Economics 811
Sample Exam Questions

1. Suppose $x(t)$ is a random variable whose movement over time is described by the stochastic differential equation

$$dx = ax(b - \ln x)\,dt + cx\,dz$$

in which a, b, c are constants and dz is the increment in a standard Weiner process (i.e., standard Brownian motion).

   (a) Let $y = \ln x$. What is the stochastic differential equation describing the process followed by y ?

   (b) Suppose that $a = 0$. Conditional on $y(0) = 0$, what is the probability distribution of $y(2)$?

2. For each pair, state which of the two bonds has the higher 'duration' (sensitivity of value to parallel shift in the yield curve). The current yield curve is assumed to be the same in all cases.

   (a) a coupon bond with 10 years to mature and 5% coupon rate versus a coupon bond with 10 years to mature and 8% coupon rate

   (b) a fully amortizing mortgage making constant monthly payments versus a standard bond of the same maturity and contractual interest rate

   (c) a pure discount bond with 2 years to mature versus a 4 year interest only loan with interest rate adjusted monthly to the then prevailing one month interest rate

3. *Briefly* explain the distinction between:

   (a) a partial differential equation and a stochastic differential equation

   (b) an asset swap and a credit default swap

   (c) put-call parity and covered interest parity

   (d) local expectations hypothesis and the unbiased expectations hypothesis

   (e) credit rating and credit spread

4. State whether the statement is true, false or uncertain and provide your reasoning.

   (a) The expected real rate of return on a nominal bond is its yield to maturity minus the expected inflation rate during its term to maturity.

   (b) Risk neutral investors would pay a higher price for an option than would risk averse investors.

   (c) The instantaneous expected rate of return on a bond is its yield to maturity.

5. Under what conditions would forward prices be the best available (i.e., unbiased and minimum error variance) forecast of future spot prices? Explain briefly why your conditions are necessary.

6. In the first homework assignment, you determined the value of a 6 month American option on a *fixed characteristics* 2 year pure discount bond. I.e., whenever exercised, the bond delivered would have 2 years to maturity as of the exercise date. Suppose instead that the American option was on a bond of *fixed maturity date* that was 2 years from now. Outline in point form (a flow diagram) a computer program that would value such an option. If it had the same exercise price, do you think the option would have higher or lower value than the fixed characteristics bond option?

7. **When to cut the forest**: Suppose you own a tree whose volume at age t will be $(1 - e^{-t})A$. Suppose that lumber prices P are random and follow the stochastic process $dP = \sigma P\, dz$. Suppose finally that the tree will rot and become worthless the instant after it is 50 years old. Describe how you would determine the rational rule for when to harvest the tree, and what you would be willing to pay for the tree just after it is planted. Be sure to state all boundary conditions that must be imposed on any partial differential equation you set down. Outline in point form a computer program to solve the problem numerically.

8. If the instantaneous arrival rate (Poisson intensity) of default is $\lambda(t)$ for times $0 \leq t \leq T$, what is the probability of defaulting before time T?

9. Consider a firm whose value $s(t)$ follows a lognormal diffusion with constant proportional volatility σ and which pays no dividends. Its only debt is a \$100 face value bond, maturing at time T, and paying coupons continuously at the rate c dollars/year. Default occurs at the first time that s falls to \$50, if that happens before T. The continuously compounded riskless interest rate is constant at r. There are no bankruptcy costs.

   Let $V(s,t)$ denote the equilibrium (arbitrage-free) market value of the bond at time t if value of the firm is s at that time. Write down the partial differential equation that $V(s,t)$ must satisfy, including any applicable boundary conditions.