

THE EVOLUTIONARY BASIS OF TIME PREFERENCE: INTERGENERATIONAL TRANSFERS AND SEX

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ABSTRACT

We consider the evolutionary basis of intertemporal choice and the rate of time preference, in particular, when there are intergenerational transfers. We show that the notion of “reproductive value” from biology provides the utility criterion for a parent to optimize the allocation of resources between transfers to offspring and for promoting her own survival to the next period. This optimization has a natural dynamic programming formulation. We show that younger individuals may well be “too impatient” but older individuals “too patient,” in a sense that agrees with observations. We compare the allocation of resources under sexual reproduction to that where there is asexual reproduction. Sex distorts the rate of time preference, but there is no general bias towards greater impatience; under plausible conditions, sex may well imply greater patience.

1 Introduction

Most models in economics take preferences as given and then derive the choices induced by these preferences. We turn this around, first characterizing the choice behavior that would result from biological evolution, and then supporting this choice behavior with preferences. That is, we identify the preferences that induce evolutionarily optimal choices.

We make the stark assumption that each choice behavior derives from a particular genetic type. Hence, the choices an individual makes during her lifetime are a characteristic that is inherited from her parents. In the simplest case, without sexual reproduction, this inheritance is without modification, so populations can be defined as a group of individuals having the same genes. Populations with different genetic types may grow at different long run rates. Only those types inducing the highest asymptotic population growth rate survive evolution.

We consider here the biological basis of intertemporal utility and time preferences, in particular. Why do we discount the future at all? What accounts for how much we discount the future and for the age profile of discount rates?

Perhaps the most basic biological model suggests that we should discount the future at the sum of the rate of population growth and the rate of mortality. (See Robson and Samuelson [21], for example.) To set the stage for the present paper, we present an example to exhibit this basic result. To simplify matters, we consider here a direct tradeoff between current and future offspring. In the main model of the paper with transfers, we consider a tradeoff between the survival probabilities of newborns and the survival probability of the parent to the next period. However, the fundamental considerations are the same.

Example 1. Suppose that an individual survives for sure from age zero to age one, and survives with probability p from age one to age two, but then dies. She has a resource endowment of I which she can split between ages one and two. In each of these periods, if she is alive, she transforms her endowment into offspring according to a continuously differentiable, strictly increasing and strictly concave reproduction function u . Assume that $u'(x) \rightarrow \infty$ as $x \rightarrow 0$. A gene is identified with a choice rule $x \in [0, I]$, the use of the endowment in the first period, so the endowment available in the second period is $I - x$. Offspring use the same rule as their parent.

Fix a choice, x , and denote the number of individuals who are one year old at time t by y_t . The following equation recursively defines the law of motion of y_t :

$$y_{t+1} = u(x) y_t + pu(I - x) y_{t-1}.$$

Indeed, the number of one year old individuals at time $t + 1$ who had one year old parents at birth is $u(x) y_t$. Similarly, $pu(I - x) y_{t-1}$ is the number of one year old individuals at time $t + 1$ whose parents were two years old when they were born.

Dividing both sides by y_t yields

$$\frac{y_{t+1}}{y_t} = u(x) + pu(I - x) \frac{y_{t-1}}{y_t}.$$

It is easy to show that there exists a value of $\lambda > 0$ such that $y_{t+1}/y_t \rightarrow \lambda$ as $t \rightarrow \infty$, no matter what the initial proportions of one-year-old and two-year-old individuals might be in the population. In the limit, therefore, the previous equation can be rewritten as

$$\lambda^2 = \lambda u(x) + pu(I - x).$$

Let $\lambda(x)$ denote the unique positive solution of this equation for λ , let x^* denote the choice generating the largest possible growth rate and set $\lambda^* = \lambda(x^*)$. Since there must be an interior solution, x^* satisfies the first-order condition $\lambda'(x^*) = 0$. Differentiating the previous displayed equation and rearranging, we obtain

$$u'(x^*) = pu'(I - x^*)/\lambda^*.$$

This equation implies that the optimal decision x^* must be the unique solution of¹

$$\max_x u(x) + \frac{pu(I - x)}{\lambda^*}. \tag{1}$$

Expression (1) means that the resource allocation that maximizes the population growth factor, which is the basic biological problem, must also be the allocation that maximizes the expected discounted number of offspring, with the discount factor equal to the probability of survival divided by the *maximal* population growth factor.² This latter problem is closely related to the simplest conventional criterion for intertemporal choice, expected discounted utility, where utility is identified with fertility, and where future fertility is deflated by the growth rate divided by the probability of survival. Equivalently, we

¹Given this uniqueness, the solution here will be supported by other criteria. We address this issue in Example 2 below.

²What is the implied attitude to risk in this example? Individuals are risk-neutral in *offspring*, since the fertility function here is best interpreted as *expected* offspring. However, it follows readily that Expression (1) is strictly concave in the total endowment, I , so individuals are strictly risk-averse in resources. The Bernoulli utility function here is a biological production function. These observations generalize straightforwardly to the models in the present paper, given all risk is idiosyncratic.

have found that the *pure* rate of time preference is equal to the rate of mortality plus the rate of population growth.³ The intuition behind the contribution of population growth is the following. If the population is in steady state growth, with growth factor λ , the value of an expected offspring tomorrow, as measured by the relative contribution to the gene pool, is less, by this factor of λ , than the value of an expected offspring today.⁴ ■

Many nonhuman species seem to be shaped by a discount rate, one of whose components is mortality. For example, birds typically suffer lower rates of predation than do comparably-sized ground-dwelling mammalian species. As an apparent consequence, they invest more in somatic maintenance and may live even longer than implied directly by the lower predation. Perhaps also as a consequence, birds invest heavily in the rearing of offspring, with the involvement not merely of the female, but also of the male, whereas male involvement is rare in ground-dwelling mammals.⁵

It is hardly surprising that mortality would also influence the rate of time discounting in humans, and well-known (see Irving Fisher [9], for example). The effect of population growth on discounting is a recent insight provided by a biological approach. This component of the rate of discounting should be constant across all ages, thus providing some support for the simplest of all economic criteria for evaluating intertemporal consumption, namely the sum of discounted utility, where the discount factor is constant.

However, this basic result is not readily squared with observations on rates of time discounting. That is, the average rate of human population growth during the 1.8 million years of our evolutionary history must have been, as a matter of arithmetic necessity, only imperceptibly greater than 0. Plausible estimates of hunter-gatherer mortality rates range from 1% for 10 year olds to 4% for 60 year olds; but these seem rather lower than plausible estimates of the pure rate of time discount. (See Gurven and Kaplan [11] for these mortality estimates; see Anderson, Harrison, Lau and Rutström [1] for experimental estimates of the pure rate of time preference.)

One of the contributions of the current paper is to evaluate a candidate for closing this gap that was proposed in a seminal paper by Rogers [25]. This candidate is sexual reproduction in the context of intergenerational transfers. Rogers argued as follows. Con-

³If, that is, $p = e^{-\delta}$, where δ is the implied continuous-time mortality rate, and $\lambda = e^g$, where g is the implied continuous-time population growth rate, then the pure rate of time preference is $\ln \lambda - \ln p = g + \delta$.

⁴For this purpose, mortality is included in the calculation of expected offspring.

⁵Even closer to home is that arboreal mammals live longer than do comparable ground dwelling mammals. Arboreality reduces predation, and has been proposed as the original circumstance leading to greater longevity in primates. See Shattuck and Williams [28].

sider a 25 year old woman who can invest resources to benefit her current newborn. For simplicity, suppose the return from the investment is received 25 years from now, when her newborn will also be 25. The marginal value of resources will be the same to her offspring 25 years from now as it is to the mother now. However, from the current mother's point of view, sex deflates the importance of her offspring by a factor of $1/2$. In the simplest case of "haploid" sex, this is because this offspring will be a carbon copy of her mother with probability $1/2$, but will be a carbon copy of her father otherwise. Given zero population growth, and apparently abstracting from mortality, a unit of resources 25 years from now will be worth $1/2$ as much to the mother as it is worth today, and Rogers uses that as the basis for calculating a plausible rate of time preference of around 2%.

There are a number of difficulties with the Rogers analysis that are discussed by Robson and Szentes [23]. One that is easy to outline is that it cannot be true that all these "same age transfers" involve interior solutions. After all, the same argument as above but now applied to a 30 year old mother contemplating an investment to favor her newborn 30 years from now would imply a lower rate of time preference. In order to address directly the difficulties with Rogers model, Robson and Szentes developed an example that permitted same age transfers. However, this requirement made the example awkward enough that it could not shed light on the core claim of Rogers—that sex is a key factor leading to impatience. The present paper develops a model that is more tractable and therefore illuminating by not being bound to allow same age transfers. Although the model is then no longer directly comparable to Rogers, it is an inherently plausible view of transfers and sex. In our model, however, sex may well reduce impatience. Thus sexual reproduction does not seem a plausible way of closing the gap between typical rates of time preference and typical mortality rates.

On the other hand, the current paper provides another way of closing this puzzling gap. Indeed, we are able to shed light on a more refined view of the puzzle. In this more refined view, it is largely children and younger adults whose rates of time preference are larger than mortality rates plus the population growth rate. The present model derives a new way of closing this impatience gap for these individuals. For younger adults, rates of time preference exceed mortality rates plus the growth rate since rising output with age seems likely to decrease the marginal productivity of resources transferred to offspring. For children, the model disconnects their rate of time preference from the mortality rate or the growth rate, so the first rate could plausibly be higher than the sum of the latter two rates.

This more refined view of the puzzle also involves a reversal of the typical gap. That is, older adults seem too patient, with moderate rates of time preference that are eventually less than their increasingly high mortality rates plus the population growth rate. The model gives an explanation of this that is the flip side of the explanation for younger adults. That is, since older adults experience decreasing output, this would likely reduce transfers to offspring, and so increase the marginal productivity of such transferred resources.

The above scenario involves a rate of time preference that is denominated in resource terms, rather than in terms of utility. That is, our measure of the rate of time preference would derive from the MRS in consumption in a conventional model rather than be the usual “pure” rate of time preference, denominated in utility terms. Our model would apply directly to a foraging society in which there were limited opportunities for intertemporal substitution. A rate of time preference that was denominated in resource terms would have been influenced by the age profile of energetic income. However, this analysis might continue to apply in a modern setting in which capital markets exist, as long as they remain rather imperfect. Consumption would then continue to track income, as indeed is observed. (See Browning and Lusardi [7], for example.) In such a modern setting, it might well then remain the case that younger adults are “too impatient” and older adults “too patient,” in the above sense.

Finally, our analysis provides formal insights. We first derive the appropriate notion of “reproductive value” for our model of transfers. This represents a generalization of the simple notion of this, originally due to RA Fisher [10], which latter is the expected discounted value of offspring from each age forwards, conditional on survival to that age, where the population growth factor is used to discount the future. This simple notion of reproductive value is only appropriate in contexts in which offspring are homogeneous.⁶ The most general notion of reproductive value stems from the relative contribution of each type of offspring to the population in the distant future. In our case, even with heterogeneous offspring, the generalized reproductive value remains an appropriate eigenvector. This approach lends itself to further generalization. A key formal result of the paper is to illuminate a close connection between utility functions and this appropriate notion of reproductive value. The utility function at each adult age is again the expected discounted sum of the *value* of descendants. Although this results in a substantially different expression from that of RA Fisher, the population growth factor still discounts the

⁶A technical flaw in the Rogers model was the application of this simple notion of reproductive value in a context with heterogeneous offspring.

future.⁷ Optimal choice at each age still involves maximizing current reproductive value. Since current reproductive value derives from the reproductive value of offspring and the reproductive value of the parent, both one period ahead, lifetime optimal choice solves a simple dynamic programming problem.

1.1 A Summary of the Paper

Section 2 of the present paper presents a benchmark model that is a stepping stone for our model of transfers. There is an asexual species in which individuals live an arbitrary number of periods. In this benchmark model, we obtain the appropriate reproductive values, which arise as an eigenvector, in a fashion reminiscent of shadow prices in the von Neumann growth model. These values are the expected discounted value of future fertility, conditional on each age, where the discount factor is the population growth factor. How these reproductive values work as utility functions is illustrated by considering a basic biological tradeoff between the number of offspring and survival to the next period. When such a tradeoff is made, an individual's decision affects only the age profile of the number of her offspring, but not their quality. The simplification that all offspring are identical is the crucial simplifying feature of this benchmark model.

We conclude Section 2 with an example that considers the circumstances under which the utility criterion is essentially uniquely determined. This involves introducing suitable idiosyncratic random shocks. In the course of the discussion of this example we find a result of some independent interest—it is the *average* population-wide growth rate that bears on an individual's choices and not any idiosyncratic realized version of this rate.

In Section 3, we go beyond the benchmark model and develop a model of transfers in which offspring cannot be identical, but vary in quality. We then examine the optimal choice of such quality by the parent or parents of each age. In Section 3.1, there is an asexual species in which parents tradeoff their own survival to the next period against the survival of their newborns. The differential survival of offspring across parental ages means that newborns are heterogeneous. In the presence of such intergenerational transfers, with differential offspring quality, it is not appropriate to maximize the discounted sum of expected offspring. However, reproductive value again arises in the same general way as in the benchmark model as an eigenvector. In addition, we retain the intuitively

⁷Reproductive value could indeed still be expressed in terms of the expected discounted sum of the *value of newborns*. Since newborns are heterogeneous, however, this is not equivalent to the expected discounted sum of the *number of newborns*, as in RA Fisher.

pleasing result that optimal choice by an individual of each age maximizes this reproductive value. The current choice made by an individual maximizes a criterion that accounts for current effects, as well as appropriate continuation values, in accordance with dynamic programming.

In Section 3, we also derive the implications of intergenerational transfers for the rate of time preference. We show that the rate of time preference for prereproductive individuals is dissociated from the sum of the mortality rate and the population growth rate. For reproductive adults, the rate of time preference includes, as one of its components, this sum. The other component reflects the motive to make transfers to newborns, and this is likely to increase the rate of time preference for younger adults, but decrease it for older adults, in rough agreement with observations.

We introduce sexual reproduction into our model in Section 3.2. Sexual reproduction implies that the choices made by the two parents interact, so that the situation is now a game. This has substantial implications for predicted behavior. To see this, suppose again that parents transfer resources to their newborn offspring, and the survival of offspring depend on these transfers. Further assume that an offspring inherits the gene of one of her parents, with equal probabilities for that of each parent, but parents cannot recognize the genetic type of the offspring.

The surviving choice behavior maximizes its growth rate *as if its frequency was zero in the population*. Indeed, consider when a homogeneous population might be robust to an invasion by a small number of mutant genes.⁸ Mutants grow fastest initially if they maximize their growth rate given that their frequency in the population is essentially zero. Mutants can also match the growth rate of the original population by adopting the same choice behavior. Hence, the only way to ensure that no mutant can grow strictly faster than the original population is for the choice behavior of the original population to maximize the growth rate of the associated gene as if its frequency were zero.

The reproductive values derive from the pattern of fertility and survival in the same way as for the asexual species. An individual of each age still maximizes a utility function that derives from reproductive value, with the key difference from the asexual case that sexual individuals deflate the importance of the survival of their offspring by a factor of $1/2$. This is because if the gene has frequency zero in the population, then the probability of the other parent having the same gene is zero, so the probability an offspring has the

⁸The situation without sex is essentially a decision-theoretic problem, so deriving the outcome there did not need such a device. It could readily also be described in these terms, however.

same gene is $1/2$. This is true even though, in the evolutionary equilibrium, all individuals have the same gene.⁹

The question then is: What effect does this have on the rate of time preference? Although sexual individuals deflate the importance of their offspring by a factor of $1/2$, they do this at all ages. Sexual individuals still derive all their reproductive value from offspring; only now all of these are worth less. It does not seem that this would increase impatience.

On the contrary, the following results are established. Every adult systematically skews her allocation of resources to favor her own survival at the expense of her offspring's survival. This is true in a myopic sense, that is, holding constant the reproductive value of the adult in the next period. This distortion will inevitably affect these reproductive values. However, even when the implications of sex for reproductive values are appropriately allowed for, the sexual type increases all adult survival rates at the expense of all newborn survival rates. Sex then unambiguously reduces the quality of offspring in this sense. However, this distortion does not entail greater impatience. Rather, sex may well imply too much patience, since greater adult survival militates in this direction.

We discuss additional relevant literature in Section 4. Section 5 concludes by discussing issues of interpretation as well as extensions that would be of definite interest, despite varying degrees of feasibility.

2 A Benchmark Model without Transfers

Consider an asexual species in which each individual lives to a maximum age of T periods. At age $\tau = 0, \dots, m - 1$, she has $u_\tau = 0$ offspring; at age $\tau = m, \dots, T$, she has $u_\tau > 0$ expected offspring. In this benchmark model, but not subsequently, the offspring produced by parents of differing ages are identical. Finally, each individual survives from age τ to age $\tau + 1$ with probability $p_\tau \in (0, 1)$, for $\tau = 0, \dots, T - 1$.

The parameters just described in general will eventually be taken to depend on the choice made by the individuals, so the issue is to derive the optimal choice. We consider a simple explicit choice problem below. As a necessary building block, we first consider the

⁹Bergstrom [3] analyzes games played by siblings. He concludes that the strategy that cannot be invaded by a mutant chooses an action as if the opponent mimics this action with probability half. This is the same principle as here. That is, although, in equilibrium, siblings have the same genes and take the same actions, they behave as if this probability were only a half.

implications of a particular fixed set of parameters for the growth rate of population.

The population then evolves as

$$n^{t+1} = n^t L,$$

where n^t is row-vector describing the population at date t so that $n^t = (n_1^t, \dots, n_T^t)$ where n_τ^t is the number of individuals of age $\tau = 1, \dots, T$ at date t .¹⁰ Also L is the Leslie matrix

$$L = \begin{bmatrix} p_0 u_1 & p_1 & 0 & \cdot & \cdot & 0 \\ p_0 u_2 & 0 & p_2 & 0 & \cdot & 0 \\ p_0 u_3 & 0 & 0 & p_3 & \cdot & \cdot \\ \dots & \cdot & \cdot & \cdot & \cdot & 0 \\ p_0 u_{T-1} & 0 & \cdot & \cdot & 0 & p_{T-1} \\ p_0 u_T & 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix}.$$

All of these columns except the first simply describe how the number of individuals of age $\tau = 2, \dots, T$ at date t is the number of individuals of age $\tau - 1$ at date $t - 1$, allowing for survival, $p_{\tau-1}$. The first column describes how the number of age 1 individuals is the total number of newborns one period ago, $\sum_{\tau=1}^T u_\tau n_\tau^{t-1}$, allowing for survival, p_0 .

The Perron-Frobenius Theorem implies that the system settles into steady state growth with the growth factor $\lambda > 0$ being the unique dominant eigenvalue.¹¹ Furthermore, the left eigenvector $q \gg 0$ (taken as a row vector) gives the limiting population proportions and the right eigenvector $v \gg 0$ (so v^T denotes the associated column vector) gives the relative values of each age class. That is, we have

$$qL = \lambda q, \quad Lv^T = \lambda v^T \quad \text{and} \quad |L - \lambda I| = 0.$$

Note that q and v are only determined up to a multiplicative constant. A straightforward proof by induction shows that the characteristic equation is

$$1 = \frac{p_0 u_1}{\lambda} + \dots + \frac{p_0 \dots p_{T-1} u_T}{\lambda^T}, \quad (2)$$

which indeed is the Euler-Lotka equation and can be obtained more directly by straightforward arguments from the steady state.

¹⁰It is convenient to consider newborns, with $\tau = 0$, only implicitly here. This asymmetric reduction in the number of age classes generates the need for a little algebra here and there, but saves a lot elsewhere, and is especially helpful in the following sections that treat transfers.

¹¹See Seneta [27]. It is enough that there exist two ages τ and $\tau' = \tau + 1$ such that u_τ and $u_{\tau'}$ are strictly positive.

The left eigenvector gives the relative proportions of individuals in each age class in the steady state, as is familiar in demography. It can be taken to be, for example, $q = (\frac{p_0}{\lambda}, \frac{p_0 p_1}{\lambda^2}, \dots, \frac{p_0 \dots p_{T-1}}{\lambda^T})$.¹²

The right eigenvector satisfies

$$v_\tau = u_\tau + \frac{p_\tau v_{\tau+1}}{\lambda}, \tau = 1, \dots, T, \quad (3)$$

the solution of which can be taken as

$$v_\tau = u_\tau + \frac{p_\tau u_{\tau+1}}{\lambda} + \dots + \frac{p_\tau \dots p_{T-1} u_T}{\lambda^{T-\tau}}, \tau = 1, \dots, T, \quad (4)$$

which are the reproductive values as defined by Ronald A. Fisher [10]. That is, v_τ is the expected discounted sum of future fertility, conditional on being alive at age τ , where the expectation includes the probability of survival to each future age, and where the discount factor is the population growth factor.

How would evolution choose between arbitrary profiles of survival probabilities and fertilities, with each profile of the form $\{p_\tau, u_{\tau+1}\}_{\tau=0}^{T-1}$? The theory of evolutionary choice can be developed in the same way as it is conventional to describe preferences in consumer theory. That is, evolutionary preferences are independent of the set of options considered. The most general biological representation of intertemporal preferences is the growth factor λ . Often, however, a more useful representation from an economic perspective is the right-hand side of the Euler-Lotka equation, Eq (2). These are connected in that it is necessary that a growth maximizing profile of demographic characteristics p_τ and $u_{\tau+1}$ for $\tau = 0, \dots, T - 1$ also maximize the right-hand side of this equation, given the *optimal* growth factor λ .

To illustrate this approach in a concrete fashion, and one congenial to economists, since it involves allocating a scarce resource, consider a very basic biological trade-off—that between the number of offspring and own survival. Considering this tradeoff allows us to remain within the benchmark model, since all offspring remain identical. (The tradeoff considered in the model of transfers, on the other hand, forces us to consider heterogeneous offspring.) Suppose now that each adult individual of some particular age $\tau \in \{m, \dots, T\}$ has resource budget or income $I_\tau > 0$. This budget is to be divided between resources used to produce offspring, $r_\tau \geq 0$, via the function $u_\tau(r_\tau)$, and resources used to promote survival to the next period, $s_\tau \geq 0$, via the function $p_\tau(s_\tau)$, so that

¹²This form of q would arise from normalizing $q_0 = 1$ if the newborns were explicitly included.

$I_\tau = r_\tau + s_\tau$. Suppose the functions $u_\tau(\cdot)$, and $p_\tau(\cdot)$ are continuously differentiable, with a strictly positive derivative everywhere, and strictly concave everywhere.

The reproductive values permit a nice view of optimal choice, as follows. From the Euler-Lotka equation, Eq (2), and the equation determining v_τ , Eq (4), the first-order condition for the optimal allocation of resources by an individual of age τ must solve the following problem

$$\max_{\substack{r_\tau, s_\tau \geq 0 \\ r_\tau + s_\tau = I_\tau}} v_\tau(r_\tau, s_\tau) \equiv \max_{\substack{r_\tau, s_\tau \geq 0 \\ r_\tau + s_\tau = I_\tau}} \left(u_\tau(r_\tau) + \frac{p_\tau(s_\tau)}{\lambda} \left[u_{\tau+1} + \frac{p_{\tau+1}u_{\tau+2}}{\lambda} + \dots + \frac{p_{\tau+1}\dots p_{T-1}u_T}{\lambda^{T-\tau-1}} \right] \right).$$

This explicitly spells out the utility criterion relevant to this age τ individual. This can be more compactly expressed as—

$$\max_{\substack{r_\tau, s_\tau \geq 0 \\ r_\tau + s_\tau = I_\tau}} v_\tau(r_\tau, s_\tau) \equiv \max_{\substack{r_\tau, s_\tau \geq 0 \\ r_\tau + s_\tau = I_\tau}} u_\tau(r_\tau) + \frac{p_\tau(s_\tau)}{\lambda} v_{\tau+1}.$$

In either case, this age τ individual solves the problem of maximizing her reproductive value $v_\tau(r_\tau, s_\tau)$ which is then interpreted as her utility function.

This model could be generalized to consider optimal choices like this by individuals of all ages, making the choice then of whether fertility is zero or strictly positive fully endogenous. Such a model could then endogenize the transition from childhood, with zero fertility, to adulthood, with positive fertility. This would not be a derivation from first principles, however, since it would be predicated on age-varying income and the age-varying survival functions. (See Kaplan and Robson [16], for a model that generates all of our key demographic characteristics without relying on time-varying parameters or functions.)

One natural question concerning this model and the subsequent ones is: Given that the earth is of a limited size, surely it is not possible that the maximal $\lambda > 1$? That is, surely finite carrying capacity forces the maximal λ to be no greater than 1? One way that the model can be adapted to this observation is to suppose that survival rates p_τ are decreasing functions of total population N , say. This dependence might plausibly be such that growth factors greater than 1 are possible at low values of N , but that growth is inevitably choked off as N grows. In the limit then a growth factor of exactly 1 will emerge in an endogenous fashion. We will return to this issue when comparing the models of transfers with and without sex.

Another natural question that arises with respect to the above model and more generally is: Since there is only one optimal choice, surely the given criterion is not the only

one that supports it? We use the following example to sketch the intuition that the addition of suitable noise means that the criterion is essentially uniquely determined. This example is a simplification of the benchmark model described above, except for the source of idiosyncratic noise.

Example 2. Suppose then that $T = 2$ and that fertility at age 1 is a continuously differentiable, strictly increasing, and strictly concave function of the resources allocated for that purpose, r , say, so that fertility is given by $u_1(r)$. If the resources used to promote survival from age 1 to age 2 is s , then this survival probability is $p(s)$, where $p(\cdot)$ is assumed to be continuously differentiable, strictly increasing and strictly concave. Suppose survival from age 0 to age 1 is certain, and that fertility at age 2 is u_2 .¹³

Suppose now that the resource endowment is a random variable, given by \tilde{I} , which has support $[0, \infty)$. Furthermore, the effectiveness of resources in promoting the survival of offspring is also a random variable given by $\tilde{\alpha}$ also with full support $[0, \infty)$. These random variables are independent of each other, and also independent across individuals.¹⁴ The budget constraint then has the form $r + \tilde{\alpha}s = \tilde{I}$.

The individual is taken to be aware of the joint realization (α, I) of the random variable pair $(\tilde{\alpha}, \tilde{I})$ and to then choose the resource allocation $(r(\alpha, I), s(\alpha, I))$, where $r(\alpha, I) + \alpha s(\alpha, I) = I$.

The basic question then is: To what extent are the functions $u_1(\cdot)$ and $p(\cdot)$ uniquely determined by the need to account for the evolutionarily optimal choices to be made here by the individual?

It is first necessary to address the question: In the presence of this random variable, and given a large population, what is the growth rate of a type that follows an arbitrary allocation rule $(r(\cdot, \cdot), s(\cdot, \cdot))$? Expected offspring of an age 1 individual is $E_{\tilde{\alpha}, \tilde{I}}[u_1(r(\tilde{\alpha}, \tilde{I}))]$; that of all age 2 individuals is $E_{\tilde{\alpha}, \tilde{I}}[p(s(\tilde{\alpha}, \tilde{I}))]u_2$, where this expectation is formed from a point of view at age 0, and so includes the survival probability from age 1 to age 2 in particular. Since all the risk here is idiosyncratic and the population is assumed to be

¹³This specification is consistent with the example of choice considered above. An alternative specification that would agree more closely with Example 1 would suppose that the survival probability, p , is fixed but that fertility at age 2 is endogenous, given by $u_2(s)$, say. This makes only notational differences to the argument here.

¹⁴The two random variables do not need to be independent of each other; it is enough that they have full support, namely $[0, \infty)^2$. However, see Robson and Samuelson [22] for an investigation of the substantial consequences of relaxing the requirement of independence across individuals.

large, the long run growth factor λ satisfies Eq (2) so that

$$1 = E_{\tilde{\alpha}, \tilde{I}} \left[u_1(r(\tilde{\alpha}, \tilde{I})) + \frac{p(s(\tilde{\alpha}, \tilde{I}))u_2}{\lambda} \right].$$

The best allocation rule maximizes λ in this version of the Euler-Lotka equation. It is then clear that the optimal allocation rule solves

$$\max_{\substack{r, s \geq 0 \\ r + \alpha s = I}} u_1(r) + \frac{p(s)u_2}{\lambda}, \quad (5)$$

for each pair of realizations (α, I) of the random variables $(\tilde{\alpha}, \tilde{I})$.

This is now entirely analogous to a familiar problem from consumer theory. Problem (5) gives rise to fully specified “demand functions” $r(\alpha, I)$ and $s(\alpha, I)$ that represent the behavior that must be generated for evolutionary optimality. Using this demand analogy, it follows that these functions can be “integrated” to obtain a utility function that is unique up to an arbitrary monotonic transformation. That is, the only overall criteria that generates the same required behavior must be of the form $\psi \left[u_1(r) + \frac{p(s)u_2}{\lambda} \right]$, for some strictly increasing function ψ . If ψ is twice continuously differentiable, and the overall criterion is required to remain additively separable, the only flexibility left is to multiply both $r(\alpha, I)$ and $s(\alpha, I)$ by a common positive constant, and to add possibly different arbitrary constants to these functions. In this straightforward sense, the functions $r(\alpha, I)$ and $s(\alpha, I)$ are essentially uniquely identified. ■

Note how the population growth factor λ that should be applied in Eq (5) is derived from average population-wide fertilities and is not customized to the particular individual. That is, an individual who obtains high realizations of $\tilde{\alpha}$ and \tilde{I} , for example, should use the same factor λ in her calculations as should an individual with low realizations. Idiosyncratic risk then does not cause varying impatience.

There is no reason to doubt that an analogous argument can also be applied in the model of the next section. That is, not only would the argument generalize to allow for the tradeoff there between survival of newborns and survival of the adult to the next period, but to allow an arbitrary number of ages as well.

3 The Model of Transfers

The point of the Rogers [25] approach was to examine how sexual reproduction was a source of impatience when transfers can be made from parents to offspring. Particular

problems with Rogers’ formulation were examined by Robson and Szentes [23]. In this section we reexamine this issue in a more natural model that is not constrained by the need to allow for “same-age” transfers as in Rogers. Indeed, we consider here the simplest kind of transfer—one from a parent to a newborn that increases the probability of that newborn surviving to the next period. The opportunity cost of this transfer to the parent is a reduction in the probability of the parent surviving to the next period. Resources cannot be explicitly saved or carried forward at all, also in contrast to Rogers. This is defensible on the grounds of realism, since there were no obvious direct ways of making commodity tradeoffs across widely separated dates in hunter-gatherer societies prior to agriculture. At the same time, it is crucial that we ultimately consider how such individuals might once have made such tradeoffs implicitly, and *would* make explicit commodity tradeoffs once they became available. We show that these implicit tradeoffs may well mean that children and younger adults have rates of time preference that exceed the sum of the mortality rate and the population growth rate; but older adults have rates that are less than this sum.

3.1 Transfers Without Sex

Consider again an asexual species. Suppose now that individuals of ages $\tau = 1, \dots, T$ have incomes $I_\tau > 0$. Newborns of age 0 have income 0. Each adult individual of age $\tau = m, \dots, T$ transfers an amount $r_\tau \geq 0$ to each of her $u_\tau > 0$ offspring, keeping $s_\tau \geq 0$ to promote her own survival to age $\tau + 1$. It is now generally inescapable that offspring from parents of different ages will be different. An offspring who is the beneficiary of a larger transfer and so survives with higher probability has higher “quality” in that sense than one with a lower transfer. This heterogeneity must be taken into account in the appropriate notion of reproductive value. The budget constraint is $s_\tau + u_\tau r_\tau = I_\tau$ for $\tau = 1, \dots, T$. Children of age $\tau = 1, \dots, m - 1$ have no offspring so that $u_\tau = 0$ and they set $r_\tau = 0$ and $s_\tau = I_\tau$. In addition, $s_T = 0$ so that $r_T = I_T/u_T$.¹⁵

The effect of the transfers is to promote the survival of newborns for one period. That is, this survival probability is endogenous, and given by $p_0(r_\tau)$, for each offspring of an age $\tau = 1, \dots, T$ parent. Survival of each age τ parent to the next period is given by $p_\tau(s_\tau)$. The functions $p_\tau(\cdot)$, $\tau = 0, \dots, T$ are continuously differentiable, strictly concave, with a strictly positive derivative everywhere, and where this derivative tends to infinity at 0. In

¹⁵For simplicity, this model does not consider transfers from parents who are no longer fertile to their older children or grandchildren. This interesting issue is taken up in the Conclusion.

this formulation, we assume for simplicity that the fertilities u_τ for $\tau = 1, \dots, T$ are fixed, in contrast to the choice considered in the context of the benchmark model.

Again we have

$$n^{t+1} = n^t L,$$

where n^t is row-vector describing the adult population at date t so that $n^t = (n_1^t, \dots, n_T^t)$, and where the Leslie matrix is now¹⁶

$$L = \begin{bmatrix} p_0(r_1)u_1 & p_1(s_1) & 0 & \cdot & \dots & 0 \\ p_0(r_2)u_2 & 0 & p_2(s_2) & 0 & \dots & 0 \\ p_0(r_3)u_3 & 0 & 0 & p_3(s_3) & \dots & \cdot \\ \dots & \cdot & \cdot & \cdot & \dots & 0 \\ p_0(r_{T-1})u_{T-1} & 0 & \cdot & \cdot & 0 & p_{T-1}(s_{T-1}) \\ p_0(r_T)u_T & 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix}.$$

All the columns except the first have a similar simple interpretation to that for the benchmark model. The first column describes how the number of individuals of age 1 at date t arises from the newborns of all individuals of age $\tau = 1, \dots, T$ at date $t - 1$ as the sum $\sum_{\tau=1}^T p_0(r_\tau)u_\tau n_\tau^{t-1}$.

The Euler-Lotka equation, or, equivalently, the characteristic equation for L , namely $|L - \lambda I| = 0$, is now

$$1 = \frac{p_0(r_1)u_1}{\lambda} + \frac{p_0(r_2)p_1(s_1)u_2}{\lambda^2} \dots + \frac{p_0(r_T)p_1(s_1)\dots p_{T-1}(s_{T-1})u_T}{\lambda^T}. \quad (6)$$

The most basic view of preferences here, as before, is λ , but a useful representation is often the right-hand side of this equation. That is, maximizing this expression, taking the *optimal* value of the growth factor, λ , as parametric, is a necessary condition for maximizing this growth factor.

The limiting population proportions, q , can be taken to be $q = (1, \frac{p_1(s_1)}{\lambda}, \dots, \frac{p_1(s_1)\dots p_{T-1}(s_{T-1})}{\lambda^{T-1}})$, with the normalization that $q_1 = 1$. The reproductive values again satisfy $Lv^T = \lambda v^T$. That is, with the normalization that $v_1 = 1$,¹⁷

$$v_\tau = \frac{p_0(r_\tau)u_\tau}{\lambda} + \frac{p_\tau(s_\tau)v_{\tau+1}}{\lambda} \text{ for } \tau = 1, \dots, T-1, \text{ with } v_T = \frac{p_0(I_T/u_T)u_T}{\lambda}. \quad (7)$$

¹⁶The advantage of suppressing explicit treatment of newborns is now significant. If newborns were included explicitly, that is, there would be T different types of them, one for each possible parental age.

¹⁷For expositional economy, children, who have $u_\tau = 0$, for $\tau = 1, \dots, m-1$, are included in this formulation.

These equations straightforwardly relate the value of an individual of age τ to the value of her offspring, derived as the expected value of these one period ahead, plus the expected value of the individual herself one period ahead. It is easy to solve these equations by backwards recursion to obtain each v_τ as follows

$$v_\tau = \frac{1}{\lambda} \left\{ p_0(r_\tau)u_\tau + \frac{p_0(r_{\tau+1})p_\tau(s_\tau)u_{\tau+1}}{\lambda} + \dots + \frac{p_0(r_T)p_\tau(s_\tau)\dots p_{T-1}(s_{T-1})u_T}{\lambda^{T-\tau}} \right\}, \quad (8)$$

for $\tau = 1, \dots, T$.¹⁸

The expression in Eq (8) in particular details the reproductive value of an age $\tau = m, \dots, T-1$ adult individual, who faces a nontrivial choice problem. Each adult individual of age $\tau = m, \dots, T$ maximizes her reproductive value which we then interpret as her utility function.

This utility function has some rather familiar properties—additive separability and the appearance of a constant discount factor, λ , for example. In addition, the term $p_0(r_\tau)$ is analogous to a age-invariant felicity function. The less familiar properties of this expression derive from the interpretation of the $p_\tau(s_\tau)$ for $\tau = 1, \dots, T$, as survival probabilities. In the first place, these probabilities are taken to be subject to choice here. Further, it is the *product* of these probabilities between the current age and any future age that enters the above criterion.

The recursive relationship in Eq (7) permits a simple dynamic programming view of these values that is tailored to the current model by highlighting the consequences of the current choice.¹⁹

Theorem 1 *In the present model of transfers without sex, the unique optimal allocation of resources by adults of age $\tau = m, \dots, T-1$ is the unique solution to the dynamic programming problem*

$$\max_{\substack{r_\tau, s_\tau \geq 0 \\ u_\tau r_\tau + s_\tau = I_\tau}} \frac{p_0(r_\tau)u_\tau}{\lambda} + \frac{p_\tau(s_\tau)v_{\tau+1}}{\lambda} \equiv \max_{\substack{r_\tau, s_\tau \geq 0 \\ u_\tau r_\tau + s_\tau = I_\tau}} v_\tau(r_\tau, s_\tau), \quad (9)$$

where Eq (7) yields the v_τ , and where Euler-Lotka equation, Eq (6), yields λ .

¹⁸By suppressing explicit treatment of newborns, we finesse the issue of their heterogeneity. If they were not suppressed, that is, we would need T additional reproductive values.

¹⁹See Houston and McNamara [14] for an excellent treatment of such dynamic programming results from a biological point of view. See, in particular, McNamara, Houston and Webb [15] for an application to kin selection.

Proof. Dynamic programming implies that the unique choices of age $\tau = m, \dots, T - 1$ adults described in Eq (9) and Eq (7) maximize the RHS of the Euler-Lotka equation, Eq (6), for an arbitrary $\lambda > 0$. If $V(\lambda)$ denotes this maximized value of the RHS of Eq (6), then it follows readily that $V(\cdot)$ is continuously differentiable; that $V(\lambda) \rightarrow \infty$, as $\lambda \rightarrow 0$; that $V(\lambda) \rightarrow 0$, as $\lambda \rightarrow \infty$; and that $V'(\lambda) < 0$, for all $\lambda > 0$. Hence there exists a unique $\lambda^* > 0$ satisfying $V(\lambda^*) = 1$. This is the maximum growth factor, since if $\lambda > \lambda^*$, so that $V(\lambda) < 1$, then there is no resource allocation profile such that λ satisfies Eq (6). ■

Given the properties of the survival functions $p_\tau(\cdot)$ for $\tau = 0, m, \dots, T$, there must be an interior solution to this problem, with $r_\tau > 0$ and $s_\tau > 0$. The first-order conditions are—

$$p'_0(r_\tau) = p'_\tau(s_\tau)v_{\tau+1}, \tau = m, \dots, T - 1,$$

which, together with Eq (7), characterize the optimal nontrivial lifetime allocations.

The rate of time preference, $\rho_\tau > -1$, for $\tau = 1, \dots, T - 1$, is defined from the marginal rate of substitution between adjacent resource income levels I_τ and $I_{\tau+1}$, say, so that

$$1 + \rho_\tau = \frac{\frac{\partial \lambda}{\partial I_\tau}}{\frac{\partial \lambda}{\partial I_{\tau+1}}}, \tau = 1, \dots, T - 1.$$

This definition incorporates mortality as a factor in the rate of time preference, as is conventional in economics since Irving Fisher [9], at least. Further, this definition also incorporates the effect of varying exogenous resource incomes, as well as that of the age-varying survival functions themselves. It is a rate of time preference denominated in resource terms, and we do not attempt to denominate it in utility terms, as in the so-called *pure* rate of time preference. Our rate is the applicable bottom line rate that would be used by an individual considering the effect of a small intertemporal rearrangement of resources and so is the rate that would be most easily observed in practice.

In the present case, it follows from the Euler-Lotka equation, Eq (6), that, for adults of age $\tau = m, \dots, T - 1$, for whom there are transfers to newborns,

$$1 + \rho_\tau = \frac{\lambda p'_0(r_\tau)}{p_\tau(s_\tau) p'_0(r_{\tau+1})}.$$

There is a familiar component of time preference given by $\frac{\lambda}{p_\tau(s_\tau)}$, which it is tempting to call the “pure rate of time preference.” We resist this temptation, but note that this component is the prediction of the basic model as sketched in Example 1. Lower survival to the next period, as reflected in a higher rate of mortality, is a familiar reason for

impatience. There is also a biological cause of impatience (which has also become more familiar recently)—population growth, as reflected in λ .

Additionally, this expression accounts for the effect of variation in the marginal product of resources used to promote the survival of newborns. This effect is analogous to how different marginal utilities of consumption affect the MRS in a simple standard model of intertemporal choice.

Consider now why younger individuals might be “too impatient ” and older individuals “too patient.” First: Why do younger individuals seem more impatient than the sum of the rate of mortality and the population growth rate? Second: Why, on the other hand, do older individuals seem more patient than this?²⁰ We start with some observations concerning adults.

The component in the rate of time preference that goes beyond this sum is $\frac{p'_0(r_\tau)}{p'_0(r_{\tau+1})}$. If the transfers r_τ increase with age, as might well be true for younger adults, this term would tend to increase impatience since

$$\frac{p'_0(r_\tau)}{p'_0(r_{\tau+1})} > 1.$$

It is of interest in this connection that average productivity for hunter-gatherers may have increased until age 45 or so. (See Figure 3 in Kaplan and Robson [16], for example.) Assuming that productivity and transfers are correlated, and a reproductive career that starts at 15, say, the model then predicts greater impatience in the age range 15-45 than implied by the sum of the mortality rate and the population growth rate.

Once the transfers r_τ decrease with age, on the other hand, this term would then reduce impatience since

$$\frac{p'_0(r_\tau)}{p'_0(r_{\tau+1})} < 1.$$

The flip side of the prediction for younger adults is then that the model predicts that older individuals, whose output is declining, would be more patient. In the light of the hunter-gatherer data, this prediction would apply to those over 45 years old.

A more general model would allow for transfers over a more protracted period of childhood. This would generate an incentive for individuals who were well beyond reproduction to still value the resources they could use to make these transfers. This issue is taken up in the Conclusion.

²⁰See Anderson, Harrison, Lau and Rutström, [1], and Bishai, [6], for example, for evidence on these two assertions.

We now complete the picture for younger individuals by considering the impatience of children. For children of age $\tau = 1, \dots, m - 2$, we have

$$1 + \rho_\tau = \frac{\lambda p'_\tau(I_\tau) v_{\tau+1}}{p_\tau(I_\tau) p'_{\tau+1}(I_{\tau+1}) v_{\tau+2}}.$$

However, since $u_\tau = 0$ for $\tau = 0, \dots, m - 1$ it follows that

$$v_\tau = \frac{p_\tau(I_\tau) v_{\tau+1}}{\lambda}, \tau = 1, \dots, m - 1,$$

so that

$$1 + \rho_\tau = \frac{p'_\tau(I_\tau) p_{\tau+1}(I_{\tau+1})}{p'_{\tau+1}(I_{\tau+1}) p_\tau(I_\tau)}, \tau = 1, \dots, m - 2.$$

This expression for the rate of time preference for children is not directly connected to the absolute level of the survival rate or to the rate of population growth, in contrast to the expression for the rate of time preference for adults.

At the transition from childhood to adulthood, at age $\tau = m - 1$, we have the special case that

$$1 + \rho_{m-1} = \frac{\lambda p'_{m-1}(I_{m-1}) v_m}{p_{m-1}(I_{m-1}) p'_0(r_m)}.$$

How impatient are children? It is helpful here to define $\delta_\tau(s_\tau) = -\ln(p_\tau(s_\tau))$ as the continuous time mortality rate implied by $p_\tau(s_\tau)$, so $\delta'_\tau(s_\tau) < 0$. It follows that the impatience of children derives from the change in the derivative of this mortality rate. That is, $\rho_\tau > 0$ for $\tau = 1, \dots, m - 2$ if and only if—

$$\delta'_\tau(I_\tau) < \delta'_{\tau+1}(I_{\tau+1}) < 0. \quad (10)$$

We assume that this plausible condition is satisfied. It requires that the marginal benefit of resources that decrease the mortality rate is higher for younger ages.

Indeed, this condition disconnects the impatience of children from the absolute level of the mortality rate and from the population growth rate. The rate of time preference of children may well then substantially exceed the sum of these two rates, complementing the results above for younger adults.²¹

For that matter, how does the impatience of children depend on age? For $\tau = 2, \dots, m - 2$, it follows readily that $1 + \rho_{\tau-1} > 1 + \rho_\tau > 0$ if and only if

$$\sqrt{\delta'_{\tau-1}(I_{\tau-1}) \delta'_{\tau+1}(I_{\tau+1})} > -\delta'_\tau(I_\tau) > 0. \quad (11)$$

²¹It is plausible but not certain that this is true for infants, since they have extremely high but hard to measure rates of time preference versus mortality rates that are high even now, but were still higher in our evolutionary past. The model need not take a stance on this.

This condition is assumed to hold in order to generate the decreasing pattern of time preference usually ascribed to children. (See Bettinger and Slonim [5].)

An apparently significant omission from the above model concerns the growth of children. That is, children allocate significant resources to somatic growth as well as to survival. Explicit treatment of this is omitted for simplicity and since the focus here is rather on adults. However, including this application for resources may not make a huge qualitative difference. That is, the high marginal product of resources in promoting growth early in life would tend to reduce the resources available for survival. This would reinforce the above observations, since the rate of time preference denominated in resources could still be taken in the form given above, except only that the resource argument would now refer to the resources devoted to survival rather than the total resources available.

To sum up the expressions obtained here for the rate of time preference—

Theorem 2 *In the current model of transfers without sex, the rate of time preference for children, those of age $\tau = 1, \dots, m - 2$ is given by*

$$1 + \rho_\tau = \frac{p'_\tau(I_\tau)p_{\tau+1}(I_{\tau+1})}{p'_{\tau+1}(I_{\tau+1})p_\tau(I_\tau)}.$$

At the transitional age of $\tau = m - 1$, a child has

$$1 + \rho_{m-1} = \frac{\lambda p'_{m-1}(I_{m-1})v_m}{p_{m-1}(I_{m-1})p'_0(r_m)}.$$

On the other hand, the rate of time preference for adults, those of age $\tau = m, \dots, T - 1$, is given by

$$1 + \rho_\tau = \frac{\lambda p'_0(r_\tau)}{p_\tau(s_\tau)p'_0(r_{\tau+1})}.$$

The predictions obtained here are intended to apply literally to a hunter-gatherer society. That is, it is plausible that individuals in such a society would be more impatient than dictated by mortality and population growth when younger, but less impatient when older. How might these predictions translate to a modern setting?

Suppose that what is hard-wired is not the specific fashion that the various functions here depend on their arguments, nor even these arguments themselves necessarily, but just their interpretation as the production and survival of offspring and own survival. These arguments are evolutionarily proximate inputs for the production and nurturing of offspring. The relevance of some of these inputs, such as sex, might have been rather constant over time, only changing rather recently. The relevance of particular commodities, on the other

hand, might have changed dramatically. Individuals are assumed to flexibly and perhaps consciously account for how the values of these functions are influenced by their choices. If roasted monkeys once promoted the survival of offspring, adults nevertheless do not have an intrinsic interest in roasted monkeys, but can shift flexibly to packaged pablum instead. This interpretation would take us far, but perhaps not all the way to the modern era, in particular because sex is no longer nearly as likely to lead to offspring.

That is, what this approach gives us is an evolutionary basis for utility, with a time-invariant component, as represented in the above model by the function $p_0(\cdot)$. Now observe that the age profile of individual output in modern societies is qualitatively very similar to that for hunter-gatherers. Moreover, although there is now a market for saving and borrowing, it remains rather imperfect, perhaps accounting for the observation that consumption still tracks income to a pronounced extent. (See Browning and Lusardi [7].) The above analysis might then well apply, perhaps in somewhat less dramatic way and *mutatis mutandis*, to modern economies.

3.2 Transfers and Sex

Reconsider the model of the previous subsection after the introduction of sexual reproduction. We wish to derive the new equilibrium allocation of resources for each adult between his/her offspring and his/her own survival. We hypothesize that there may be multiple types, where each type is characterized by a particular age profile of allocations. We suppose that mating pairs must be the same age and mating is random with respect to type. We simplify the process of sexual reproduction by supposing that each offspring inherits each of the resource allocation profiles of her parents with probability 1/2. That is, there is so-called “haploid sex.”²² An equilibrium resource allocation profile will have the property that a small number of mutants with any alternative allocation profile will do worse. We will consider then the growth process for a small number of mutants with an arbitrary resource allocation profile in a population with some given allocation profile.²³

²²Humans are actually diploid, so having two copies of each gene at each of a large number of “loci.” At each locus, each offspring gets a randomly chosen gene from mother and a randomly chosen gene from father. Diploid sex introduces a number of complications that seem largely tangential to the present purpose.

²³This general “non-invadability by mutants” condition could also be applied to the model of transfers without sex, and would generate the same results already found more directly for that case. It is needed here due to the game theoretic nature of the problem with sex; it was not needed without sex, since that issue is merely decision-theoretic.

As before, individuals of ages $\tau = 1, \dots, T$ have incomes $I_\tau > 0$. Newborns have income zero. Each adult of age $\tau = m, \dots, T$ now transfers an amount $r_\tau/2$ to each of the $2u_\tau > 0$ joint offspring, keeping s_τ to promote her own survival to age $\tau + 1$. The budget constraint is $s_\tau + u_\tau r_\tau = I_\tau$ for $\tau = 1, \dots, T$. As before, children use all their resources on their own survival, given $u_\tau = 0$, so that $s_\tau = I_\tau$ for $\tau = 0, \dots, m - 1$. Also $s_T = 0$ so that $r_T = I_T/u_T$. The survival functions $p_\tau(\cdot)$ for $\tau = 0, \dots, T - 1$ are as specified for the model with transfers but without sex in the previous subsection. Thus the model of transfers with sex has been constructed to be strictly comparable to the model without sex. Indeed, the outcome in the model without sex represents the optimum for the model with sex.²⁴

Suppose then that the population resource allocation profile is $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=1}^T$ and consider a rare mutant with profile $\{s_\tau, r_\tau\}_{\tau=1}^T$.²⁵ This mutant is subject to the growth process given by

$$n^{t+1} = n^t L,$$

where n^t is row-vector describing the mutant adult population at date t so that $n^t = (n_1^t, \dots, n_T^t)$. The Leslie matrix is now

$$L = \begin{bmatrix} p_0\left(\frac{\bar{r}_1+r_1}{2}\right)u_1 & p_1(s_1) & 0 & \cdot & \cdot & 0 \\ p_0\left(\frac{\bar{r}_2+r_2}{2}\right)u_2 & 0 & p_2(s_2) & 0 & \cdot & 0 \\ \cdots & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdots & \cdot & \cdot & \cdot & \cdot & 0 \\ p_0\left(\frac{\bar{r}_{T-1}+r_{T-1}}{2}\right)u_{T-1} & 0 & \cdot & \cdot & 0 & p_{T-1}(s_{T-1}) \\ p_0\left(\frac{\bar{r}_T+r_T}{2}\right)u_T & 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix}.$$

With the exception of the first column, this is identical to the Leslie matrix for the case without sex, and holds for the same reason—each mutant of age $\tau = 1, \dots, T - 1$ survives to age $\tau + 1$ with probability $p_\tau(s_\tau)$. Consider then the first column. Since the mutant is rare, essentially all the pairings of mutants are with nonmutants. In each such pairing, each of the $2u_\tau$ offspring receive $\frac{\bar{r}_\tau+r_\tau}{2}$ but only u_τ of these offspring are also mutants. The τ th term in the first column then reflects the contribution of mutant age τ parents at date t to mutant one year olds at date $t + 1$.

²⁴There is no advantage to sex in the present context. See Perry, Reny and Robson [17] for a discussion of the puzzle posed by the need to find the advantage of sex, of biparental sex in particular.

²⁵For expositional simplicity, we restrict attention to pure population allocation profiles. However, even if there is a mixture of profiles in the population, the best reply by a rare mutant will always be a pure profile. It follows that the only possible evolutionarily stable equilibria are then pure.

The limiting growth rate of the mutant type under the assumption that the mutant remains a small fraction of the population is relevant to evaluate whether the original type is robust to invasion by the mutant. The Euler-Lotka equation becomes

$$1 = \frac{p_0(\frac{\bar{r}_1+r_1}{2})u_1}{\lambda} + \frac{p_0(\frac{\bar{r}_2+r_2}{2})p_1(s_1)u_2}{\lambda^2} \dots + \frac{p_0(\frac{\bar{r}_T+r_T}{2})p_1(s_1)\dots p_{T-1}(s_{T-1})u_T}{\lambda^T}, \quad (12)$$

where λ is this limiting growth rate. The most basic biological view of preferences here, as before, is λ , but a useful economically relevant representation is the right-hand side of this equation. That is, maximizing this expression, given the *optimal* growth factor, is a necessary condition for maximizing this growth factor.

The limiting population proportions, q , can again be taken to be $q = (1, \frac{p_1(s_1)}{\lambda}, \dots, \frac{p_1(s_1)\dots p_{T-1}}{\lambda^{T-1}})$, with the normalization that $q_1 = 1$. Further, the vector of reproductive values still satisfies $Lv^T = \lambda v^T$, and we set $v_1 = 1$. In this case—

$$v_\tau = \frac{p_0(\frac{\bar{r}_\tau+r_\tau}{2})u_\tau}{\lambda} + \frac{p_\tau(s_\tau)v_{\tau+1}}{\lambda} \text{ for } \tau = 1, \dots, T-1, \text{ with } v_T = \frac{p_0(\frac{\bar{r}_T+r_T}{2})u_T}{\lambda}. \quad (13)$$

These equations can again be solved by backwards recursion to obtain each v_τ as follows

$$v_\tau = \frac{1}{\lambda} \left\{ p_0 \left(\frac{\bar{r}_\tau + r_\tau}{2} \right) u_\tau + \frac{p_0(\frac{\bar{r}_{\tau+1}+r_{\tau+1}}{2})p_\tau(s_\tau)u_{\tau+1}}{\lambda} + \dots + \frac{p_0(\frac{\bar{r}_T+r_T}{2})p_\tau(s_\tau)\dots p_{T-1}(s_{T-1})u_T}{\lambda^{T-\tau}} \right\}, \quad (14)$$

for $\tau = 1, \dots, T-1$.

This expression fully spells out the reproductive value of an age $\tau = m, \dots, T-1$ adult. However, dynamic programming again provides a more compact and elegant formulation of the problem. In this formulation, individuals recursively maximize the appropriate concept of reproductive value, which we then interpret as their utility—

Theorem 3 *The unique nontrivial allocations $\{s_\tau, r_\tau\}_{\tau=m}^{T-1}$ that satisfy the dynamic programming problem*

$$\max_{\substack{r_\tau, s_\tau \geq 0 \\ u_\tau r_\tau + s_\tau = I_\tau}} \frac{u_\tau p_0(\frac{\bar{r}_\tau+r_\tau}{2})}{\lambda} + \frac{p_\tau(s_\tau)v_{\tau+1}}{\lambda} \equiv \max_{\substack{r_\tau, s_\tau \geq 0 \\ u_\tau r_\tau + s_\tau = I_\tau}} v_\tau(r_\tau, s_\tau), \quad (15)$$

along with Eq (13), are the unique solution to the problem of maximizing the limiting growth rate of a “small” number of mutants with allocations $\{s_\tau, r_\tau\}_{\tau=m}^{T-1}$ embedded in a population using an arbitrary allocation $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$. That is, Eqs (13) and (15) characterize the unique “best reply mutant allocation.”²⁶

²⁶It is not hard to show that this best reply mutant allocation profile would remain unique against an arbitrary population mixture of types with various allocation profiles. Hence the evolutionarily stable equilibrium derived below remains unique even if mixtures are permitted.

Proof. Dynamic programming implies that the unique solution to Eqs (13) and (15) maximizes the RHS of Eq (12), for an arbitrary $\lambda > 0$. If $W(\lambda)$ denotes this maximized value of the RHS of Eq (12), then it follows readily that $W(\cdot)$ is continuously differentiable; that $W(\lambda) \rightarrow \infty$, as $\lambda \rightarrow 0$; that $W(\lambda) \rightarrow 0$, as $\lambda \rightarrow \infty$; and that $W'(\lambda) < 0$, for all $\lambda > 0$. Hence there exists a unique $\bar{\lambda}$ satisfying $W(\bar{\lambda}) = 1$. This is the maximum growth factor, since if $\lambda > \bar{\lambda}$, so that $W(\lambda) < 1$, then there is no profile of resource allocations such that λ satisfies Eq (12). ■

Under the conditions imposed on the functions $p_\tau(\cdot)$, $\tau = 0, m, \dots, T-1$, if these best reply allocations are interior, they must satisfy the first-order conditions—

$$\frac{p'_0\left(\frac{\bar{r}_\tau + r_\tau}{2}\right)}{2} = p'_\tau(s_\tau)v_{\tau+1}. \quad (16)$$

Conversely, if these first-order conditions are satisfied for the mutant allocations $\{s_\tau, r_\tau\}_{\tau=m}^{T-1}$, then these allocations characterize the mutant best reply to the allocations $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$.

We now characterize the unique evolutionarily stable outcome for the present model with transfers and sex. It is sex, of course, that implies that there are now strategic interactions, in contrast to the version of this model with no sex. For the resource allocation profile $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=1}^T$ to be an equilibrium, it is sufficient that the unique best choice of mutant profile $\{s_\tau, r_\tau\}_{\tau=1}^T$ against the profile $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=1}^T$ is simply $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=1}^T$ itself.

Consider then the nontrivial interior population allocation $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$ and reproductive values \bar{v}_τ , characterized by

$$\frac{p'_0(\bar{r}_\tau)}{2} = p'_\tau(\bar{s}_\tau)\bar{v}_{\tau+1}, \tau = 1, \dots, T-1. \quad (17)$$

and the recursive relationship, Eq (13). Suppose that $\bar{\lambda}$ is the implied growth rate from Eq (12). It is clear from Eq (16) that setting $\{s_\tau, r_\tau\}_{\tau=m}^{T-1} = \{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$ is then the unique mutant best reply to a population that chooses $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$. Of course, it also follows that $\{v_\tau\}_{\tau=1}^T = \{\bar{v}_\tau\}_{\tau=1}^T$ and $\lambda = \bar{\lambda}$. Altogether, we then have—

Theorem 4 *The nontrivial allocations $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$ satisfying Eqs (12), (13) and (17), with $\{s_\tau, r_\tau\}_{\tau=m}^{T-1} = \{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$ characterize the unique evolutionarily stable equilibrium allocation of the present model of transfers and sex.²⁷*

Note that Equation (17) implies that the unique equilibrium choice can be characterized as actually maximizing the “wrong” criterion from the point of view of each couple,

²⁷This construction implicitly restricts attention to symmetric Nash equilibria, as usual.

namely, $\frac{u_\tau p_0(\bar{r}_\tau)}{2} + p_\tau(\bar{s}_\tau)v_{\tau+1}$. Indeed, sex leads to an attenuation of the incentive to transfer resources to offspring. In general, with diploid sex, this is because each of these offspring have only probability 1/2 of sharing a rare mutant gene present in one of the parents. With haploid sex, this takes the simple form that the probability of each offspring being a carbon copy of the mutant parent is 1/2. Since the parent is unable to distinguish these copies from the others, transfers cannot condition on this. This result is familiar in biology as Hamilton's rule (see Hamilton [12], but also Bergstrom [3]). (Note that this biological motive for undercontribution arises despite all individuals being genetically identical in the equilibrium.) From an economic point of view, children are a public good to their parents, and the undercontribution that arises is also to be expected.

Although the myopic bias against transfers to newborns is now clear, the ultimate effect of such a bias is less clear. After all, the underlying rationale for promoting your own survival instead of that of your newborns can only be future reproduction, and each of these future offspring will also only be 1/2 relatives. However, it is now shown that the overall effect may still be that transfers to newborns are reduced by sex.

It should now be recognized that sex will reduce the growth rate, other things equal. This is because we set up the models in such a way that the income constraints were equivalent, so the "social optimum" for maximizing λ with sex is the problem as posed without sex.²⁸

However, perhaps the most relevant comparison to make is one where both the asexual and the sexual types have reached carrying capacity so that the growth factor in either case is 1. Other things cannot be equal now. There must be less selection pressure with sexual reproduction than without. A plausible possibility is that this appears as a multiplicative improvement in the survival functions. In the end, we will see that the result of this improvement is merely to amplify the effects.

More specifically, suppose that the survival probabilities now have the form $B(N)\hat{p}_\tau(\cdot)$, for $\tau = 1, \dots, T - 1$, where N is total population, and where the functions $\hat{p}_\tau(\cdot)$ have the same qualitative properties as $p_\tau(\cdot)$, for $\tau = 1, \dots, T - 1$. The function $B(\cdot)$ is continuously differentiable and strictly decreasing. Further, we assume that $B(\cdot)$ is such that the growth factor of either the nonsexual or sexual population is greater than 1 for small enough N and less than 1 for large enough N . It follows that either population will achieve a steady state

²⁸Suppose we take this observation at face value, so that $\bar{\lambda} < \lambda^*$, say, where $\bar{\lambda}$ and λ^* are the population growth rates with and without sex, respectively. A suitable modification of the argument to follow would show that it is again true that sex decreases impatience.

with zero growth at uniquely determined population levels, N^* and \bar{N} , for the nonsexual and sexual case, respectively. Since, at a population level of \bar{N} , the nonsexual population would grow, it follows that $N^* > \bar{N}$.

What this boils down to is that, if we denote the survival probabilities with sex by $\bar{p}_\tau(\cdot)$ and those without sex by $p_\tau(\cdot)$, then there exists $\beta > 1$ such that²⁹

$$\bar{p}_\tau(s_\tau) = \beta p_\tau(s_\tau) \in [0, 1] \quad (18)$$

for all $s_\tau \geq 0$ and for $\tau = 1, \dots, T-1$.³⁰ We assume these shifts do not affect newborns, so $\bar{p}_0(r) = p_0(r)$ for all $r \geq 0$.

For the case of transfers without sex, we showed that

$$p'_\tau(r_\tau^*) = p'_\tau(s_\tau^*)v_{\tau+1}^*, \tau = m, \dots, T-1,$$

where these optimal choices are now denoted by r_τ^* and s_τ^* and the notation v_τ^* is introduced to distinguish the two sets of shadow prices.

With sex, we have that

$$\frac{p'_0(\bar{r}_\tau)}{2} = \beta p'_\tau(\bar{s}_\tau)\bar{v}_{\tau+1}, \tau = m, \dots, T-1.$$

In order to show that $\bar{r}_\tau < r_\tau^*$, and $\bar{s}_\tau > s_\tau^*$, it is then enough to show that $\bar{v}_{\tau+1} > \frac{v_{\tau+1}^*}{2}$, for $\tau = m, \dots, T-1$. Indeed, this claim holds at $\tau = T-1$ because $v_T^* = \bar{v}_T = p_0(I_T/u_T)u_T$. Adopt then the inductive hypothesis that $\bar{v}_{\tau+1} > \frac{v_{\tau+1}^*}{2}$ for some $\tau+1 \in \{m+1, \dots, T-1\}$. It follows that

$$\begin{aligned} \bar{v}_\tau &= p_0(\bar{r}_\tau)u_\tau + \bar{p}_\tau(\bar{s}_\tau)\bar{v}_{\tau+1} \\ &> \frac{p_0(\bar{r}_\tau)u_\tau}{2} + \bar{p}_\tau(\bar{s}_\tau)\bar{v}_{\tau+1}. \end{aligned}$$

Further, since $(\bar{r}_\tau, \bar{s}_\tau)$ maximizes this last expression, it now follows that

$$\begin{aligned} \bar{v}_\tau &> \frac{p_0(r_\tau^*)u_\tau}{2} + \bar{p}_\tau(s_\tau^*)\bar{v}_{\tau+1} \\ &> \frac{p_0(r_\tau^*)u_\tau}{2} + \frac{p_\tau(s_\tau^*)v_{\tau+1}^*}{2} = \frac{v_\tau^*}{2}, \end{aligned}$$

²⁹Of course, $\beta = \frac{B(\bar{N})}{B(N^*)} > 1$.

³⁰The key results here concern the implications of the resource allocations of adults for impatience. These results hold under considerably weaker conditions concerning the effect of reduced selection pressure on survival. However, the multiplicative shift of survival probabilities considered here simplifies the incidental results for children.

completing the inductive proof. Hence sex unambiguously reduces all flexible transfers, those at ages $\tau = m, \dots, T - 1$, and increases the corresponding amounts used to promote own survival to the next period.

To sum up—

Theorem 5 *In the present model of transfers with sex, suppose that the same growth factor, $\lambda = 1$, is attained as in the model without sex by means of the shift described in Eq (18). It follows that all adults of age $\tau = m, \dots, T - 1$ transfer an amount \bar{r}_τ , say, to their newborn offspring which is strictly less than the amount transferred in the model of transfers without sex, r_τ^* , say. Equally, the amount such adults use for their own survival \bar{s}_τ , say, strictly exceeds that in the model without sex, s_τ^* , say.*

We have shown that sex reduces the quality of offspring, in that it involves reduced transfers to newborns and hence reduced survival of all of these to age one. The flip side of the coin is that adult survival rates are increased by sex relative to the model with no sex.

The rate of time preference in the present model with sex is of the same form as the model of the previous subsection without sex, as described in Theorem 2. Denoting the resource allocation choices made with sex by \bar{r}_τ and \bar{s}_τ for $\tau = 1, \dots, T$, the rate of time preference for adults, those of age $\tau = m, \dots, T - 1$, is—

$$1 + \bar{\rho}_\tau = \frac{\frac{\partial \lambda}{\partial I_\tau}}{\frac{\partial \lambda}{\partial I_{\tau+1}}} = \frac{p'_0(\bar{r}_\tau)}{p'_0(\bar{r}_{\tau+1})\bar{p}_\tau(\bar{s}_\tau)}, \tau = 1, \dots, T - 1, \quad (19)$$

It is important to note that the derivation of this expression for the rate of time preference essentially involves canceling a factor of $1/2$ from the numerator and the denominator. That is, since the individual deflates the value of resources transferred to all offspring, there is no direct effect of sex on impatience. We will show, however, that there remains an indirect effect that operates through enhanced survival of the adult from one age to the next.

Eq (19) has a similar interpretation as obtained for the model without sex. Furthermore, the same qualitative observations apply concerning how this rate of time preference changes with age. That is, the model with sex can also readily generate children and younger adults who are more impatient than implied by the mortality rate and the population growth rate, but older adults who are more patient.

Arguing exactly as in the model without sex, but now using Eq (18), the rate of time

preference for children of age $\tau = 1, \dots, m - 2$ is given by

$$1 + \bar{\rho}_\tau = \frac{\bar{p}'_\tau(I_\tau)\bar{p}_{\tau+1}(I_{\tau+1})}{\bar{p}'_{\tau+1}(I_{\tau+1})\bar{p}_\tau(I_\tau)} = \frac{p'_\tau(I_\tau)p_{\tau+1}(I_{\tau+1})}{p'_{\tau+1}(I_{\tau+1})p_\tau(I_\tau)}.$$

For a child at the transition, at age $\tau = m - 1$, we have the special case that

$$1 + \bar{\rho}_{m-1} = \frac{2\bar{p}'_{m-1}(I_{m-1})\bar{v}_m}{\bar{p}_{m-1}(I_{m-1})p'_0(\bar{r}_m)} = \frac{2p'_{m-1}(I_{m-1})\bar{v}_m}{p_{m-1}(I_{m-1})p'_0(\bar{r}_m)}.$$

We again assume, in the interest of realism, that children are impatient, as captured by Eq (10) from the previous subsection. Further, the impatience of children is decreasing as in Eq (11).

We are now in a position to answer the question: What is the effect of sex on the rate of time preference? There is no effect of sex on the rate of time preference of children of age $\tau = 1, \dots, m - 2$. The effect on impatience of the transitional child of age $\tau = m - 1$ may be ambiguous without further assumptions, but is of limited interest.

Consider then the effect of sex on the rate of time preference of adults. Let the resource allocation choices made by adults in the model without sex be then given by r_τ^* and s_τ^* , for $\tau = m, \dots, T$. Recall their rate of time preference is then—

$$1 + \rho_\tau^* = \frac{\frac{\partial \lambda}{\partial I_\tau}}{\frac{\partial \lambda}{\partial I_{\tau+1}}} = \frac{p'_0(r_\tau^*)}{p'_0(r_{\tau+1}^*)p_\tau(s_\tau^*)}, \tau = m, \dots, T - 1. \quad (20)$$

When we compare the expressions in Eq (19) and Eq (20), there is a clear effect of sex on one component of the adult rate of time preference. That is, we have that

$$\frac{1}{\beta p_\tau(\bar{s}_\tau)} < \frac{1}{p_\tau(s_\tau^*)}, \tau = m, \dots, T - 1,$$

since sex unambiguously increases adult survival probabilities. The effect of the remaining component of time preference, which concerns the derivatives of the survival rates of newborns, is less clear in general, but might sometimes work in the opposite direction. In any case, one case where this remaining component has no effect is if the survival rate of newborns is linear in resources. That is, in addition to the hypotheses of Theorem 4, suppose that $p_0(r_\tau) = \alpha r_\tau$, for some $\alpha > 0$ and that adults of age $\tau = m, \dots, T - 1$ have interior optimal resource allocations, with or without sex. Now, sexual reproduction strictly decreases the rate of time preference of adults of age $\tau = m, \dots, T - 1$, but it has no effect on the rate of time preference of children of age $\tau = 1, \dots, m - 2$.

The central message of Rogers was that sex is a key factor leading to impatience. In this model, however, this conclusion does not generally hold.

4 Related Literature

We now discuss some closely related papers that help form the backdrop to the present investigation. First, note that the rate of time preference is implicit in several models that are partly based on the huge intergenerational transfers and social exchanges that are characteristic of our species. These models explain how individuals who are beyond reproductive age may still be selected to have limited mortality. This is because they retain biological value as the source of resource transfers to their young. Such transfers make such older adults more patient—indeed, without transfers, an adult who is beyond reproductive age has zero biological marginal product of resources (see Robson and Kaplan [20] and Kaplan and Robson [16], for example).

Consider, in a little more detail, the model of Robson and Kaplan [20]. This considers an asexual population in long run demographic equilibrium, with a population distribution that reflects the probability of survival to each age, and the population growth rate. There is assumed to be social budget constraint under which the aggregate resource deficits of the young are covered by the aggregate resource surpluses of older individuals. This social budget constraint smoothes over lumpiness and risk that would be evident on the level of each family. The population is asexual which finesses the issue of how sex would attenuate the incentives to make such intergenerational transfers.

From the point of view of each individual, the social budget constraint can be reinterpreted as a lifetime budget constraint. The rate of time preference is the rate at which resources at different ages would be traded off in the budget constraint, and is just the rate of population growth plus the rate of mortality. This argument concerning the rate of time preference extends to individuals who are beyond the last age of reproduction. Without transfers, on the other hand, such individuals would have had no use for resources at all at these advanced ages. Allowing transfers therefore helps explain why older individuals are not extremely impatient.³¹

Finally, in this vein of closely related papers, consider two recent complementary papers due to Robson and Samuelson. Robson and Samuelson [21] reviews how expected discounted utility arises in a simple setting with only idiosyncratic risk but also shows that non-exponential discounting might arise for large trade-offs. Robson and Samuelson [22] show that an alternative way of closing the gap between typical rates of time preference

³¹There is perhaps a sense in which this result then comes from the supply side for resources, rather than the demand side.

and typical mortality rates is to introduce aggregate mortality shocks. If all age classes are subject to a common aggregate shock to survival, there is still geometric discounting. However, even if the population growth rate is zero on average, the rate of discount will exceed the mortality rate derived from expected survival. Furthermore, they show that imperfectly correlated aggregate shocks to survival may produce a decreasing pure rate of time preference, holding other factors constant, but would not produce preference reversals. However, they do not investigate the more refined version of the puzzle posed by this gap in comparable detail to the present paper.

Taking a more general perspective, there is a small but growing literature arguing that evolution can help explain preferences.³² Most of these papers focus on attitudes towards risk and altruism. Overviews of the theories on the relationship between biology and economic behavior can be found in Robson [18] and [19]. Here we review only the handful of papers concerned with the evolution of time preferences that are additional to those already discussed.

Hansson and Stuart [13] consider a neoclassical growth model in which clans compete for shares of resources. A clan's production depends on the stock of clan-specific capital and on the total population of all clans. The clans indeed face a carrying capacity constraint, meaning that per capita production decreases as total population rises. An individual is active for only one period, and each clan faces a consumption-saving decision. Higher consumption results in higher immediate population growth but a smaller capital stock, and hence smaller output, in the next generation. The carrying capacity constraint means that each clan's size must be constant in the long-run equilibrium. As a result, evolution selects clans with a zero rate of time preference. Agents maximize the per capita steady state consumption of current and future generations.

Robson and Wooders [24] also derive a zero rate of time preference in a growth model where total output depends on capital and labor. Both labor and capital in the model can be of many types. The per capita income of each type of labor determines its growth rate. The authors show that when the balanced growth rate is maximized, income must be distributed across individuals in accordance with marginal product pricing.

Hansson and Stuart [13] and Robson and Wooders [24] were motivated by concerns additional to time preference. Both papers are led then to consider choice problems that are too simple in the relevant dimensions for the present focus on time preference.

³²A key early paper is Becker [2], who argues that economic arguments might be used instead of evolutionary ones in explaining altruistic behavior between relatives, for example.

Two subsequent papers conclude that evolution might produce time-inconsistent preferences. First, Samuelson and Swinkels [26] assume that agents cannot process information perfectly. Agents make decisions based on incorrect priors. As a result, time-inconsistent preferences can survive evolution because they compensate for faulty information processing.

Dasgupta and Maskin [8] argue that, during the evolutionary process, individuals faced the following typical choice problem. There are two random options, A and B . At time zero, option A is more attractive than option B . However, if A has not paid off after some time, option B becomes more attractive. Thus it is optimal to choose option A at time zero but switch to option B after some time. The authors argue that in modern times, individuals face situations they did not face during the evolutionary process. They associate each modern option with an option available before, A or B , for example. This may generate actual time inconsistency. In other words, Dasgupta and Maskin take the view that individuals behave suboptimally in a modern society. Unlike Samuelson and Swinkels (2006) or Maskin and Dasgupta (2005), we assume that neither the information structure nor individuals' choice problems change over time.

5 Conclusions

The model of transfers adopted here is one of the more tractable possible. Indeed, the model is mathematically similar to the benchmark model presented in Section 2. It is not formally very different to suppose that newborns have an endogenous survival rate than it is to suppose the number of these is endogenous. In both cases, there is an endogenous number of surviving age one individuals. Indeed, since all individuals who are one or more are identical, the notion of reproductive value can be straightforwardly applied to all individuals of age one or more, despite the differences among newborns. However, the issues here are conceptually important and revealing. It is generally true that differentiated offspring require a notion of reproductive value that is not the simple expected discounted value of newborns. Further, it is only when the effect of transfers is confined to newborns that matters are as straightforward as they are here.³³

In terms of accounting for the age profiles of empirically observed rates of time preference, it might be that the ideal model would combine intergenerational transfers, along

³³However, a model in which transfers were made at birth, but had effects that lasted more than one period, would seem likely to remain tractable.

the lines developed here, and aggregate mortality shocks, as in Robson and Samuelson [22]. Any model that did this, however, would likely skirt the limits of analytic feasibility.

It would also be of interest to instead generalize the system of transfers substantially. It is realistic, after all, to allow these transfers to be made by parents to older children as well as to newborns. Further, it is also realistic to allow transfers from parents who are beyond the age of last reproduction, not only to their children but also to their grandchildren. Such a generalization would form a useful counterpart to the investigations of Kaplan and Robson [16]. Whereas Kaplan and Robson consider a full model of social intergenerational transfers with no sex; such a generalization would involve intra-family transfers and sex.

Such more general models of transfers often become rapidly intractable. It is not hard to write an apparently simple model in which the transfer that a parent will make to a child depends not merely on the age of the parent herself, but on the age of her parent when she was born, the age of her grandparent when her parent was born, *et cetera*. Even if strong simplifying assumptions are made that prevent such snowballing complications, there remain a huge number of transfers that might be made in principle, not all of which will have interior solutions, so that the model may be rather complex to analyze.

In this light, the present model is valuable as illustrating that some central results are likely to be robust—such as the maximization of the appropriate concept of reproductive value in a dynamic programming context. More specifically, the model suggests reasons why children and younger adults may be too impatient and older adults too patient, relative to the basic model. Finally, it provides a counterexample to any general claim that sex would generate an inappropriately high rate of time preference. That is, although sex distorts the rate of time preference, it may well reduce it. Indeed, the reason why sex fails to sharply increase impatience seems likely to be a feature of many more general models.

It is important to note that despite varying rates of time preference, there can be no preference reversals in a frictionless model like the models used here. In the first place, time preference is tied here to age, rather than to time into the future, as is conventional in economics in general and in discussions of hyperbolic discounting in particular. Even more fundamentally, if a particular choice involving two remote ages were optimal when considered at some young age, evolved preferences would operate in models like these to ensure that such a choice would always be made in the same way even when considered at ages that were closer to the ages in question.

Another issue that arises with respect to this model is as follows. There is here a unique

best or evolutionarily stable outcome. What then accounts for the heterogeneity in time preference actually observed? Most basically, such questions trouble biologists also. That is, why do we typically observe substantially dispersed phenotypes? One intriguing answer is provided by models of aggregate uncertainty in which the optimal genotype involves idiosyncratic randomization.³⁴ Another answer that might be especially relevant here is: Time preference is not entirely genetically controlled but is influenced by experience as well. That is, for example, individuals who survive traumatic adverse economic outcomes might well become more risk-averse and more patient. Observed heterogeneity of preferences simply then reflects heterogeneity in personal economic histories. However, this approach takes us even further from conventional economics and is left for future research.

Finally, the model of transfers with sex raises the following interesting issue. The game between the two parents is one where offspring survival is a public good. The undercontribution that results is not surprising in that economic light and it is in complete agreement with the biological perspective as in “Hamilton’s rule.” However, this prediction does not seem iron-clad here. It would be more convincing in a model where the male contributes nothing to raising offspring, as is typical of most mammalian species other than our own. That is, suppose that the resources are contributed to offspring only by the female, in the complete absence of the male. Although this absence would induce her to contribute more, she would still have a marginal incentive to make transfers that was reduced on account of sex by $1/2$. When both parents are present and contribute, on the other hand, there is an opportunity to find ways around this undercontribution problem. These range from the biological at one end—endocrinology forging a pair bond between parents (“falling in love”), for example—to selfishly rational behavior at the other—as in the familiar ways of inducing cooperation in repeated games.

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³⁴See Bergstrom [4] for example.

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