an expected time $T$ trading price of 100 if not dividend impaired.

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[^0]:    ${ }^{1}$ Although rationality might appear to dictate calling the preferreds immediately if new shares could be issued at a lower dividend rate, this is not necessarily the case if there are significant issuance costs and the option is to call is American. As anecdotal evidence, there are preferred shares currently outstanding of several top Canadian banks which have not been called despite over-

[^1]:    ${ }^{3}$ Recovery rates are market values approximately one month after dividend impairment or default occurs.
    ${ }^{4}$ For modest number of firms $n$ (up to 31), it appears feasible to compute $P$ directly. There are $2^{n}$ impairment configurations, corresponding to the integers from 0 to $2^{n}-1$. With just 15 firms, the number of cases is 32768 . In the $n$-digit binary representation of these integers, each digit can

[^2]:    ${ }^{8}$ If the debt-related information is in the form of a credit default swap spread, consider the bond to have be initially trading at par and have a coupon rate equal to the swap spread plus reference $T$ year default-free rate.

[^3]:    ${ }^{9}$ In the current analysis, we assume that our premium flow is unaltered by debt default prior to $T$. If premium flow is reduced as we go off-risk due to default, the required fair market pricing would be slightly higher.

