Interest Rate Volatility and Real Investment

W. John Heaney and Robert A. Jones

Timing becomes an important aspect of investment decisions whenever proceeding with an investment alters future investment opportunities. In such cases the decision maker should consider the present value of displaced future opportunities as part of the cost of the current investment.[1] When this cost is properly accounted for, it may be optimal to delay economically viable investments, conditional on reconsidering the decision at a later date. The traditional approach to capital budgeting,[2] which involves discounting the risk adjusted expected incremental cash flows that would result from the current investment, is unsuitable for handling the timing problem under uncertainty. That problem is to determine the current value of the future market values of the displaced investment opportunities, assuming that investment decisions will be optimal at all future times. The traditional approach may be able to handle this optimal management problem in principle.[3] However 'real option theory' offers a more efficient approach.

The application of option theory to the capital budgeting problem was pioneered by Brennan (1973), and has been applied to the investment timing problem by McDonald and Siegel (1986) and Heaney and Jones (1988). In that earlier paper we suggest that long standing presumptions about the effect of real interest rate changes on investment may have to be qualified when account is taken of displaced future opportunities. In the absence of timing considerations, a rise in interest rates has a detrimental effect on capital spending since real investment becomes less attractive when compared to alternatives available in the bond market. This is the conventional net present value effect. However when timing considerations are present this effect is ameliorated. Higher rates reduce the current value of displaced futures opportunities, and thus the cost, rationally considered, of going ahead. This is the option value effect. For short duration investments, this latter effect may so

[1] Future investment opportunities may be enhanced by current investments. However in this paper we focus on the case where they are impaired by current investment.

[2] The most popular approach to capital budgeting (e.g., Brealey and Myers (1984), Ross and Westerfield (1988)) involves discounting expected incremental cash flows at a risk adjusted discount rate. However it is difficult to justify this approach in a multiperiod context (see for example Bogue and Roll (1974), Fama (1977)). A consistent multiperiod scheme for valuation involving discounting risk adjusted expected cash flows fusing the term structure of interest rates has been developed by Rubinstein (1976). A state contingent price approach to the multiperiod capital budgeting problem has been developed by Pye (1960) and Bantz and Miller (1978).

[3] See for example Lusztig and Schwab (1972).

dominate the net present value effect that a rise in interest rates may actually increase aggregate current investment.[4]

In our earlier paper decision makers took interest rates to be fixed. A change in rates would come as a complete surprise. This paper extends that analysis to an environment where interest rate and output price fluctuations are both rationally anticipated. We examine the optimal policy for managing a simple point input – point output investment opportunity that may proceed only once. The policy is described by the set of interest rate – price states in which the project proceeds. Several conclusions emerge. First, the anticipation that interest rates may change does not undo the suggestion of the earlier paper. That is, short to medium term payoff investment may be spurred by rises in rates. Second, the volatility of interest rates itself can add to the option value, inducing investors to hold out for higher net present values than with just output price volatility. Indeed, this effect can be so strong for long term investments that, even with *no* uncertainty about output prices, it can induce projects to be postponed at low interest rates that would proceed at higher rates. This emphasizes that, when timing considerations are present, one cannot separately embody output price uncertainty in the risk adjustment of project cash flows and interest rate uncertainty in a current yield curve, then combine these two bits of information to determine the optimal policy. The two sources of uncertainty must be considered jointly when assessing foregone option value. Third, even simple projects of the type we consider may, *ceterus paribus*, have two levels of real interest rates at which the manager would be just indifferent to investing. Expressed in terms of the firm's demand curve for capital goods, the demand curve may be backward bending. This leads to interesting macroeconomic implications, suggesting the possibility of multiple equilibria: one with low levels of aggregate investment despite low real interest rates, another with high investment despite high real rates.

1. The Investment Opportunity and Environment

We consider the simplest possible investment opportunity. In return for purchasing inputs costing C one obtains a single unit of output T periods later. There is no scrap value to the capital. The project may be started at any point in time, however going ahead precludes similar investment at any future date. C and T are given constants. What varies over time is the interest rate and the price of the output.

Interest rate uncertainty is assumed to be one dimensional, in the form of the single state

[4] As an extreme example, recall that in a world of certainty higher interest rates imply that trees will optimally harvested earlier if their growth rate is a declining function of age. Incurring the expense of harvesting could be considered a zero duration 'investment' project in which going ahead today precludes going ahead with the same project for a considerable period of time.

variable model of the term structure of Cox, Ingersoll and Ross (1985). With $r(t)$ denoting the instantaneous interest rate at time t on default free loans, we suppose r follows the continuous time stochastic process described by

$$dr = \kappa_1(\bar{r} - r)\,dt + \sigma_1 r^{1/2}\,dz_1 \tag{1}$$

in which $\kappa_1, \bar{r}, \sigma_1$ are constant parameters and dz_1 is the increment in a standard Brownian motion. The resulting short term interest rates are non-negative, have a tendency to return to a 'normal' level of $\bar{r}$, and are more volatile when rates are high than low. There is also an associated 'price of r–risk', reflecting economy wide risk preferences and production opportunities in equilibrium which will show up in asset valuation relations below. In the CIR context it took the form $\lambda_1 r$, which we also adopt even though our production setting is not identical. By a now standard arbitrage argument, the above implies an equilibrium term structure of interest rates at time t that is a deterministic function of $r(t)$. The value of a one dollar T-period discount bond in state r will be denoted by $B(r, T)$.

We minimize clutter in what follows by taking the output price state variable $s(t)$ to be the equilibrium *forward* price prevailing at time t for output to be delivered at time $t + T$. It is the price of output that could be locked in, if the firm so chose, at the time of project inception. The relation between spot and forward prices at time t depends on many things: the extent to which spot prices covary with aggregate wealth, interactions between spot price changes and interest rates, the storability of the commodity and service flows emanating from it, etc. However if markets are complete with respect to spot price risk, s is the relevant risk adjusted expected cash flow from the project. Conditioning our project valuation on r and s gives a particularly simple form for the market value of the project at its moment of inception, which is all that matters for determining the optimal investment policy. All that said, we add some arbitrary structure by assuming that s follows the stochastic process

$$ds = \kappa_2(s/\bar{s})^{\beta}(\bar{s} - s)\,dt + \sigma_2 s^{1/2}\,dz_2 \tag{2}$$

This is almost exactly the same type of process assumed for r. Parameters $\kappa_2, \beta, \bar{s}, \sigma_2$ are constants. The parameter $0 < \beta < 1$ has been added only to force $s = 0$ to be an absorbing state even when $\kappa_2 > 0$. I.e., is s ever goes to 0 it will remain there, assuring that the investment opportunity is worthless. This is useful later on in providing a boundary value. A positive value of κ_2 would be appropriate if there is a normal, or long run, price of the output good relative to the assumed fixed price of inputs.

Two further parameters complete the description of the decision maker's environment. There will be a prevailing price of forward commodity price risk, which we arbitrarily suppose takes the form $\lambda_2 s$ simply for symmetry with the price of interest rate risk (note that we are supposing the risk prices are independent of the level of the other state variable).

And we allow the two driving Weiner processes, z_1 and z_2, to have a constant instantaneous correlation coefficient ρ. One can make a plausible case for this being of either sign. If the output commodity is storable and held in positive quantities in equilibrium, and if changes in its spot price are uncorrelated with changes in interest rates, then the equilibrium forward price should rise with interest rates (the opportunity cost of storing the good), implying positive ρ. Alternatively, if the spot demand for this commodity is negatively correlated with interest rates (e.g., higher interest rates bring on a recession), then ρ could well be negative. For most of our examples both ρ and λ_2 will be set at 0 as they are not central to the points we wish to make.

2. Asset Values, Option Values and the Investment Policy

Assume the manager of the investment opportunity can trade continuously and without transaction cost in competitive securities markets that effectively span the two dimensions of uncertainty (i.e., would allow the construction of self-financing portfolios that replicate the payoffs of assets whose cash flows are functions of s, r, t). In such an environment, the value $V(r, s, t)$ rationally placed on an asset known to make continuous cash payments at a rate $q(r, s, \tau)$ for $t < \tau < T$, and terminal payment $f(r, s)$ at time T, satisfies the second order partial differential equation[5]

$$\begin{aligned} &rV + \lambda_1 r V_r + \lambda_2 s V_s = \\ &\quad \tfrac{1}{2}\sigma_1^2 r V_{rr} + \rho\sigma_1\sigma_2 r^{1/2} s^{1/2} V_{rs} + \tfrac{1}{2}\sigma_2^2 s V_{ss} + \kappa_1(\bar{r} - r)V_r + \kappa_2(s/\bar{s})^{\beta}(\bar{s} - s)V_s + V_t + q \end{aligned} \tag{3}$$

This states that the expected total return per unit time from holding the asset equals the riskless interest opportunity cost of holding it, plus the prevailing market excess returns for the associated exposure to r and s risk. Many functions satisfy (3). The desired solution is the one satisfying boundary conditions appropriate for the asset — in this case

$$V(r, s, T) = f(r, s) \qquad \text{for all } r, s \text{ at time } T$$

Equation (3) follows from the standard arbitrage argument, assuming that no other random factors in the economy impinge on either the boundary conditions characterizing the asset or the prevailing premiums for r or s risk at any time (we sweep sunspots under the rug). We have also inserted our assumed forms for the prevailing excess returns with no assurance they could jointly be supported in any reasonable equilibrium.

[5] Unless otherwise indicated, r and s refer to the values of these state variables at the same time t that is the other argument of the functions V, q, etc. State variables appearing as subscripts on functions indicate partial derivatives with respect to those state variables.

Two particular asset types concern us: the project once put in place and the unexercised option to put such a project in place. Consider first the project in place. To simplify matters, since all we shall use is the project value at inception, suppose the project was put in place at time t' and that a default-free forward contract was entered into at that time to sell the output at price $\tilde{s} \equiv s(t')$. Let the value at time t of this project – forward contract package be denoted by $P(r, s, t; \tilde{s})$. There are no cash flows prior to $t' + T$. At that time the output is delivered and payment $\tilde{s}$ is received. Fluctuations in s after t' have no effect on any cash flows. Hence $P_s = 0$ and (3) reduces to

$$\tfrac{1}{2}\sigma_1^2 r P_{rr} + (\kappa_1(\bar{r} - r) - \lambda_1 r)P_r + P_t - rP = 0 \tag{4}$$

subject to $P(r, s, t' + T; \tilde{s}) = \tilde{s}$. This is simply the value of a pure discount bond with face value $\tilde{s}$ maturing T periods after the project is put in place. At time t' the value of the package is thus

$$P(r, s, t') = sB(r, T) \tag{5}$$

But since the forward price at time t' is, by definition, the price that makes the value of the forward contract 0, this must also be the value of the project by itself at that time. I.e., it is the appropriately risk adjusted and discounted expected cash flow from the new investment.

Consider next the opportunity to acquire the above claim in return for the fixed payment C. It is equivalent to a perpetual American call option on a randomly varying quantity s of T period pure discount bonds. It is important to recognize that this is *not* an option on a traded security, both because the quantity of these bonds is fluctuating and because the calendar date of their ultimate maturity keeps moving forward as exercise is postponed. The result that one should never prematurely exercise an American call on a non-dividend paying security does not apply here.

The value of the investment option depends on the policy governing its exercise. Let the exercise policy be described by a subset $D \subset R \times S$ of possible r, s states, with the rule that investment proceeds immediately when $(r, s) \in D$ but is postponed when $(r, s) \notin D$.[6] If not initially in D, the project proceeds at the random time t' when r, s first hits the boundary ∂D of the acceptance region. While in $\tilde{D}$, the complement of D, the value of the option satisfies (3). Upon hitting the boundary the option is relinquished, cost C is paid, and the claim worth $P(r, s, t')$ is obtained. The term q in (3) is 0 if the opportunity is costless to maintain. The term V_t vanishes because of the time stationarity. The value $V(r, s; D)$ of the option for given D thus satisfies the elliptic partial differential equation

$$\tfrac{1}{2}\sigma_1^2 r V_{rr} + \rho\sigma_1\sigma_2 r^{1/2} s^{1/2} V_{rs} + \tfrac{1}{2}\sigma_2^2 s V_{ss} + [\kappa_1(\bar{r} - r) - \lambda_1 r]V_r$$

[6]Since the stochastic processes and risk prices are time stationary and the option never expires, the optimal policy will also be stationary. We thus consider only time independent policies.

$$+ [\kappa_2 (s/\bar{s})^{\beta} (\bar{s} - s) - \lambda_2 s] V_s - rV \;\; = \;\; 0 \tag{6}$$

in D with boundary condition

$$V(r, s; D) = sB(r, T) - C \qquad \text{on } \partial D \tag{7}$$

It remains to establish the optimal investment policy D^*, the one maximizing $V(r, s; D)$. It is characterized by the Merton–Samuelson 'high contact', or 'smooth joining', condition (see van Moerbeke, 1976) at the boundary ∂D^*:

$$\begin{aligned} V_r(r, s; D^*) &= sB_r(r, T) \\ V_s(r, s; D^*) &= B(r, T) \qquad \text{on } \partial D^* \end{aligned} \tag{8}$$

To find the optimal investment policy one must find the function V and region D^* that jointly satisfy (6), (7) and (8). This is called a free boundary value problem since the location of the boundary ∂D^* is not fixed in advance. It is the shape and location of this boundary that we seek, since that is what determines the current investment response to changes in product prices and interest rates.

An alternative condition that is equivalent to (7) and (8) in this context reflects the dynamic programming nature of the problem. The value of the option, optimally managed, can never be less than its value if exercised immediately:

$$V(r, s; D^*) \geq sB(r, T) - C \qquad \text{for all } r, s \tag{9}$$

Or equivalently,

$$sB(r, T) \leq C + V(r, s; D^*) \tag{10}$$

When equality holds it is optimal to proceed immediately with the investment. If investment now did not alter future investment opportunities, then V would be 0 and (10) reduces the conventional net present value rule. B is the discount factor explicit in current default free term structure; s is the appropriate risk adjusted expected cash flow. If the present value of this flow exceeds the cost C the project should proceed. Given B, the fact that interest rates fluctuate would be irrelevant were it not for V.

3. Method of Solution

We have been unable to obtain a closed form solution to the system (6)–(8) and resort to numerical methods to provide illustrative results. One way to solve the elliptic pde is to reintroduce the V_t term into into (6), add a terminal value condition, and solve the resulting parabolic pde backwards to $t = -\infty$ (i.e., until the solution ceases to change).[7] In effect,

[7]See Lapidus and Pinder, 1982, chapter 5, for a survey of methods.

the option is treated as having an expiry date that recedes further and further into the future. This is what we do.

This parabolic pde is solved on a discrete grid of r, s, t values using a finite difference alternating direction scheme. This is computationally efficient with two state variables since it involves no matrix inversion and is numerically stable with large time steps. We use a 41×41 grid of r, s states with real interest rates ranging 0–20% and the forward prices 0–20. When one's only concern is the steady state solution, convergence can be speeded by using an appropriate rotating sequence of time steps — some quite large — followed by a batch of short steps when one feels the solution is near.

This still leaves the free (and moving with time reintroduced) boundary value issue embodied in (7) and (8). Rather than treat it in its current form, we exploit the stochastic dynamic programming nature of this optimal stopping time problem. The condition $V \geq sB - C$ is imposed at each time step by comparing the solution so far with the value that could be realized by immediate exercise, state by state. One obtains a crude estimate of the location of the optimal exercise boundary by noting the gridpoints between which there is a switch in whether $sB - C$ is the larger.[8]

The function $sB(r, T) - C$ was obtained by numerically solving the single state variable pde (4), with terminal value 1, for unit discount bond prices B using a Crank–Nicholson procedure. Alternatively one could have used the explicit solution for bond prices given in Cox, Ingersoll and Ross (1985).

The remaining fixed boundaries were treated as follows. As mentioned before, $s = 0$ is an absorbing state, so $V(r, 0, t) = 0$ along that edge of the grid. The $r = 0$ and $r = 20\%$ boundaries were fudged. We imposed the condition that the values at those edges were quadratic extrapolations in the r direction of the adjacent interior values. This is clearly not the true boundary condition. However with mean reversion in the r process the probability of the state attaining these boundaries can be small, and thus interior solution values are little influenced by them.

4. Numerical Illustrations

Even this simple environment has an unfortunately large number of parameters that could be varied. The situations examined here are limited to the following:

Project type: In all cases the initial capital cost C of the project is fixed at 1. The project term is either 1 year (short term) or 8 years (long term).

Output price process: In all cases zero drift in s is used. I.e., $\kappa_2 = 0$. This makes

[8] See John Crank, 1984, for methods that utilize (7) and (8) directly to obtain an accurate estimate of the location of the free boundary.

the values of $\bar{s}$ and β irrelevant. Furthermore risk-neutrality with respect to s-risk is assumed. I.e., $\lambda_2 = 0$. The volatility parameter σ_2 is either 0 (no volatility) or 1 (volatile). The latter value corresponds to an instantaneous *proportional* standard deviation of 50%/year at $s = 4$, dropping with the square root of s to 25%/year at $s = 16$.

Interest rate process: Three interest rate environments are used.

a) Fixed interest rates with a flat yield curve. I.e., $\sigma_1 = \kappa_1 = 0$. This provides a reference point from which we can see what happens as the environment changes.

b) Driftless but volatile interest rates. $\sigma_1 = 0.1$, $\kappa_1 = 0$. This results in yield curves that are fairly flat but can shift.[9] This gives an instantaneous *absolute* standard deviation of r of 2.2%/year at an interest rate of 5%, rising with the square root of r.

c) A 'realistic' interest rate process. As in b) but with $\kappa_1 = 0.3$ and $\bar{r} = .05$. This means that short term rates are drawn toward a normal level of 5%, with the gap expected to close at the rate of 0.3/year.

In all three cases we assume a zero price of r-risk. The yield curves in case c) thus all start at the current r but tend toward a rate of 4.75% at infinite time to maturity.[10]

Except for one specific illustration, r and s and assumed uncorrelated: $\rho = 0$.

Nine solution are reported in the rather cluttered cross between graphs and tables that follow. The horizontal axes measure the current forward price of the output good, the vertical axes the current short term interest rate. The table entries are the values of $V(r, s)$, which will be either pure option value or the project net present value depending on whether it is in the exercise region or not. The blank space is filled with +++ in the region of the state space where the project optimally proceeds. The exercise boundary lies somewhere in the first set of these as one moves to the right along a row. Parameter values used are listed below the graph/table.

Case 1 depicts a world in which interest rates and product prices are constant, and the project is long term. The exercise region is those r, s combinations for which the project has positive net present value. The latter is simply s discounted for T years at the continuously

[9] The pure discount bond yield curve eventually tends toward 0 very far out since the now absorbing state $r = 0$ will be attained with probability one.

[10] See Cox, Ingersoll and Ross, 1985 eq.26. This does not imply risk neutral individuals in their setting, but rather that the constant returns to scale production activity in which virtually all real wealth is invested in their economy is reversible and locally riskless. The current productivity of capital is known with certainty, though its productivity a year from now is not.

compounded rate r on the vertical axis, less the project cost of 1. These are the values tabulated inside the exercise region. The opportunity has zero option value outside the region. A move to higher r can only move one out of the exercise region, cancelling the investment.

Case 2 depicts the same world but with volatile product prices. Note that the horizontal axis has been shifted to accomodate the exercise region. Three properties stand out. First, the exercise boundary lies far to the right of that of Case 1. The manager waits until the net present value is substantially positive before proceeding. Much higher product prices are required to induce investment at all levels of r. Second, although the value of the opportunity monotonically declines as r rises, the project is postponed at low rates for s levels where it would proceed at higher rates — there is a negatively sloped portion to the exercise boundary. This occurs because the opportunity cost (applied to the exercise value available immediately) of waiting for still higher NPV is low. At still higher rates the project is again postponed, this time because the option is so little 'in-the-money' because of discounting that again there is little to lose by waiting. Thus there are two interest rate levels at which the manager is just indifferent to investing. Finally, there is substantial option value to the investment opportunity outside the exercise region, including points where the project had negative NPV in Case 1.

Case 3 is identical to Case 2 except that the project is short term (1 year). The negative slope of the exercise boundary extends over the entire region depicted, emphasizing that the incentive to postpone investment at low interest rates is stronger the shorter the project duration.

Case 4 depicts a world of 'realistically' volatile interest rates but constant product prices. The exercise boundary again lies to the right of Case 1's, the manager holds out for noticeably positive NPV's, and there is value to the option even when it is out-of-the-money. Interest rate volatility by itself appears to discourage long term investment at any given level of rates since it encourages waiting for more attractive financing costs. The exercise boundary here is positively sloped but flatter than in Case 1. This indicates greater sensitivity of this type of investment to rate changes than in the fixed rate environment.

Cases 5 and 6 depict the long and short term projects with both product price volatility and the realistic interest rate environment. The observations made in Cases 4 and 3 respectively continue to hold. Having both sources of uncertainty further increases option values and shifts the exercise boundary to the right. The impact of interest rate uncertainty is least on the short term project's values. However this verifies that interest rate increases can be rationally anticipated, yet still result in a perverse response of current capital spending (what about the rush to buy houses when mortgage rates start rising?).

Case 7 is our benchmark Case 5 (long term project with both uncertainties) with strong positive correlation between the forward output price and interest rates put in: $\rho = 0.8$. This might be appropriate for a readily storable output. It expands the exercise region and renders the project much less sensitive to interests (steeper exercise boundary).

Case 8 is Case 5 with the deterministic drift in interest rates back toward $\bar{r}$ removed. This increases the option's value at low interest rates, decreases it at high rates, and moves the exercise boundary to the right. The effect is so strong that the boundary develops a backwards bend like Case 2 at rates below 9%. Thus even long term investment can exhibit the perverse interest rate response.

Finally, Case 9 depicts the long term project with driftless volatile interest rates but fixed product prices. This reproduces the case of Ingersoll and Ross (1992) in which there is interest rate uncertainty only, for which they obtain a closed form solution. The numerical results agree with theirs for this case, validating the numerical procedure.

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19.0     0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00+++0.09+++0.20+++0.31+++0.42+++0.53+++0.64+++0.75+++0.86+++0.97
18.0     0.00   0.00   0.00   0.00   0.00   0.00   0.00+++0.07+++0.19+++0.30+++0.42+++0.54+++0.66+++0.78+++0.90+++1.01+++1.13
17.0     0.00   0.00   0.00   0.00   0.00   0.00+++0.03+++0.16+++0.28+++0.41+++0.54+++0.67+++0.80+++0.93+++1.05+++1.18+++1.31
16.0     0.00   0.00   0.00   0.00   0.00   0.00+++0.11+++0.25+++0.39+++0.53+++0.67+++0.81+++0.95+++1.09+++1.23+++1.36+++1.50
15.0     0.00   0.00   0.00   0.00   0.00+++0.05+++0.21+++0.36+++0.51+++0.66+++0.81+++0.96+++1.11+++1.26+++1.41+++1.56+++1.71
14.0     0.00   0.00   0.00   0.00   0.00+++0.14+++0.31+++0.47+++0.63+++0.80+++0.96+++1.12+++1.28+++1.45+++1.61+++1.77+++1.94
13.0     0.00   0.00   0.00   0.00+++0.06+++0.24+++0.41+++0.59+++0.77+++0.94+++1.12+++1.30+++1.47+++1.65+++1.83+++2.01+++2.18
12.0     0.00   0.00   0.00   0.00+++0.15+++0.34+++0.53+++0.72+++0.91+++1.11+++1.30+++1.49+++1.68+++1.87+++2.06+++2.26+++2.45
11.0     0.00   0.00   0.00+++0.04+++0.24+++0.45+++0.66+++0.87+++1.07+++1.28+++1.49+++1.70+++1.90+++2.11+++2.32+++2.53+++2.73
10.0     0.00   0.00   0.00+++0.12+++0.35+++0.57+++0.80+++1.02+++1.25+++1.47+++1.70+++1.92+++2.15+++2.37+++2.60+++2.82+++3.04
 9.0     0.00   0.00   0.00+++0.22+++0.46+++0.70+++0.95+++1.19+++1.43+++1.68+++1.92+++2.16+++2.41+++2.65+++2.89+++3.14+++3.38
 8.0     0.00   0.00+++0.05+++0.32+++0.58+++0.85+++1.11+++1.37+++1.64+++1.90+++2.16+++2.43+++2.69+++2.96+++3.22+++3.48+++3.75
 7.0     0.00   0.00+++0.14+++0.43+++0.71+++1.00+++1.28+++1.57+++1.86+++2.14+++2.43+++2.71+++3.00+++3.28+++3.57+++3.86+++4.14
 6.0     0.00   0.00+++0.24+++0.55+++0.86+++1.17+++1.48+++1.78+++2.09+++2.40+++2.71+++3.02+++3.33+++3.64+++3.95+++4.26+++4.57
 5.0     0.00+++0.01+++0.34+++0.68+++1.01+++1.35+++1.68+++2.02+++2.35+++2.69+++3.02+++3.36+++3.69+++4.03+++4.36+++4.70+++5.03
 4.0     0.00+++0.09+++0.45+++0.82+++1.18+++1.54+++1.90+++2.27+++2.63+++2.99+++3.36+++3.72+++4.08+++4.45+++4.81+++5.17+++5.54
 3.0     0.00+++0.18+++0.57+++0.97+++1.36+++1.75+++2.15+++2.54+++2.93+++3.33+++3.72+++4.11+++4.51+++4.90+++5.29+++5.69+++6.08
 2.0     0.00+++0.28+++0.70+++1.13+++1.56+++1.98+++2.41+++2.83+++3.26+++3.69+++4.11+++4.54+++4.96+++5.39+++5.82+++6.24+++6.67
 1.0     0.00+++0.38+++0.85+++1.31+++1.77+++2.23+++2.69+++3.15+++3.61+++4.08+++4.54+++5.00+++5.46+++5.92+++6.38+++6.85+++7.31
 0.      0.00+++0.50+++1.00+++1.50+++2.00+++2.50+++3.00+++3.50+++4.00+++4.50+++5.00+++5.50+++6.00+++6.50+++7.00+++7.50+++8.00
  R
  / S    1.00   1.50   2.00   2.50   3.00   3.50   4.00   4.50   5.00   5.50   6.00   6.50   7.00   7.50   8.00   8.50   9.00

                 SIGR:  0.00    KAPR:  0.00    RBAR:  0.05    LAMR:  0.      RHO :  0.      T   :  8.00
                 SIGS:  0.00    KAPS:  0.00    SBAR:  4.00    LAMS:  0.      BETA:  0.10    C   :  1.00
```

CASE 1

```
19.0     0.11   0.15   0.19   0.24   0.29   0.35   0.41   0.49   0.57   0.65   0.75+++0.86+++0.97+++1.08+++1.19+++1.30+++1.41
18.0     0.14   0.19   0.23   0.29   0.35   0.42   0.50   0.58   0.68   0.78+++0.90+++1.01+++1.13+++1.25+++1.37+++1.49+++1.61
17.0     0.18   0.23   0.29   0.35   0.42   0.51   0.60   0.70   0.81+++0.93+++1.05+++1.18+++1.31+++1.44+++1.57+++1.70+++1.82
16.0     0.22   0.28   0.35   0.42   0.51   0.60   0.71   0.82   0.95+++1.09+++1.23+++1.36+++1.50+++1.64+++1.78+++1.92+++2.06
15.0     0.26   0.33   0.41   0.50   0.60   0.71   0.83   0.97+++1.11+++1.26+++1.41+++1.56+++1.71+++1.86+++2.01+++2.16+++2.31
14.0     0.31   0.40   0.49   0.60   0.71   0.84   0.97   1.12+++1.28+++1.45+++1.61+++1.77+++1.94+++2.10+++2.26+++2.43+++2.59
13.0     0.38   0.47   0.58   0.70   0.83   0.97   1.13+++1.30+++1.47+++1.65+++1.83+++2.01+++2.18+++2.36+++2.54+++2.71+++2.89
12.0     0.44   0.56   0.68   0.82   0.97   1.13   1.30+++1.49+++1.68+++1.87+++2.06+++2.26+++2.45+++2.64+++2.83+++3.02+++3.21
11.0     0.52   0.65   0.79   0.95   1.12   1.30+++1.49+++1.70+++1.90+++2.11+++2.32+++2.53+++2.73+++2.94+++3.15+++3.36+++3.56
10.0     0.61   0.76   0.92   1.09   1.28   1.48+++1.70+++1.92+++2.15+++2.37+++2.60+++2.82+++3.04+++3.27+++3.49+++3.72+++3.94
 9.0     0.71   0.88   1.06   1.25   1.46   1.68+++1.92+++2.16+++2.41+++2.65+++2.89+++3.14+++3.38+++3.62+++3.87+++4.11+++4.35
 8.0     0.83   1.02   1.22   1.43   1.66   1.91+++2.16+++2.43+++2.69+++2.96+++3.22+++3.48+++3.75+++4.01+++4.27+++4.54+++4.80
 7.0     0.96   1.17   1.39   1.63   1.88   2.15+++2.43+++2.71+++3.00+++3.28+++3.57+++3.86+++4.14+++4.43+++4.71+++5.00+++5.28
 6.0     1.10   1.33   1.58   1.84   2.12   2.41+++2.71+++3.02+++3.33+++3.64+++3.95+++4.26+++4.57+++4.88+++5.19+++5.50+++5.81
 5.0     1.26   1.52   1.80   2.08   2.38   2.70+++3.02+++3.36+++3.69+++4.03+++4.36+++4.70+++5.03+++5.37+++5.70+++6.04+++6.37
 4.0     1.45   1.74   2.04   2.35   2.67   3.01   3.36+++3.72+++4.08+++4.45+++4.81+++5.17+++5.54+++5.90+++6.26+++6.62+++6.99
 3.0     1.66   1.98   2.31   2.65   3.00   3.36   3.73   4.11+++4.51+++4.90+++5.29+++5.69+++6.08+++6.47+++6.87+++7.26+++7.65
 2.0     1.92   2.27   2.63   3.00   3.38   3.77   4.16   4.56   4.97   5.39+++5.82+++6.24+++6.67+++7.09+++7.52+++7.95+++8.37
 1.0     2.26   2.65   3.05   3.46   3.86   4.28   4.70   5.12   5.55   5.99   6.43   6.87   7.32   7.77+++8.23+++8.69+++9.15
  R
  / S    3.00   3.50   4.00   4.50   5.00   5.50   6.00   6.50   7.00   7.50   8.00   8.50   9.00   9.50  10.00  10.50  11.00

                 SIGR:  0.00    KAPR:  0.00    RBAR:  0.05    LAMR:  0.      RHO :  0.      T   :  8.00
                 SIGS:  1.00    KAPS:  0.00    SBAR:  4.00    LAMS:  0.      BETA:  0.10    C   :  1.00
```

CASE 2

```
19.0     0.34   0.57   0.83   1.14   1.49+++1.89+++2.31+++2.72+++3.13+++3.55+++3.96+++4.38+++4.79+++5.20+++5.62+++6.03+++6.44
18.0     0.35   0.59   0.85   1.17   1.52+++1.92+++2.34+++2.76+++3.18+++3.59+++4.01+++4.43+++4.85+++5.26+++5.68+++6.10+++6.52
17.0     0.37   0.60   0.88   1.19   1.55+++1.95+++2.37+++2.80+++3.22+++3.64+++4.06+++4.48+++4.91+++5.33+++5.75+++6.17+++6.59
16.0     0.38   0.62   0.90   1.22   1.58+++1.98+++2.41+++2.83+++3.26+++3.69+++4.11+++4.54+++4.97+++5.39+++5.82+++6.24+++6.67
15.0     0.39   0.64   0.93   1.25   1.61+++2.01+++2.44+++2.87+++3.30+++3.73+++4.16+++4.59+++5.02+++5.46+++5.89+++6.32+++6.75
14.0     0.41   0.66   0.95   1.28   1.64+++2.04+++2.48+++2.91+++3.35+++3.78+++4.22+++4.65+++5.09+++5.52+++5.95+++6.39+++6.82
13.0     0.42   0.68   0.98   1.31   1.67+++2.07+++2.51+++2.95+++3.39+++3.83+++4.27+++4.71+++5.15+++5.59+++6.02+++6.46+++6.90
12.0     0.44   0.71   1.01   1.34   1.70   2.11+++2.55+++2.99+++3.43+++3.88+++4.32+++4.77+++5.21+++5.65+++6.10+++6.54+++6.98
11.0     0.46   0.73   1.04   1.37   1.74   2.14+++2.58+++3.03+++3.48+++3.93+++4.38+++4.82+++5.27+++5.72+++6.17+++6.61+++7.06
10.0     0.48   0.76   1.07   1.41   1.78   2.18+++2.62+++3.07+++3.52+++3.98+++4.43+++4.88+++5.33+++5.79+++6.24+++6.69+++7.14
 9.0     0.49   0.78   1.10   1.45   1.82   2.22+++2.66+++3.11+++3.57+++4.03+++4.48+++4.94+++5.40+++5.85+++6.31+++6.77+++7.23
 8.0     0.52   0.81   1.14   1.49   1.86   2.27   2.69+++3.15+++3.62+++4.08+++4.54+++5.00+++5.46+++5.92+++6.38+++6.85+++7.31
 7.0     0.54   0.84   1.18   1.53   1.91   2.31   2.74+++3.20+++3.66+++4.13+++4.59+++5.06+++5.53+++5.99+++6.46+++6.93+++7.39
 6.0     0.56   0.88   1.22   1.58   1.96   2.37   2.79   3.24+++3.71+++4.18+++4.65+++5.12+++5.59+++6.06+++6.53+++7.01+++7.48
 5.0     0.59   0.92   1.27   1.63   2.02   2.43   2.85   3.29+++3.76+++4.23+++4.71+++5.18+++5.66+++6.13+++6.61+++7.09+++7.56
 4.0     0.63   0.97   1.32   1.70   2.09   2.50   2.92   3.36   3.81+++4.28+++4.76+++5.25+++5.73+++6.21+++6.69+++7.17+++7.65
 3.0     0.67   1.02   1.39   1.78   2.17   2.58   3.00   3.44   3.89   4.35+++4.82+++5.31+++5.79+++6.28+++6.76+++7.25+++7.73
 2.0     0.72   1.10   1.48   1.88   2.28   2.70   3.12   3.56   4.00   4.45   4.91   5.38   5.86+++6.35+++6.84+++7.33+++7.82
 1.0     0.80   1.20   1.61   2.03   2.45   2.88   3.31   3.75   4.19   4.64   5.09   5.55   6.02   6.48   6.96   7.44   7.92
 R
 / S     1.00   1.50   2.00   2.50   3.00   3.50   4.00   4.50   5.00   5.50   6.00   6.50   7.00   7.50   8.00   8.50   9.00

                    SIGR:  0.00    KAPR:  0.00    RBAR:  0.05    LAMR:  0.      RHO :  0.      T   :  1.00
                    SIGS:  1.00    KAPS:  0.00    SBAR:  4.00    LAMS:  0.      BETA:  0.10    C   :  1.00
```

CASE 3

```
19.0     0.00   0.03   0.17   0.34   0.52   0.70   0.90   1.10   1.30   1.51   1.72   1.93   2.15   2.37+++2.59+++2.82+++3.04
18.0     0.00   0.03   0.18   0.35   0.54   0.73   0.93   1.14   1.35   1.57   1.79   2.01   2.24+++2.47+++2.70+++2.93+++3.16
17.0     0.00   0.04   0.19   0.37   0.56   0.76   0.97   1.19   1.41   1.64   1.86   2.10+++2.33+++2.57+++2.81+++3.05+++3.28
16.0     0.00   0.04   0.20   0.38   0.59   0.80   1.02   1.24   1.47   1.71   1.94+++2.19+++2.43+++2.68+++2.92+++3.17+++3.41
15.0     0.00   0.04   0.21   0.40   0.61   0.83   1.06   1.30   1.53   1.78   2.03+++2.28+++2.53+++2.79+++3.04+++3.29+++3.54
14.0     0.00   0.04   0.22   0.42   0.64   0.87   1.11   1.35   1.60   1.86+++2.12+++2.38+++2.64+++2.90+++3.16+++3.42+++3.68
13.0     0.00   0.04   0.23   0.44   0.67   0.91   1.16   1.41   1.68+++1.94+++2.21+++2.48+++2.75+++3.01+++3.28+++3.55+++3.82
12.0     0.00   0.05   0.24   0.46   0.70   0.95   1.21   1.48+++1.76+++2.03+++2.31+++2.58+++2.86+++3.13+++3.41+++3.68+++3.96
11.0     0.00   0.05   0.25   0.48   0.74   1.00   1.27+++1.55+++1.84+++2.12+++2.41+++2.69+++2.97+++3.26+++3.54+++3.82+++4.11
10.0     0.00   0.05   0.26   0.51   0.77   1.05+++1.34+++1.63+++1.92+++2.21+++2.51+++2.80+++3.09+++3.38+++3.68+++3.97+++4.26
 9.0     0.00   0.05   0.28   0.54   0.82   1.11+++1.41+++1.71+++2.01+++2.31+++2.61+++2.91+++3.21+++3.51+++3.81+++4.12+++4.42
 8.0     0.00   0.06   0.29   0.57   0.86+++1.17+++1.48+++1.79+++2.10+++2.41+++2.72+++3.03+++3.34+++3.65+++3.96+++4.27+++4.58
 7.0     0.00   0.06   0.31   0.60+++0.91+++1.23+++1.55+++1.87+++2.19+++2.51+++2.83+++3.15+++3.47+++3.79+++4.11+++4.42+++4.74
 6.0     0.00   0.06   0.33   0.64+++0.97+++1.30+++1.63+++1.96+++2.29+++2.61+++2.94+++3.27+++3.60+++3.93+++4.26+++4.59+++4.91
 5.0     0.00   0.07   0.36+++0.69+++1.03+++1.37+++1.71+++2.05+++2.38+++2.72+++3.06+++3.40+++3.74+++4.08+++4.41+++4.75+++5.09
 4.0     0.00   0.08+++0.39+++0.74+++1.09+++1.44+++1.79+++2.14+++2.48+++2.83+++3.18+++3.53+++3.88+++4.23+++4.58+++4.92+++5.27
 3.0     0.00   0.09+++0.44+++0.79+++1.15+++1.51+++1.87+++2.23+++2.59+++2.95+++3.31+++3.66+++4.02+++4.38+++4.74+++5.10+++5.46
 2.0     0.00+++0.11+++0.48+++0.85+++1.22+++1.59+++1.96+++2.33+++2.70+++3.06+++3.43+++3.80+++4.17+++4.54+++4.91+++5.28+++5.65
 1.0     0.00+++0.14+++0.52+++0.90+++1.28+++1.66+++2.04+++2.42+++2.80+++3.19+++3.57+++3.95+++4.33+++4.71+++5.09+++5.47+++5.85
 R
 / S     1.00   1.50   2.00   2.50   3.00   3.50   4.00   4.50   5.00   5.50   6.00   6.50   7.00   7.50   8.00   8.50   9.00

                    SIGR:  0.10    KAPR:  0.30    RBAR:  0.05    LAMR:  0.      RHO :  0.      T   :  8.00
                    SIGS:  0.00    KAPS:  0.00    SBAR:  4.00    LAMS:  0.      BETA:  0.10    C   :  1.00
```

CASE 4

19.0	0.83	0.99	1.15	1.32	1.49	1.67	1.85	2.04	2.24	2.44	2.64	2.85	3.06	3.27	3.49+++	3.71+++	3.94
18.0	0.86	1.02	1.19	1.36	1.54	1.73	1.92	2.12	2.32	2.52	2.73	2.95	3.17	3.39+++	3.62+++	3.85+++	4.08
17.0	0.89	1.05	1.23	1.41	1.60	1.79	1.99	2.19	2.40	2.62	2.84	3.06	3.29+++	3.52+++	3.76+++	4.00+++	4.24
16.0	0.92	1.09	1.27	1.46	1.65	1.85	2.06	2.27	2.49	2.71	2.94	3.17	3.41+++	3.66+++	3.90+++	4.15+++	4.39
15.0	0.95	1.13	1.32	1.51	1.71	1.92	2.13	2.35	2.58	2.81	3.05	3.29+++	3.54+++	3.80+++	4.05+++	4.30+++	4.55
14.0	0.98	1.17	1.36	1.57	1.77	1.99	2.21	2.44	2.68	2.92	3.16	3.42+++	3.68+++	3.94+++	4.20+++	4.46+++	4.72
13.0	1.01	1.21	1.41	1.62	1.84	2.06	2.29	2.53	2.77	3.03	3.28+++	3.55+++	3.82+++	4.08+++	4.35+++	4.62+++	4.89
12.0	1.05	1.25	1.46	1.68	1.90	2.14	2.38	2.62	2.88	3.14+++	3.41+++	3.68+++	3.96+++	4.24+++	4.51+++	4.79+++	5.06
11.0	1.09	1.30	1.51	1.74	1.97	2.21	2.46	2.72	2.99	3.26+++	3.54+++	3.82+++	4.11+++	4.39+++	4.68+++	4.96+++	5.24
10.0	1.12	1.34	1.57	1.80	2.04	2.29	2.55	2.82	3.10+++	3.38+++	3.68+++	3.97+++	4.26+++	4.55+++	4.84+++	5.14+++	5.43
9.0	1.16	1.39	1.62	1.86	2.12	2.38	2.65	2.93	3.22+++	3.51+++	3.81+++	4.12+++	4.42+++	4.72+++	5.02+++	5.32+++	5.62
8.0	1.20	1.44	1.68	1.93	2.19	2.47	2.75	3.04+++	3.34+++	3.65+++	3.96+++	4.27+++	4.58+++	4.89+++	5.20+++	5.51+++	5.82
7.0	1.25	1.49	1.74	2.00	2.27	2.56	2.85	3.15+++	3.47+++	3.79+++	4.11+++	4.42+++	4.74+++	5.06+++	5.38+++	5.70+++	6.02
6.0	1.29	1.54	1.80	2.07	2.36	2.65	2.96	3.27+++	3.60+++	3.93+++	4.26+++	4.59+++	4.91+++	5.24+++	5.57+++	5.90+++	6.23
5.0	1.33	1.59	1.87	2.15	2.44	2.75	3.07+++	3.40+++	3.74+++	4.08+++	4.41+++	4.75+++	5.09+++	5.43+++	5.77+++	6.11+++	6.44
4.0	1.38	1.65	1.93	2.23	2.53	2.85	3.18+++	3.53+++	3.88+++	4.23+++	4.58+++	4.92+++	5.27+++	5.62+++	5.97+++	6.32+++	6.67
3.0	1.43	1.71	2.00	2.31	2.63	2.96+++	3.31+++	3.66+++	4.02+++	4.38+++	4.74+++	5.10+++	5.46+++	5.82+++	6.18+++	6.54+++	6.89
2.0	1.48	1.77	2.08	2.39	2.73	3.07+++	3.43+++	3.80+++	4.17+++	4.54+++	4.91+++	5.28+++	5.65+++	6.02+++	6.39+++	6.76+++	7.13
1.0	1.53	1.83	2.15	2.48	2.83	3.19+++	3.57+++	3.95+++	4.33+++	4.71+++	5.09+++	5.47+++	5.85+++	6.23+++	6.61+++	6.99+++	7.37
R / S	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50	10.00	10.50	11.00

SIGR: 0.10 KAPR: 0.30 RBAR: 0.05 LAMR: 0. RHO : 0. T : 8.00
SIGS: 1.00 KAPS: 0.00 SBAR: 4.00 LAMS: 0. BETA: 0.10 C : 1.00

CASE 5

19.0	0.45	0.70	0.98	1.28	1.61	1.97+++	2.37+++	2.79+++	3.22+++	3.64+++	4.06+++	4.48+++	4.90+++	5.32+++	5.74+++	6.17+++	6.59
18.0	0.46	0.71	0.99	1.30	1.63	2.00+++	2.40+++	2.83+++	3.25+++	3.68+++	4.10+++	4.53+++	4.95+++	5.38+++	5.80+++	6.23+++	6.65
17.0	0.47	0.73	1.01	1.32	1.66	2.03+++	2.43+++	2.86+++	3.29+++	3.72+++	4.15+++	4.58+++	5.00+++	5.43+++	5.86+++	6.29+++	6.72
16.0	0.48	0.75	1.04	1.35	1.69	2.06+++	2.46+++	2.89+++	3.33+++	3.76+++	4.19+++	4.62+++	5.06+++	5.49+++	5.92+++	6.35+++	6.79
15.0	0.49	0.76	1.06	1.37	1.72	2.09+++	2.49+++	2.93+++	3.36+++	3.80+++	4.24+++	4.67+++	5.11+++	5.54+++	5.98+++	6.42+++	6.85
14.0	0.50	0.78	1.08	1.40	1.75	2.12	2.52+++	2.96+++	3.40+++	3.84+++	4.28+++	4.72+++	5.16+++	5.60+++	6.04+++	6.48+++	6.92
13.0	0.51	0.80	1.10	1.43	1.78	2.15	2.56+++	3.00+++	3.44+++	3.88+++	4.33+++	4.77+++	5.21+++	5.66+++	6.10+++	6.55+++	6.99
12.0	0.52	0.81	1.12	1.45	1.81	2.19	2.59+++	3.03+++	3.48+++	3.93+++	4.37+++	4.82+++	5.27+++	5.72+++	6.16+++	6.61+++	7.06
11.0	0.54	0.83	1.15	1.48	1.84	2.22	2.63+++	3.06+++	3.52+++	3.97+++	4.42+++	4.87+++	5.32+++	5.77+++	6.23+++	6.68+++	7.13
10.0	0.55	0.85	1.17	1.51	1.87	2.26	2.66+++	3.10+++	3.56+++	4.01+++	4.47+++	4.92+++	5.38+++	5.83+++	6.29+++	6.74+++	7.20
9.0	0.56	0.87	1.20	1.54	1.91	2.29	2.70+++	3.14+++	3.60+++	4.05+++	4.51+++	4.97+++	5.43+++	5.89+++	6.35+++	6.81+++	7.27
8.0	0.58	0.89	1.22	1.57	1.94	2.33	2.74	3.18+++	3.63+++	4.10+++	4.56+++	5.03+++	5.49+++	5.95+++	6.42+++	6.88+++	7.34
7.0	0.59	0.91	1.25	1.60	1.98	2.37	2.78	3.22+++	3.68+++	4.14+++	4.61+++	5.08+++	5.55+++	6.01+++	6.48+++	6.95+++	7.42
6.0	0.60	0.93	1.27	1.64	2.01	2.41	2.83	3.26+++	3.72+++	4.19+++	4.66+++	5.13+++	5.60+++	6.07+++	6.54+++	7.02+++	7.49
5.0	0.62	0.95	1.30	1.67	2.05	2.45	2.87	3.31	3.76+++	4.23+++	4.71+++	5.18+++	5.66+++	6.13+++	6.61+++	7.09+++	7.56
4.0	0.63	0.97	1.33	1.70	2.09	2.50	2.92	3.35	3.81+++	4.28+++	4.76+++	5.24+++	5.72+++	6.20+++	6.68+++	7.16+++	7.64
3.0	0.65	1.00	1.36	1.74	2.13	2.54	2.97	3.40	3.86	4.32+++	4.81+++	5.29+++	5.77+++	6.26+++	6.74+++	7.23+++	7.71
2.0	0.67	1.02	1.39	1.78	2.18	2.59	3.02	3.46	3.91	4.38+++	4.86+++	5.35+++	5.83+++	6.32+++	6.81+++	7.30+++	7.79
1.0	0.68	1.05	1.42	1.81	2.22	2.64	3.07	3.51	3.97	4.44	4.91+++	5.40+++	5.89+++	6.39+++	6.88+++	7.37+++	7.86
R / S	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00

SIGR: 0.10 KAPR: 0.30 RBAR: 0.05 LAMR: 0. RHO : 0. T : 1.00
SIGS: 1.00 KAPS: 0.00 SBAR: 4.00 LAMS: 0. BETA: 0.10 C : 1.00

CASE 6

19.0	0.63	0.78	0.94	1.11	1.30	1.49	1.70+++	1.92+++	2.14+++	2.37+++	2.59+++	2.82+++	3.04+++	3.26+++	3.49+++	3.71+++	3.94
18.0	0.66	0.82	0.98	1.16	1.35	1.56	1.78+++	2.00+++	2.24+++	2.47+++	2.70+++	2.93+++	3.16+++	3.39+++	3.62+++	3.85+++	4.08
17.0	0.69	0.85	1.03	1.21	1.41	1.63+++	1.86+++	2.09+++	2.33+++	2.57+++	2.81+++	3.05+++	3.28+++	3.52+++	3.76+++	4.00+++	4.24
16.0	0.72	0.89	1.07	1.27	1.48	1.70+++	1.94+++	2.19+++	2.43+++	2.68+++	2.92+++	3.17+++	3.41+++	3.66+++	3.90+++	4.15+++	4.39
15.0	0.75	0.93	1.12	1.33	1.55	1.78+++	2.03+++	2.28+++	2.53+++	2.79+++	3.04+++	3.29+++	3.54+++	3.80+++	4.05+++	4.30+++	4.55
14.0	0.79	0.97	1.17	1.39	1.61	1.86+++	2.12+++	2.38+++	2.64+++	2.90+++	3.16+++	3.42+++	3.68+++	3.94+++	4.20+++	4.46+++	4.72
13.0	0.82	1.02	1.22	1.45	1.69+++	1.94+++	2.21+++	2.48+++	2.75+++	3.01+++	3.28+++	3.55+++	3.82+++	4.08+++	4.35+++	4.62+++	4.89
12.0	0.86	1.06	1.28	1.51	1.76+++	2.03+++	2.31+++	2.58+++	2.86+++	3.13+++	3.41+++	3.68+++	3.96+++	4.24+++	4.51+++	4.79+++	5.06
11.0	0.90	1.11	1.34	1.58	1.84+++	2.12+++	2.41+++	2.69+++	2.97+++	3.26+++	3.54+++	3.82+++	4.11+++	4.39+++	4.68+++	4.96+++	5.24
10.0	0.94	1.16	1.40	1.65	1.92+++	2.21+++	2.51+++	2.80+++	3.09+++	3.38+++	3.68+++	3.97+++	4.26+++	4.55+++	4.84+++	5.14+++	5.43
9.0	0.99	1.21	1.46	1.73+++	2.01+++	2.31+++	2.61+++	2.91+++	3.21+++	3.51+++	3.81+++	4.12+++	4.42+++	4.72+++	5.02+++	5.32+++	5.62
8.0	1.03	1.27	1.53	1.80+++	2.10+++	2.41+++	2.72+++	3.03+++	3.34+++	3.65+++	3.96+++	4.27+++	4.58+++	4.89+++	5.20+++	5.51+++	5.82
7.0	1.08	1.33	1.59	1.88+++	2.19+++	2.51+++	2.83+++	3.15+++	3.47+++	3.79+++	4.11+++	4.42+++	4.74+++	5.06+++	5.38+++	5.70+++	6.02
6.0	1.13	1.39	1.67	1.97+++	2.29+++	2.61+++	2.94+++	3.27+++	3.60+++	3.93+++	4.26+++	4.59+++	4.91+++	5.24+++	5.57+++	5.90+++	6.23
5.0	1.18	1.45	1.74	2.05+++	2.38+++	2.72+++	3.06+++	3.40+++	3.74+++	4.08+++	4.41+++	4.75+++	5.09+++	5.43+++	5.77+++	6.11+++	6.44
4.0	1.24	1.52	1.82	2.14+++	2.48+++	2.83+++	3.18+++	3.53+++	3.88+++	4.23+++	4.58+++	4.92+++	5.27+++	5.62+++	5.97+++	6.32+++	6.67
3.0	1.29	1.59	1.90	2.23+++	2.59+++	2.95+++	3.31+++	3.66+++	4.02+++	4.38+++	4.74+++	5.10+++	5.46+++	5.82+++	6.18+++	6.54+++	6.89
2.0	1.35	1.66	1.98	2.33+++	2.70+++	3.06+++	3.43+++	3.80+++	4.17+++	4.54+++	4.91+++	5.28+++	5.65+++	6.02+++	6.39+++	6.76+++	7.13
1.0	1.42	1.73	2.07	2.43+++	2.80+++	3.19+++	3.57+++	3.95+++	4.33+++	4.71+++	5.09+++	5.47+++	5.85+++	6.23+++	6.61+++	6.99+++	7.37
R / S	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50	10.00	10.50	11.00

SIGR: 0.10 KAPR: 0.30 RBAR: 0.05 LAMR: 0. RHO : 0.80 T : 8.00
SIGS: 1.00 KAPS: 0.00 SBAR: 4.00 LAMS: 0. BETA: 0.10 C : 1.00

CASE 7

19.0	0.42	0.49	0.57	0.66	0.74	0.83	0.91	1.00	1.10	1.19	1.29	1.39	1.49	1.59	1.70	1.81	1.92
18.0	0.44	0.52	0.60	0.69	0.78	0.87	0.97	1.06	1.16	1.27	1.37	1.48	1.60	1.71	1.83	1.95	2.08
17.0	0.47	0.55	0.64	0.73	0.83	0.93	1.03	1.14	1.25	1.36	1.48	1.60	1.73	1.86	1.99	2.13	2.27
16.0	0.50	0.59	0.69	0.79	0.90	1.00	1.12	1.23	1.36	1.48	1.61	1.75	1.89	2.03	2.18+++	2.33+++	2.49
15.0	0.54	0.64	0.75	0.86	0.98	1.09	1.22	1.35	1.48	1.62	1.76	1.91	2.07	2.23+++	2.40+++	2.57+++	2.74
14.0	0.59	0.71	0.82	0.94	1.07	1.20	1.34	1.48	1.63	1.78	1.94	2.11	2.28+++	2.47+++	2.65+++	2.83+++	3.01
13.0	0.65	0.78	0.91	1.04	1.18	1.32	1.47	1.63	1.79	1.96	2.14	2.33+++	2.52+++	2.72+++	2.91+++	3.11+++	3.31
12.0	0.72	0.86	1.00	1.15	1.30	1.46	1.62	1.80	1.98	2.17	2.37+++	2.57+++	2.78+++	2.99+++	3.20+++	3.41+++	3.62
11.0	0.80	0.95	1.11	1.27	1.44	1.61	1.80	1.99	2.19	2.39+++	2.61+++	2.84+++	3.06+++	3.29+++	3.52+++	3.74+++	3.97
10.0	0.89	1.06	1.23	1.41	1.60	1.79	1.99	2.20	2.42	2.64+++	2.88+++	3.12+++	3.37+++	3.61+++	3.85+++	4.09+++	4.34
9.0	1.00	1.18	1.37	1.57	1.77	1.98	2.20	2.43	2.67	2.91+++	3.17+++	3.43+++	3.69+++	3.95+++	4.21+++	4.47+++	4.74
8.0	1.11	1.32	1.52	1.74	1.96	2.19	2.43	2.68	2.94	3.21+++	3.48+++	3.76+++	4.04+++	4.32+++	4.60+++	4.88+++	5.16
7.0	1.24	1.47	1.70	1.94	2.18	2.43	2.69	2.96	3.24	3.52+++	3.82+++	4.12+++	4.42+++	4.72+++	5.02+++	5.33+++	5.63
6.0	1.39	1.64	1.89	2.15	2.42	2.69	2.97	3.26	3.56	3.87+++	4.18+++	4.50+++	4.83+++	5.15+++	5.48+++	5.80+++	6.12
5.0	1.55	1.83	2.11	2.39	2.69	2.98	3.29	3.60	3.92	4.24	4.58+++	4.92+++	5.27+++	5.61+++	5.96+++	6.31+++	6.66
4.0	1.74	2.04	2.35	2.66	2.98	3.31	3.64	3.97	4.31	4.66	5.01	5.37	5.74+++	6.11+++	6.48+++	6.86+++	7.23
3.0	1.94	2.28	2.62	2.97	3.32	3.67	4.03	4.39	4.76	5.13	5.50	5.88	6.27	6.66	7.05+++	7.45+++	7.85
2.0	2.18	2.55	2.93	3.31	3.69	4.08	4.47	4.86	5.26	5.66	6.06	6.46	6.87	7.28	7.70	8.12	8.54
1.0	2.44	2.86	3.28	3.70	4.12	4.54	4.97	5.39	5.82	6.26	6.69	7.13	7.57	8.01	8.45	8.89	9.34
R / S	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50	10.00	10.50	11.00

SIGR: 0.10 KAPR: 0.00 RBAR: 0.05 LAMR: 0. RHO : 0. T : 8.00
SIGS: 1.00 KAPS: 0.00 SBAR: 4.00 LAMS: 0. BETA: 0.10 C : 1.00

CASE 8

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18.0     0.00   0.04   0.08   0.12   0.17   0.22   0.27   0.32   0.38   0.49+++0.63+++0.76+++0.90+++1.04+++1.17+++1.31+++1.44
17.0     0.00   0.04   0.09   0.14   0.19   0.25   0.31   0.37   0.46+++0.61+++0.75+++0.90+++1.04+++1.19+++1.34+++1.48+++1.63
16.0     0.00   0.05   0.10   0.16   0.22   0.29   0.35   0.43+++0.57+++0.73+++0.88+++1.04+++1.20+++1.35+++1.51+++1.67+++1.82
15.0     0.00   0.06   0.12   0.18   0.25   0.33   0.40   0.52+++0.69+++0.86+++1.02+++1.19+++1.36+++1.53+++1.70+++1.87+++2.04
14.0     0.00   0.07   0.14   0.21   0.29   0.38   0.47+++0.63+++0.81+++1.00+++1.18+++1.36+++1.54+++1.72+++1.90+++2.08+++2.26
13.0     0.00   0.08   0.16   0.24   0.34   0.43   0.56+++0.75+++0.95+++1.14+++1.34+++1.53+++1.73+++1.92+++2.12+++2.31+++2.51
12.0     0.00   0.09   0.18   0.28   0.39   0.49+++0.68+++0.89+++1.10+++1.31+++1.52+++1.73+++1.94+++2.14+++2.35+++2.56+++2.77
11.0     0.00   0.10   0.21   0.32   0.44   0.58+++0.80+++1.03+++1.25+++1.48+++1.70+++1.93+++2.16+++2.38+++2.61+++2.83+++3.06
10.0     0.00   0.12   0.24   0.37   0.50+++0.70+++0.94+++1.18+++1.42+++1.67+++1.91+++2.15+++2.39+++2.63+++2.88+++3.12+++3.36
 9.0     0.00   0.14   0.28   0.43   0.58+++0.82+++1.08+++1.34+++1.61+++1.87+++2.13+++2.39+++2.65+++2.91+++3.17+++3.43+++3.69
 8.0     0.00   0.16   0.32   0.49   0.68+++0.96+++1.24+++1.52+++1.80+++2.08+++2.36+++2.64+++2.92+++3.20+++3.48+++3.76+++4.04
 7.0     0.00   0.18   0.37   0.56+++0.81+++1.11+++1.41+++1.71+++2.01+++2.31+++2.61+++2.91+++3.22+++3.52+++3.82+++4.12+++4.42
 6.0     0.00   0.21   0.42   0.64+++0.94+++1.27+++1.59+++1.91+++2.24+++2.56+++2.88+++3.21+++3.53+++3.86+++4.18+++4.50+++4.83
 5.0     0.00   0.24   0.49   0.74+++1.09+++1.44+++1.78+++2.13+++2.48+++2.83+++3.18+++3.52+++3.87+++4.22+++4.57+++4.92+++5.27
 4.0     0.00   0.28   0.56   0.87+++1.25+++1.62+++1.99+++2.37+++2.74+++3.12+++3.49+++3.86+++4.24+++4.61+++4.99+++5.36+++5.74
 3.0     0.00   0.32   0.65+++1.01+++1.41+++1.82+++2.22+++2.62+++3.02+++3.43+++3.83+++4.23+++4.63+++5.03+++5.44+++5.84+++6.24
 2.0     0.00   0.37   0.75+++1.16+++1.59+++2.03+++2.46+++2.89+++3.32+++3.76+++4.19+++4.62+++5.05+++5.49+++5.92+++6.35+++6.78
 1.0     0.00   0.42   0.86+++1.32+++1.79+++2.25+++2.72+++3.18+++3.65+++4.11+++4.58+++5.04+++5.51+++5.97+++6.44+++6.90+++7.37
 0.0     0.00+++0.50+++1.00+++1.50+++2.00+++2.50+++3.00+++3.50+++4.00+++4.50+++5.00+++5.50+++6.00+++6.50+++7.00+++7.50+++8.00
 R
  / S    1.00   1.50   2.00   2.50   3.00   3.50   4.00   4.50   5.00   5.50   6.00   6.50   7.00   7.50   8.00   8.50   9.00

                SIGR:  0.10    KAPR:  0.00    RBAR:  0.05    LAMR:  0.      RHO :  0.      T   :  8.00
                SIGS:  0.00    KAPS:  0.00    SBAR:  4.00    LAMS:  0.      BETA:  0.10    C   :  1.00
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CASE 9