On Extremes and Crashes

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Apocryphal Story

It is the early evening of Friday the 16th October 1987. In the equity markets it has been an unusually turbulent week which has seen the S&P 500 index fall by 9.21%. On that Friday alone the index is down 5.25% on the previous day, the largest one-day fall since 1962. Against this background, a young employee in a risk management division of a major bank is asked to calculate a worst case scenario for a future fall in the index. He has at his disposal all daily closing values of the index since 1960 and can calculate from these the daily percentage returns (figure 1).

The employee is fresh out of university where he followed a course in extreme value theory as part of his mathematics degree. He therefore decides to undertake an analysis of annual maximal percentage falls in the daily index value. He reduces his data to 28 annual maxima, corresponding to each year since 1960 and including the unusually large percentage fall of the present day. These maxima are:

1960	1961	1962	1963	1964	1965	1966	1967	1968	1969
2.268	2.083	6.676	2.806	1.253	1.758	2.460	1.558	1.899	1.903
1970	1971	1972	1973	1974	1975	1976	1977	1978	1979
2.768	1.522	1.319	3.052	3.671	2.362	1.797	1.626	2.009	2.958
1980	1981	1982	1983	1984	1985	1986	1987		
3.007	2.886	3.997	2.697	1.821	1.455	4.817	5.254		

To these data he fits a Fréchet distribution and attempts to calculate estimates of various return levels. A return level is an old concept in extreme value theory, popular with hydrologists and engineers who must build

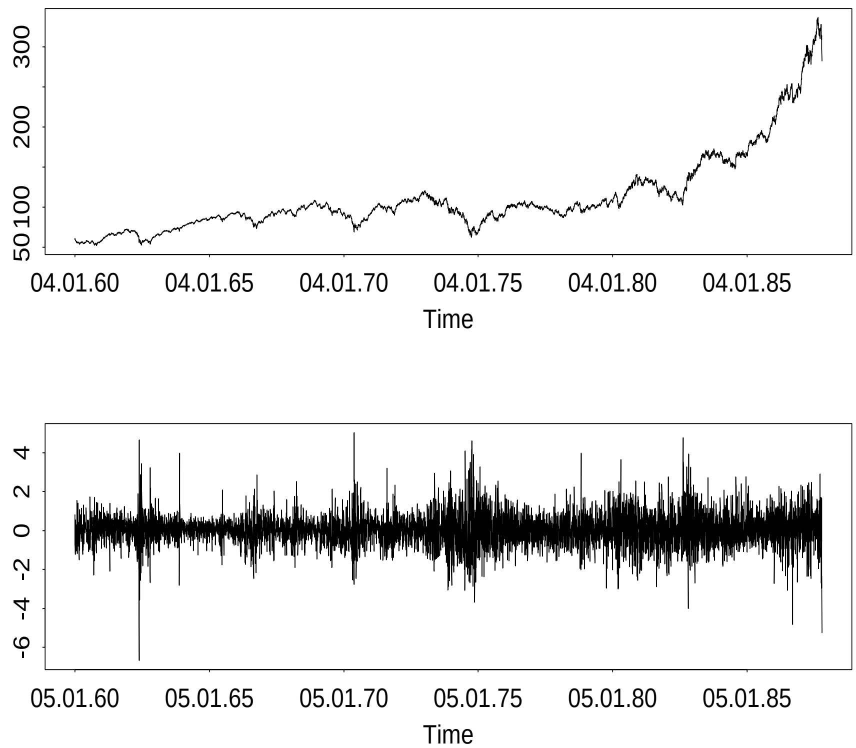

Figure 1: S&P 500 index from 1960 to 16th October 1987; raw values in upper picture, percentage returns in lower

structures to withstand extreme winds or extreme water levels. The 50-year return level is a level which, on average, should only be exceeded in one year every fifty years.

Note that this is not the same as saying that the level will be exceeded only once every fifty years on average. When a level is exceeded in a year there may or may not be a tendency for it to be exceeded more than once. This depends on the dependencies in the underlying daily return series and the propensity of the series to form clusters. But that is another story...

Our employee uses his Fréchet model to calculate return levels. Having received a good statistical education he also calculates a 95% confidence interval for the return levels. He recognizes that he is using only 28 data points and that his estimates of the parameters of the Fréchet model are prone to error. Figure 2 shows his results for the 50 year return level. The most likely value is 7.4, but there is much uncertainty in the analysis and the confidence interval is approximately (4.9, 24).

Being a prudent person, it is the value of 24% which the employee brings

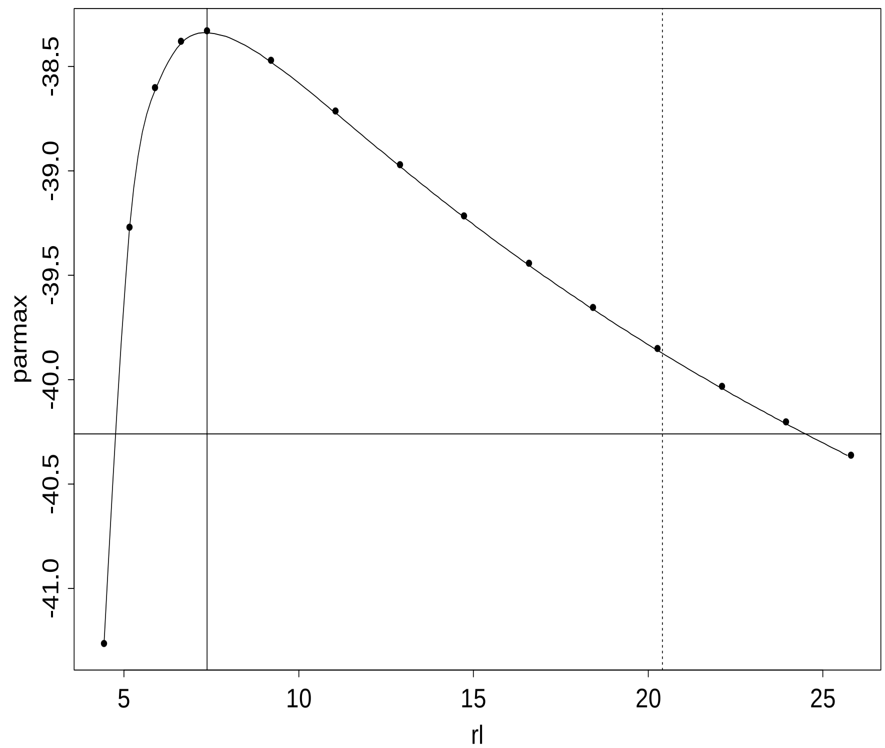

Figure 2: 95% confidence interval for 50 year return level is given by the intersections of the profile likelihood curve with the horizontal line; maximum likelihood estimate given by solid vertical line.

to his supervisor as a worst case fall in the index. He could of course have calculated the 100 or 1000 year return levels, but somewhere a line has to be drawn and a decision has to be taken. So he brings his most conservative estimate of the 50 year return level.

His supervisor is sceptical and points out that 24% is more than three times as large as the previous record daily fall since 1960. The employee replies that he has done nothing other than analyse the available data with a natural statistical model and give a conservative estimate of a well-defined rare event.

On Monday the 19th October 1987 the S&P 500 closed down 20.4% on its opening value (figure 3).

Extreme Events and Risk Management

To our knowledge the above story never took place, but it could have. There is a notion that the crash of 19th October 1987 represents an event that

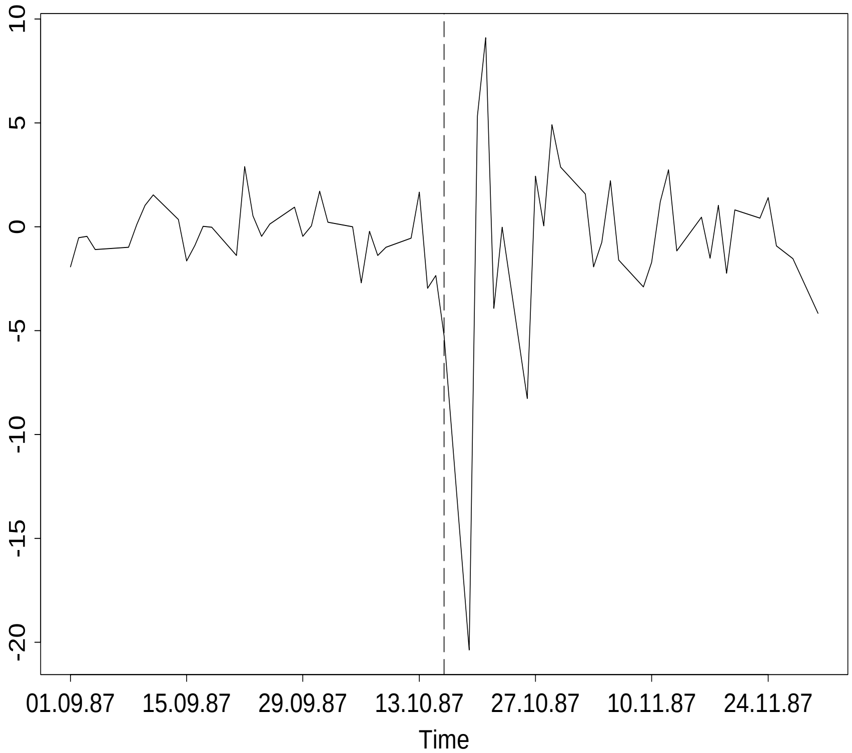

Figure 3: What happened next. Percentage returns on S&P 500 index from September to November 1987. Vertical line marks day of analysis.

cannot be reconciled with previous and subsequent market price movements. According to this view, normal daily movements and crashes are things of an entirely different nature [1]. One point of the above story is to show that a process generating normal daily returns is not necessarily inconsistent with occasional crashes.

Extreme value theory (EVT) is a branch of probability theory which focusses explicitly on extreme outcomes and which provides a series of natural models for them. EVT has a long history of application in engineering, and in particular hydrology, but has only more recently come to the intention of the finance world [2]. There is growing interest in the subject among insurance companies, particularly in high layer excess-of-loss reinsurance business [3, 4], and several parallels can be drawn between insurance and finance concerns.

The chief message is that EVT has a role to play in risk management [5]. The return level computed in the story is an example of a risk measure. The reader may have detected an element of hindsight in the choice of the 50 year return level so that the crash lay near the boundary of the estimated confidence interval. Before the event the choice of level would, however, have been a risk management decision. We define a worst case by considering how often we could tolerate it occurring; this is exactly the kind of consideration that goes into the determination of dam heights and oil-rig component strengths.

Of course the logical process can be inverted. We can imagine a so-called scenario which we believe to be extreme, say a 20% fall in the value of something, and then use EVT to attempt to quantify how extreme, in the sense of how infrequent, the scenario might be.

EVT offers other measures of risk not touched upon in the story, but described, for instance, in reference [2]. The high quantile of a return distribution, commonly called the value at risk or VaR, can be estimated using various techniques for modelling the tail of a potentially heavy-tailed distribution. Deficiencies of common VaR estimation methods are their reliance on normal distributional assumptions and neglect of the issue of fat tails. A further measure is the shortfall or beyond VaR risk measure, the amount by which VaR may be exceeded in the rare event that it is exceeded. EVT is able to offer a very natural distributional approximation for the shortfall.

There is a further important point embedded in the story, and that is the necessity of considering uncertainty on various levels. Only one model was fitted, a Fréchet model for annual maxima. The Fréchet distributional form is well-supported by theoretical arguments but the choice of annual aggregation is somewhat arbitrary; why not semesterly or quarterly maxima? This issue is sometimes labelled model risk and in a full analysis would be addressed. The next level of uncertainty is parameter risk. Even supposing the model in the story is a good one, parameter values could only be established roughly and this was reflected in a wide range of values for the return level.

In summary one can say that EVT does not predict the future with certainty; in no way should the story have suggested this. It is more the case that EVT provides sensible natural models for extreme phenomena and a framework for assessing the uncertainty which surrounds rare events. In finance these models could be pressed into service as benchmarks for measuring risk.

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