THE EVOLUTION OF DECISION AND EXPERIENCED UTILITIES*

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Been Down So Long It Looks Like Up To Me—Richard Fariña.

Abstract. Psychologists report that people make choices on the basis of "decision utilities" that routinely overestimate the "experienced utility" consequences of these choices. This paper argues that this dichotomy between decision and experienced utilities may be the solution to an evolutionary design problem. We examine a setting in which evolution designs agents with utility functions that must mediate intertemporal choices, and in which there is an incentive to condition current utilities on the agent's previous experience. Anticipating future utility adjustments can distort intertemporal incentives, a conflict that is attenuated by separating decision and experienced utilities.

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The Evolution of Decision and Experienced Utilities

1 Introduction

People who contemplate living in California routinely report that they expect to be significantly happier there, primarily on the strength of California's blissful weather. People who actually live in California are no happier than the rest of us (Schkade and Kahneman [15]). Far from being a California quirk, this "focussing illusion" is sufficiently widespread as to prompt the conclusion that "Nothing ... will make as much difference as you think." [15, p. 345][1]

Psychologists interpret these findings by drawing a distinction between *decision utility* and *experienced utility* (e.g., Kahneman and Thaler [7]). Decision utilities are the utilities that determine (or at least describe, in a revealed-preference interpretation) our choices. For Schkade and Kahneman [15], these are the utilities people reveal when they contemplate living in California. Experienced utilities are the rewards we realize once the choices are made. For Schkade and Kahneman, these are reflected in the satisfaction reports from people living in California.

Experienced utilities are of no interest to a fiercely neoclassical economist—decision utilities suffice to describe behavior. However, if we are to consider welfare questions, the difference may be critically relevant. If experienced utilities do not match decision utilities, should we persevere with the standard economists' presumption that decision utilities are an appropriate guide to well-being? Or should we exhort people to work more diligently in discerning their future experienced utilities, and then use these to override their decision utilities (as Schkade and Kahneman [15] imply)? Once we have contending utilities (or contending selves, in the common parlance of behavioral economics), such questions are both inevitable and vexing.

We adopt a positive perspective in this paper, answering the following question: Why might we have both decision and experienced utilities in the first place? We take an evolutionary approach. We assume that evolution has equipped agents with utility functions designed to induce fitness-maximizing choices. An agent in our model must make choices in each of two periods that will (along with random events) determine his fitness. Moreover, these choices give rise to an intertemporal trade-off, in the sense that the optimal second-period choice depends upon the alternative chosen in the first period. The first-period choice may determine the agent's health or wealth or skill or status,

[1]The term "focussing illusion" (e.g., Loewenstein and Schkade [9]) refers to a tendency to overestimate either the salutary or detrimental effects of current choices. This phenomenon was thrust into the spotlight by Brickman, Coates and Janoff-Bulman's [1] study of lottery winners and paraplegics, and has become the subject of a large literature. See Loewenstein and Schkade [9] for an introduction and Gilbert [6] for an entertaining popular account.

Attention has also been devoted to the related prospect that people may exhibit a *projection bias* (Loewenstein, O'Donoghue and Rabin [8], Conlin, O'Donoghue and Vogelsang [3]). An agent exhibits a projection bias if he expects his future preferences to be more similar to his current preferences than will actually be the case.

for example, which may in turn affect how aggressive the agent should be in seeking second-period consumption. Evolution equips the agent with a first-period utility function providing the decision utilities shaping the first-period choice. Evolution also equips the agent with a second-period utility function determining the hedonic rewards he experiences as a result of his first-period and second-period choices. This latter function is interpreted as experienced utility, though it also provides the relevant decision utility for the second period.[2] We show that in general, the decision utility shaping the first-period choice does not match the resulting second-period experienced utility. Evolution systematically misleads the agent as to the future implications of his choices.

Why should evolution build an agent to do anything other than maximize fitness, without resorting to conflicting utility notions? Evolution's design problem is complicated by two constraints. First, there are limits on how large and how small are the hedonic utilities evolution can give us.[3] By themselves, bounds on utility pose no obstacles. All that matters is that better alternatives get higher utilities, and we can accommodate this no matter how tight the range of possible utilities. However, our second assumption is that the agent is likely to make mistakes when utilities are too close. When alternative 1 provides only a slightly higher utility than alternative 2, the agent may mistakenly choose alternative 2. As a result, there is an evolutionary advantage to having the utility function be as steep as possible, so that the agent is dealing with large utility differences that seldom induce mistakes. This goal conflicts with the bounds on utility. Evolution's response is to make the utility function very steep in the range of decisions the agent is most likely to face, where such steepness is particularly important in avoiding mistaken decisions, and relatively flat elsewhere.[4] For this is to be effective, the steep spot of the utility function must be in the right place. In the second period, the "right place" depends on what happens in the first period. Evolution thus has a incentive to adjust second-period or experienced utilities in response to first-period outcomes. But if this is to be done without distorting first-period decisions, the agent must not anticipate this adjustment—the experienced utilities guiding second-period decisions must not match the decision utilities shaping first-period decisions.

Section 2 introduces the evolutionary environment. Section 3 presents the analysis of decision and experienced utility in the context of a simple model. Section 4 considers extensions and implications.

[2]It is relevant in this connection that Carter and McBride [2] argue that experienced utility has similar empirical properties to decision utility.

[3]Notice that in taking this position, we are following much of the current literature in behavioral economics in viewing utility maximization as a neurological process by which we make choices, rather that simply a description of consistent choices. In particular, our view is that utilities are induced by chemical processes within our brains that are subject to physical constraints.

[4]Robson [13] argues that utility bounds and limited discrimination between utilities will induce evolution to strategically position the steep part of the utility function. See Friedman [5] for a precursor and Netzer [10] and Wolpert and Leslie [18] for more recent contributions. Rayo and Becker [12] develop this idea in a model more closely related to that here. Tremblay and Schultz [17] provide evidence that the neural system encodes relative rather than absolute preferences, which might be a natural consequence of such limited discrimination.

2 The Evolution of Utility

2.1 The Evolutionary Environment

There are two periods. The agent makes a choice x_1 in the first period and x_2 in the second. These choices would be multidimensional in a more realistic model, but here are taken for simplicity to be elements of $[0, 1]$. Whenever it is helpful in conveying intuition, we (temporarily) adopt particular interpretations of x_1 and x_2, such as levels of first-period and second-period consumption, or as a decision to move to California (or not) and a subsequent decision of how much time to spend surfing (whether in California or Iowa). We recognize that our stark one-dimensional variables cannot capture all the subtleties of such decisions.

The agent's fitness is determined by his choices x_1 and x_2 as well as the realizations s_1 and s_2 of independent random variables $\tilde{s}_1$ and $\tilde{s}_2$, reflecting environmental shocks in the first and second periods. For example, the agent's health may depend not only on effort he invests in procuring food, but also on vagaries of the weather or stock market affecting the productivity of these efforts. We assume that $\tilde{s}_1$ and $\tilde{s}_2$ have zero means (i.e., we measure fitness in terms of increments from a base fitness level). The agent's first-period choice x_1 must be made in ignorance of the realization s_1, while x_2 is chosen knowing s_1 but not s_2.

In the absence of any constraints, evolution's task of designing an agent to make fitness-maximizing choices would be trivial. The agent's problem has a maximizer $(x_1^*, x_2^*(\cdot))$, where $x_2^*(\cdot)$ is the optimal mapping from first-period outcomes to second-period choices. Why would evolution not simply "hard-wire" agents to make this optimal decision?

The point of departure for our analysis is the assumption that evolution *cannot* hard-wire the alternative $(x_1^*, x_2^*(\cdot))$, as trivial as this sounds in the context of this model. Our interpretation here is that what it means to choose a particular value of x_1 or x_2 changes with the context in which the decision is made. The agent's choice may consist of an investment in status that sometimes involves hiding food and other times acquiring education, that sometimes involves cultivating social relationships with neighbors and other times driving neighbors away. Moreover, the relevant context fluctuates too rapidly for evolution to adapt. The dominant form of investment can change from clearing fields to learning *C++* too quickly for mutation and selection to keep pace. As a result, evolution must recognize that the agent will frequently face problems that are novel from an evolutionary perspective.[5]

[5] Rayo and Becker [12] similarly address the question of why evolution cannot hard-wire agents to make optimal choices. They assume that the evolutionarily optimal action depends upon an environmental state, and that there are so many possible values of this state that it is prohibitively expensive for evolution to hard-wire the agent to condition actions on every value. Our assumption that the state is entirely novel is equivalent, differing from Rayo and Becker primarily in emphasis. Rayo and Becker explicitly include the state variable within their model, while we sweep it into the background, simply assuming that evolution cannot dictate optimal choices, in order to simplify the notation. Their simplest model, which corresponds to

To capture this constraint, we need to specify the technology by which the agent's decisions are converted into fitnesses. Our point of departure is the relationship

$$z = z_1 + \delta z_2,$$

defining the agent's realized total fitness z as the sum of realized first-period fitness z_1 and the discounted value of realized second-period fitness z_2, with the discount factor δ perhaps reflecting a nonunitary survival probability. At this point, however, we note that it requires only a change in the units in which z and z_1 are measured to normalize the discount factor to be unity, and hence to rewrite this equation as $z = z_1 + z_2$. This significantly simplifies the following notation and so we we adopt this convention throughout. We then write

$$\begin{aligned} z &= z_1 + z_2 && (1)\\ &= \hat{\zeta}_1(x_1, s_1) + \hat{\zeta}_2(z_1, x_1, x_2, s_2) && (2)\\ &= [\zeta_1(x_1) + s_1] + [\gamma z_1 + \zeta_2(x_1, x_2) + s_2]. && (3) \end{aligned}$$

The first line presents our normalized accounting of fitness. The next line indicates that first-period fitness is a function of the first-period action x_1 and shock s_1. For example, x_1 may reflect an investment in skills, and z_1 the resulting expertise, or x_1 may reflect actions taken in pursuit of status, and z_1 the resulting place in the social order. Second period fitness is similarly a function of the second-period action x_2 and shock s_2, and also a function of both the first-period action x_1 and fitness z_1. A relatively large value of x_1 may reflect a first-period investment that enhances the productivity of x_2 in the second period. In addition, a relatively large first-period fitness z_1 may carry over directly into a higher second-period fitness, regardless of how z_1 is achieved. An agent who is better-nourished in the first period may enjoy the salutary effects of good health in the second. The final line simplifies the analysis by making the fitness functions quasi-linear. Section 4.2 describes how the quasi-linearity assumption can be generalized.

Technically, the key distinction is that, while evolution cannot attach utilities to the agent's choices x_1 and x_2, she can attach utilities to total fitnesses. Times have changed too quickly for evolution to attach utility to passing through the drive-through coffee line in the morning, but she can reward the resulting feeling of alertness.

We assume that ζ_2 is strictly concave in x_2, and that $\zeta_1(x_1) + [\gamma\zeta(x_1) + \max_{x_2} \zeta_2(x_1, x_2)]$ is strictly concave in x_1. This ensures the existence of unique expected fitness maximizers x_1^* and $x_2^*(x_1)$, which we take to be interior. We further assume that each ζ_i is continuous and therefore bounded. We assume that $\tilde{s}_1$ and $\tilde{s}_2$ have differentiable, symmetric unimodal densities g_1 and g_2, with zero derivatives only at 0, that are positive on supports $[\underline{s}_1, \overline{s}_1]$ and $[\underline{s}_2, \overline{s}_2]$, with symmetry implying that $\underline{s}_1 = -\overline{s}_1$ and $\underline{s}_2 = -\overline{s}_2$. Our results go through

our basic model, then makes the analysis more tractable by assuming that the state variable affects optimal actions but not maximal fitness.

unchanged, and with somewhat simpler technical arguments, if $\tilde{s}_1$ and $\tilde{s}_2$ have unbounded supports.

Finally, we should be clear on our view of evolution. We adopt throughout the language of principal-agent theory, viewing evolution as a principal who "designs" an incentive scheme in order to induce (constrained) optimal behavior from an agent. However, we do not believe that evolution literally or deliberately solves a maximization problem. We have in mind an underlying model in which utility functions are the heritable feature defining an agent. These utility functions give rise to frequency-independent fitnesses. Under a simple process of natural section respecting these fitnesses, expected population fitness is a Lyapunov function, ensuring that the type maximizing expected fitness will dominate the population. If the mutation process generating types is sufficiently rich, the outcome of the evolutionary process can then be approximated by examining the utility function that maximizes expected fitness, allowing our inquiry to focus on the latter.

2.2 Utility Functions

Evolution can endow the agent with a nondecreasing utility function $V_1(z)$ in the first period, and in the second period with a nondecreasing utility function $V_2(z|z_1)$. In the first period, the agent considers the realized total fitness z produced by the agent's first-period and anticipated second-period choice, reaping utility $V_1(z)$. In the second period, the agent's choice induces a realized total fitness z and hence corresponding utility $V_2(z|z_1)$. Notice in particular that evolution can condition second-period utilities on the realization of the first-period intermediate fitness z_1. Through the technology given by (1)–(2), V_1 and V_2 implicitly become utility functions on x_1, x_2, s_1 and s_2.[6]

To interpret these utility functions, let us return to our moving-to-California decision. We think of $V_1(z)$ as representing the utility the agent contemplates should he move to California, taking into account his projections of how much surfing he will do once there. $V_2(z|z_1)$ is the utility the agent realizes, once he has moved to California. We think of the former as the decision utility mediating the first choice, and the latter as the resulting experienced utility. If these functions are identical, we have no focussing illusion.

In the absence of any additional constraints (beyond the inability to write utilities directly over x_1 and x_2), evolution's utility-function design problem is still trivial. She need only give the agent the utility functions

$$\begin{aligned} V_1(z) &= z \\ V_2(z|z_1) &= z. \end{aligned}$$

As straightforward as this result is, we believe it violates crucial evolutionary constraints that we introduce in two steps. Our first assumption is that evolution

[6]Our view here is that the agent effectively learns which values of x_1, x_2, s_1 and s_2 lead to high utilities, in the process coming to act "as if" the agent "knows" the functions ζ_1 and ζ_2. For the same reason the evolution cannot make utility a function of x_1 and x_2, evolution cannot simply tell the agent to maximize $\zeta_1(x_1) + s_1 + [\gamma z_1 + \zeta_2(x_1, x_2) + s_2]$.

faces limits on how large or small a utility she can induce. Our view here is that utilities must be produced by physical processes, presumably the flow of certain chemicals in the brain. The agent makes choices leading to a fitness level z, and receives pleasure from the resulting cerebral chemistry. There are then bounds on just how strong (or how weak) the resulting sensations can be. Without loss, we assume that utilities must be drawn from the interval $[0,1]$.[7]

These constraints alone pose no difficulties. Essentially, evolution need simply recognize that utility functions are unique only up to linear transformations. In particular, in this case evolution need only endow the agent with the utility functions

$$\begin{aligned} V_1(z) &= A + Bz \\ V_2(z|z_1) &= A + Bz, \end{aligned}$$

where A and B are chosen (in particular, with B sufficiently small) so as to ensure that utility is drawn from the unit interval, no matter what the feasible values of x_1, x_2, s_1 and s_2.

We now add a second constraint to evolution's problem—there are limits to the ability of the agent to perceive differences in utility. When asked to choose between two alternatives whose utilities are very close, the agent may be more likely to choose the alternative with the higher utility, but is not certain to do so. This is in keeping with our interpretation of utility as reflecting physical processes within the brain. A very slightly higher dose of a neurotransmitting chemical may not be enough to ensure the agent flawlessly chooses the high-utility alternative.[8] In particular, we assume that there is a possibly very small ε_i such that in each period i, the agent can be assured only of making a choice that brings him within ε_i of the maximal utility. We will then be especially interested in the limits as the utility errors $\varepsilon_i \to 0$. It may well be, of course, that such errors are not small in practice. However, we are interested in the role of utility constraints in driving a wedge between decision and experienced utilities, and especially interested in the possibility that such a wedge could arise despite arbitrarily small errors.

[7]Evidence for bounds on the strength of hedonic responses can be found in studies of how the firing rate of neurons in the pleasure centers of the brain respond to electrical stimulation. Over an initial range, this response is roughly linear, but eventually high levels of stimulation cause no further increase. See, for example, Simmons and Gallistel [16].

[8]Very small utility differences pose no problem for classical economic theory, where differences in utility indicate that one alternative is preferred to another, with a small difference serving just as well as a large one. However, it is a problem when utilities are induced via physical processes. The psychology literature is filled with studies documenting the inability of our senses to reliably distinguish between small differences. (For a basic but vivid textbook treatment, see Foley and Matlin [4].) If the difference between two chemical flows is arbitrarily small, we cannot be certain that the agent will invariably choose the larger.

3 Decision and Experienced Utility

3.1 The Second Period

The agent enters the second period having made a choice x_1 and realized a first-period fitness of z_1. The agent cannot distinguish any pair of choices whose expected utilities are within $\varepsilon_2 > 0$ of each other. Hence, instead of certainly choosing the maximizer $x_2^*(x_1)$ of $E_{\tilde{s}_2} V_2(z_1 + (\gamma z_1 + \zeta_2(x_1, x_2) + \tilde{s}_2)|z_1)$ in the second period, the agent may choose any x_2 with the property that

$$E_{\tilde{s}_2} V_2(z_1 + (\gamma z_1 + \zeta_2(x_1, x_2^*) + \tilde{s}_2)|z_1) - E_{\tilde{s}_2} V_2(z_1 + (\gamma z_1 + \zeta_2(x_1, x_2) + \tilde{s}_2)|z_1) \leq \varepsilon_2.$$

To keep things simple, we assume the agent chooses uniformly over the resulting satisficing set $[\underline{x}_2, \overline{x}_2]$, where $\underline{x}_2 < x_2^* < \overline{x}_2$ and[9]

$$\begin{aligned} & E_{\tilde{s}_2} V_2((1+\gamma)z_1 + \zeta_2(x_1, \underline{x}_2) + \tilde{s}_2|z_1) \quad (4) \\ = \; & E_{\tilde{s}_2} V_2((1+\gamma)z_1 + \zeta_2(x_1, \overline{x}_2) + \tilde{s}_2|z_1) \\ = \; & E_{\tilde{s}_2} V_2((1+\gamma)z_1 + \zeta_2(x_1, x_2^*) + \tilde{s}_2|z_1) - \varepsilon_2. \quad (5) \end{aligned}$$

It would be more realistic to model the utility perception error ε_2 as proportional to the maximized expected fitness, rather than as an absolute error. Doing so has no effect on our analysis. In particular, we can interpret ε_2 as the "just noticeable difference" in utilities induced by the equilibrium of the proportional-errors model, and then simplify the notation by writing the constraints as in (4)–(5), while retaining the proportional interpretation of the errors.

Evolution chooses the utility functions V_2 to maximize fitness, subject to (4)–(5), giving:

Lemma 1 *There exist functions $\underline{Z}_2(z_1)$ and $\overline{Z}_2(z_1)$, with $\underline{Z}_2(z_1) \leq \overline{Z}_2(z_1)$, such that the optimal second-period utility function satisfies*

$$\begin{aligned} V_2(z|z_1) &= 0, \text{ for all } z < \underline{Z}_2(z_1) \quad (6) \\ V_2(z|z_1) &= 1, \text{ for all } z > \overline{Z}_2(z_1). \quad (7) \end{aligned}$$

In the limit as $\varepsilon_2 \to 0$, the agent's second-period choice x_2 approaches $x_2^*(x_1)$.

If $\varepsilon_1 > 0$, then x_1 arises out of random satisficing behavior in the first period, but the second-period choice (when $\varepsilon_2 \to 0$) is $x_2^*(x_1)$, for each realization x_1.

This result leaves open the question of how the utility function is specified on the potentially nonempty interval $(\underline{Z}_2(z_1), \overline{Z}_2(z_1))$. In the course of examining the first period, we will show that this gap shrinks to zero as does ε_1, the first-period utility-perception error. In particular, the gap $(\underline{Z}_2(z_1), \overline{Z}_2(z_1))$ arises because evolution faces uncertainty concerning agent's first-period choice x_1.

[9] We could work with a more general assumption about how the choice from the satisficing set is made, but must preclude the possibility that an attempt by evolution to improve the agent's decisions by increasing $\underline{x}_2$ and decreasing $\overline{x}_2$ is thwarted by agent's pushing more and more of her choice probability toward these boundaries.

As $\varepsilon_1 \to 0$, this uncertainty disappears, and in the process $\underline{Z}_2(z_1)$ and $\overline{Z}_2(z_1)$ converge. We thus approach a bang-bang utility function, equalling 0 for small fitnesses and 1 for large fitnesses.

The values $\underline{Z}_2(z_1)$ and $\overline{Z}_2(z_1)$ depend on z_1. This allows evolution to adjust the second-period utility function in order to maximize its effective slope, minimizing the incidence of mistaken decisions.

The bang-bang limiting character of this utility function may appear extreme, dooming the agent to being either blissfully happy or woefully depressed. Notice, however, that the expected utilities with which the agent evaluates his choices do not have this property. The utility function $E_{\tilde{s}_2}V_2((1+\gamma)z_1 + \zeta_1(x_1,x_2) + \tilde{s}_2|z_1)$ is a continuous function of x_2 (given x_1 and z_1).

The remainder of this section presents and discusses the argument behind this result. Suppose first (temporarily) that evolution could condition the second-period utility function on the agent's first-period choice x_1 as well as his first-period fitness z_1 (with the assumption concerning x_1 dropped in the last paragraph of this section). Let $x_2^*(x_1)$ be the unique (since ζ_2 is strictly concave) maximizer of $\zeta_2(x_1,x_2)$. Since V_2 is nondecreasing in z, $E_{\tilde{s}_2}V_2$ is strictly increasing in ζ_2. From (4)–(5), we then have $\zeta_2(x_1,\underline{x}_2) = \zeta_2(x_1,\overline{x}_2)$, with the agent choosing x_2 from a uniform distribution over $[\underline{x}_2,\overline{x}_2]$. The agent's second-period fitness will then be higher the smaller is the satisficing set $[\underline{x}_2(x_1),\overline{x}_2(x_1)]$, or equivalently the larger is $\zeta_2(x_1,\underline{x}_2(x_1)) = \zeta_2(x_1,\overline{x}_2(x_1))$.

The problem is then one of maximizing $\zeta_2(z_1,\overline{x}_2) = \zeta_2(z_1,\underline{x}_2)$, subject to the constraints given by (4)–(5). The constraints given by (4)–(5) can be written as[10]

$$\varepsilon_2 = \int V_2(z|z_1)[g_2(z-[(1+\gamma)z_1+\zeta_2(x_1,x_2^*)]) - g_2(z-[(1+\gamma)z_1+\zeta_2(x_1,\underline{x}_2)])]dz \quad (8)$$

$$= \int V_2(z|z_1)[g_2(z-[(1+\gamma)z_1+\zeta_2(x_1,x_2^*)]) - g_2(z-[(1+\gamma)z_1+\zeta_2(x_1,\overline{x}_2)])]dz. \quad (9)$$

Now let us fix a candidate value $\zeta_2(x_1,\underline{x}_2(x_1)) = \zeta_2(x_1,\overline{x}_2(x_1))$ and ask if it could be part of an optimal solution. If we could choose a utility function $V_2(z|z_1)$ so as make (8) and (9) exceed ε_2, then the candidate value $\zeta_2(x_1,\underline{x}_2(x_1)) = \zeta_2(x_1,\overline{x}_2(x_1))$ would give us slack in the constraints (4)–(5), and the utility function in question would in fact induce a larger value of $\zeta_2(x_1,\underline{x}_2(x_1)) = \zeta_2(x_1,\overline{x}_2(x_1))$ than our candidate (since the right sides of (8)–(9) are decreasing in $\zeta_2(x_1,\underline{x}_2(x_1))$ and $\zeta_2(x_1,\overline{x}_2(x_1))$). This would imply that our candidate value does *not* correspond to an optimal utility function. Hence, the optimal utility function must maximize (8) (and equivalently, (9)), for the optimal $\zeta_2(x_1,\underline{x}_2(x_1))$ and $\zeta_2(x_1,\overline{x}_2(x_1))$, in the process giving a maximum

[10] To arrive at (8)–(9), we first expand the expectations in (4)–(5) to obtain

$$\varepsilon_2 = \int V_2((1+\gamma)z_1+\zeta_2(x_1,x_2^*)+s_2|z_1)g_2(s_2)ds_2 - \int V_2((1+\gamma)z_1+\zeta_2(x_1,\underline{x}_2)+s_2|z_1)g_2(s_2)ds_2$$

$$= \int V_2((1+\gamma)z_1+\zeta_2(x_1,x_2^*)+s_2|z_1)g_2(s_2)ds_2 - \int V_2((1+\gamma)z_1+\zeta_2(x_1,\overline{x}_2)+s_2|z_1)g_2(s_2)ds_2.$$

A change of the variable of integration from s_2 to z then gives (8)–(9).

equal to ε_2. We now need only note that (8) and (9) are both maximized by setting the utility $V_2(z|z_1)$ as small as possible when $g_2(z - [(1+\gamma)z_1 + \zeta_2(x_1, x_2^*)]) - g_2(z - [(1+\gamma)z_1 + \zeta_2(x_1, \underline{x}_2)]) < 0$ and by setting the utility $V_2(z|z_1)$ as large as possible when this inequality is reversed, and hence the optimal utility function must have this property. Because g_2 has a symmetric, unimodal density with nonzero derivative (except at 0), there is a threshold $\hat{Z}_2(z_1) \in [(1+\gamma)z_1 + \zeta_2(x_1, \underline{x}_2), (1+\gamma)z_1 + \zeta_2(x_1, x_2^*)]$ such these differences are negative for lower values of z and positive for higher values of z. This gives us a utility function $V_2(z|z_1)$ that takes a jump from 0 to 1 at $\hat{Z}_2(z_1)$. As $\varepsilon_2 \to 0$ and hence the agent's satisficing set shrinks, $\hat{Z}_2(z_1)$ converges to $\zeta_2(x_1, x_2^*(x_1))$ and the agent flawlessly maximizes $\zeta_2(x_1, x_2)$by choosing $x_2^*(x_1)$.

Let us pause to gain some intuition for this argument. Think of evolution as a principal designing incentive schemes for the agent, and note that the optimal utility function exhibits features familiar from principal-agent problems. Consider a hidden-action principal-agent problem with two effort levels. It is a standard result that the optimal payment attached to an outcome is increasing in the outcome's likelihood ratio, or (intuitively) in the relative likelihood of that outcome having come from high vs. low effort. Much the same property appears here. Evolution would like expected utility to fall off as rapidly as possible as the agent moves away from the optimal decision $x_2^*(x_1)$, thereby "steepening" the utility function and reducing the possibility of a mistakenly suboptimal choice. Evolution does so by attaching high payments to fitnesses with high likelihood ratios, or (intuitively) outcomes that are relatively likely to have come from an optimal rather than a suboptimal choice.

The key property in characterizing the utility function in our case is then a single-crossing property, namely that likelihood ratios fall short of one for small fitnesses and exceed one for large fitnesses. The likelihood comparison appears in difference rather than ratio form in (8)–(9), but the required single-crossing property is implied by the familiar monotone likelihood ratio property, that $\frac{g_2(z-\alpha)}{g_2(z)}$ is increasing in z, for $\alpha > 0$.

This would give us Lemma 1 (and more), were it not for our counterfactual assumption that evolution can "observe" x_1 as well as z_1. If unable to observe x_1, evolution must now form a posterior expectation over the likely value of x_1, given her observation of z_1.[11] She would then choose a utility function $V_2(z|z_1)$ that maximizes the agent's expected fitness, given this posterior. In particular, for each possible value of x_1, the agent will mix over a set $[\underline{x}_2(x_1), \overline{x}_2(x_1)]$, being the satisficing set corresponding to (4)–(5) (for that value of x_1). Evolution is concerned with the resulting expected value of the total fitness $(1+\gamma)z_1 + \zeta_2(\tilde{x}_1, \tilde{x}_2) + \tilde{s}_2$, where the expectation is taken over the likely value of x_1 (given z_1), over the choice of x_2 (from the resulting satisficing set), and the draw of s_2 (governed by g_2). Evolution increases expected fitness by reducing the size of the satisficing sets $[\underline{x}_2(x_1), \overline{x}_2(x_1)]$. In general, comparing two potential

[11]We emphasize that evolution does not literally form posterior beliefs over the agent's actions and then solve an optimization problem. The results follow from the observation that fitness will be maximized by that utility function that would be optimal given the appropriate posterior beliefs.

utility functions may require some careful calculations, since one may give a larger satisficing set for some values of x_1 and a smaller satisficing set for other values of x_2. Fortunately, we can obtain partial results by relying on dominance arguments that avoid these calculations. Because ζ_2 is bounded, so are the values $[(1+\gamma)z_1+\zeta_2(x_1,x_2^*(x_1))]$ and $[(1+\gamma)z_1+\zeta_2(x_1,\underline{x}_2(x_1))] = [(1+\gamma)z_1 + \zeta_2(x_1,\overline{x}_2)]$, over the set of possible values of x_1, with the former larger than the latter. There is accordingly a value $\underline{Z}_2(z_1)$ such that $g_2(z-[(1+\gamma)z_1 + \zeta_2(x_1,x_2^*)]) - g_2(z-[(1+\gamma)z_1+\zeta_2(x_1,\underline{x}_2)])$ is negative for $z < \underline{Z}_2(z_1)$, for every x_1, as well as a value $\overline{Z}_2(z_1)$ such that these differences are all positive for all $z > \overline{Z}_2(z_1)$. It thus decreases the size of *every* possible satisficing set to set $V_2(z|z_1) = 0$ for $z < \underline{Z}_2(z_1)$ and $V_2(z|z_1) = 1$ for $z > \underline{Z}_2(z_1)$. This leaves us without having determined what happens on the set $[\underline{Z}_2(z_1), \overline{Z}_2(z_1)]$, and if there is a wide range of possible x_1 values, this gap might be large. As ε_1 gets small, however, the first-period satisficing set will shrink, causing the gap $[\underline{Z}_2(z_1), \overline{Z}_2(z_1)]$ to disappear (cf. Lemma 2). Finally, even for fixed (but small) $\varepsilon_1 > 0$, it follows from the fact that $V_2(z|z_1)$ is increasing and the continuity of ζ_2 that as ε_2 approaches zero, the agent's second-period satisficing sets collapse on $x_2^*(x_1)$, for each realization x_1 of the first-period random satisficing choice.

3.2 The First Period

Attention now turns to the first period. For simplicity, we take the limit $\varepsilon_2 \to 0$ before considering the optimal first-period utility function, returning to this assumption at the end of this section.

The agent has a utility function $V_1(z)$ with $V_1 \in [0,1]$. In addition, the agent cannot distinguish any pair of choices whose expected utilities are within $\varepsilon_1 > 0$ of each other. This again leads to a random choice from a satisficing set $[\underline{x}_1, \overline{x}_1]$, where

$$\begin{aligned} & E_{\tilde{s}_1,\tilde{s}_2} V_1(\zeta_1(\underline{x}_1) + \tilde{s}_1 + [\gamma(\zeta_1(\underline{x}_1)+\tilde{s}_1) + \zeta_2(x_1, x_2^*(\underline{x}_1)) + \tilde{s}_2]) \qquad (10) \\ = \; & E_{\tilde{s}_1,\tilde{s}_2} V_1(\zeta_1(\overline{x}_1) + \tilde{s}_1 + [\gamma(\zeta_1(\overline{x}_1)+\tilde{s}_1) + \zeta_2(x_1, x_2^*(\overline{x}_1)) + \tilde{s}_2]) \\ = \; & E_{\tilde{s}_1,\tilde{s}_2} V_1(\zeta_1(x_1^*) + \tilde{s}_1 + [\gamma\zeta_1(x_1^*)+\tilde{s}_1) + \zeta_2(x_1, x_2^*(x_1^*)) + \tilde{s}_2]) - \varepsilon_1. \qquad (11) \end{aligned}$$

In the first period, the agent randomizes uniformly over the set $[\underline{x}_1, \overline{x}_1]$. Evolution chooses the utility function $V_1(z)$ to maximize expected fitness, subject to (10)–(11).

Once again, we have a bang-bang function in realized utilities, with the expected utility function $E_{\tilde{s}_1,\tilde{s}_2} V_1(\zeta_1(x_1)+\tilde{s}_1+[\gamma(\zeta_1(x_1)+\tilde{s}_1)+\zeta_2(x_1,x_2^*(x_1))+\tilde{s}_2])$ being a continuous function of x_1. Section 5.1 uses arguments parallelling those applied to the second period to prove:

Lemma 2 *There exists a value $\hat{Z}_1$ such that the optimal first-period utility function is given by*

$$\begin{aligned} V_1(z) &= 0, \text{ for all } z < \hat{Z}_1 \\ V_1(z) &= 1, \text{ for all } z > \hat{Z}_1. \end{aligned}$$

In the limit as $\varepsilon_1 \to 0$*, we have*

$$\hat{Z}_1 = (1+\gamma)\zeta_1(x_1^*) + \zeta_2(x_1^*, x_2^*(x_1^*)),$$

as well as

$$\begin{aligned}\underline{Z}_2(z_1) &\to (1+\gamma)z_1 + \zeta_2(x_1^*, x_2^*(x_1^*)) \\ \overline{Z}_2(z_1) &\to (1+\gamma)z_1 + \zeta_2(x_1^*, x_2^*(x_1^*)).\end{aligned}$$

The final part of this lemma resolves a lingering question from the preceding analysis of the second period, showing that the intermediate range $[\underline{Z}_2, \overline{Z}_2]$ on which we had not pinned down the second-period utility function disappears as ε_1 tends to zero and hence the randomness in the agent's first-period choice disappears.

The ideas behind this result parallel those of the second period. The utility perception error ε_1 causes the agent to choose x_1 randomly from a satisficing set $[\underline{x}_1, \overline{x}_1]$, and evolution's task is to choose the utility function to reduce the size of this satisficing set. Total fitness is now affected by the random variable $\tilde{s}_1$ as well as $\tilde{s}_2$, and the key to the result is to show that the resulting distribution over total fitness exhibits a single-crossing property, with larger total fitnesses relatively more likely to have come from the fitness-maximizing choice x_2^* than from either of the choices $\underline{x}_1$ or $\overline{x}_1$.

Putting our two intermediate results together, we have shown:

Proposition 1 *In the limit as the "utility-perception errors"* ε_2 *and then* ε_1 *approach zero, the optimal utility functions are given by*

$$\begin{aligned}V_1(z) &= 0, \text{ for all } z < \hat{Z}_1 = (1+\gamma)\zeta_1(x_1^*) + \zeta_2(x_1^*, x_2^*(x_1^*)) \\ V_1(z) &= 1, \text{ for all } z > \hat{Z}_1 = (1+\gamma)\zeta_1(x_1^*) + \zeta_2(x_1^*, x_2^*(x_1^*))\end{aligned}$$

$$\begin{aligned}V_2(z|z_1) &= 0, \text{ for all } z < \hat{Z}_2(z_1) = (1+\gamma)z_1 + \zeta_2(x_1^*, x_2^*(x_1^*)) \\ V_2(z|z_1) &= 1, \text{ for all } z > \hat{Z}_2(z_1) = (1+\gamma)z_1 + \zeta_2(x_1^*, x_2^*(x_1^*)).\end{aligned}$$

We thus have bang-bang utility functions in each period. The jump in the second-period function is adjusted in response to the agent's first-period fitness outcome, shifting upward whenever the agent has a more favorable first-period outcome z_1 and shifting downward in response to disappointing first-period outcomes. Intuitively, this allows evolution to adjust the steep part of second-period expected utility to occur in the range of decisions likely to be relevant in the second period, in the process strengthening the second-period incentives. As the utility-perception errors ε_1 and ε_2 get small, the agent's choices collapse around the optimal choices x_1^* and $x_2(x_1^*)$.

Our argument can be easily adapted to establish Proposition 1 under the assumption that ε_2 goes to zero sufficiently fast relative to ε_1 (as opposed to taking $\varepsilon_2 \to 0$ first). Indeed, we can establish Proposition 1 no matter how the utility-perception errors ε_1 and ε_2 go to zero, with additional technical assumptions and a somewhat more involved argument. In order to evaluate the utility consequences of his first-period actions, the agent must know what his subsequent second-period actions will be. Taking ε_2 to zero before examining the first period, as we have done, simplifies the argument by allowing the agent to unambiguously anticipate the choice $x_2^*(x_1)$ in the second period. What should the agent anticipate if $\varepsilon_2 > 0$? His second-period choice will now be a random draw from a satisficing set. An apparently natural assumption would give the agent rational expectations about his second-period choice. However, the satisficing set is determined by the second-period utility function, and under the separation of decision and experienced utilities, the agent does not correctly anticipate the second-period utility function governing the choice of x_2.[12] It is then conceptually problematic to assume rational expectations. Whatever rule evolution gives the agent for anticipating second-period choices, we will obtain the results given in Proposition 1 as long as random second-period choices do not reverse first-period fitness rankings. In particular, the fitness-maximizing first-period choice x_1^* gives a distribution of total fitnesses that first-order stochastically dominates the distribution induced by the suboptimal choices $\underline{x}_1$ or $\overline{x}_2$, when each is paired with the corresponding optimal second-period choice $x_2^*(\cdot)$. It would suffice for a general limit result that for $\varepsilon_2 > 0$ (but small), the optimal choice x_1^* still gives fitnesses that first-order stochastically dominate those of $\underline{x}_1$ or $\overline{x}_2$; given the rule used by the agent to anticipate second-period choices.[13] One obvious sufficient condition for this to hold is that $\zeta_2(x_1, x_2)$ be separable in x_1 and x_2 (with the agent's anticipated second-period choice then naturally being independent of x_1). Other sufficient conditions would allow more flexible technologies, at the cost of more cumbersome statements.

3.3 Comparing Decision and Experienced Utilities

We now compare the agent's decision and experienced utilities—are the utilities guiding the agent's decision the same as those the agent will experience when the resulting outcome is realized?

To answer this question, suppose the agent considers the possible outcome (x_1, s_1, x_2, s_2). For example, the agent may consider moving to California (the choice of x_1), learning to surf (the choice of x_2), finding a job (the realization s_1) and enjoying a certain amount of sunshine (the realization s_2). Let us assume the agent anticipates choosing x_2 optimally in the second period, so that $x_2 = x_2^*(x_1)$. Then fix x_1 and look at utility as s_1 and s_2 vary. If the outcome considered by the agent gives $\zeta_1(x_1) + s_1 + \gamma[\zeta_1(x_1) + s_1] + \zeta_2(x_1, x_2) + s_2 > \hat{Z}_1$, then he attaches the maximal utility of one to that outcome. However, if the scenario contemplated by the agent at the same time involves a value $s_2 < 0$ (the agent contemplates a good job realization and hence a success without relying

[12]Notice that in the limit as $\varepsilon_2 \to 0$, it need only be the case that second-period expected utility will be increasing in fitness to ensure that $x_2^*(x_1)$ will be chosen in the second period, making rational expectations straightforward.

[13]Total fitness would then continue to exhibit the appropriate version of the single-crossing property given by (20)–(21), with the agent's belief about x_2 as well as those about $\tilde{s}_1$ and $\tilde{s}_2$ now being random.

on outstanding weather), then his realized experienced utility will be zero, since then

$$
\begin{aligned}
z &= (1+\gamma)z_1 + \zeta_2(x_1, x_2^*(x_1)) + s_2 \ < \ (1+\gamma)z_1 + \zeta_2(x_1, x_2^*(x_1)) \\
\implies & V_2(z|z_1) \ = \ 0.
\end{aligned}
$$

The agent's decision utility of one thus gives way to an experienced utility of zero.

Alternatively, if the agent considers a situation where $\zeta_1(x_1)+s_1+\gamma[\zeta_1(x_1)+s_1] + \zeta_2(z_1, x_2) + s_2 < \hat{Z}_1$, then this generates a decision utility level of zero. However, if, at the same time $s_2 > 0$, his experienced utility will be one.

The agent's decision and experienced utilities will thus sometimes agree, but the agent will sometimes believe he will be (maximally) happy, only to end up miserable, and sometimes he will believe at the start that he will be miserable, only to turn out happy. The agent will be mistaken about his experienced utility whenever his utility projection depends more importantly on the first-period choice than second-period uncertainty (i.e., anticipating a good outcome because he is moving to a great location, regardless of the weather; or anticipating a bad outcome because his location is undesirable, despite good weather). The agent's decision utilities fail to take into account that once the first-period choice has been realized, his utility function will adjust to focus on the second period, bringing second-period realizations to heightened prominence.

Could this focussing illusion in realized outcomes be washed out in the process of taking expected values? Suppose we know simply that the agent contemplates a first-period utility $V_1(z)$ for some specific z. This is the type of information typically provided by empirical studies that begin by soliciting decision utilities. What expectations should we have concerning this person's second-period utility? Let us suppose the agent chose x_1^* in the first period and will choose $x_2^*(x_1^*)$ in the second, both because we expect to observe people who have made optimal choices (given their decision utilities), and because the continued existence of the focussing illusion in the presence of optimal choices is of key interest. This leaves us uncertain as to the likely values of s_1 and s_2. We can let $E_{\tilde{s}_1,\tilde{s}_2}\left\{V_2(z|\tilde{z}_1)|z\right\}$ represent our expectation of the agents' second-period utility, given the observation of z. Then in general,

$$
\begin{aligned}
V_1(z) &\neq E_{\tilde{s}_1,\tilde{s}_2}\left\{V_2(z|\tilde{z}_1)|z\right\} && (12)\\
&= \Pr\left\{V_2(z|\tilde{z}_1) = 1|z\right\} && (13)\\
&= \Pr\left\{\tilde{s}_2 \geq 0|z\right\}. && (14)
\end{aligned}
$$

The larger is z, the more likely it is that $\tilde{s}_2 > 0$. As a result, $E_{\tilde{s}_1,\tilde{s}_2}\left\{V_2(z|\tilde{z}_1)|z\right\}$ increases from 0 to 1 as z increases from its minimum to its maximum value. Figure 1 illustrates.

An agent's view of the utilities guiding his first-period decisions thus give way to a more moderate view of second-period experienced utilities.

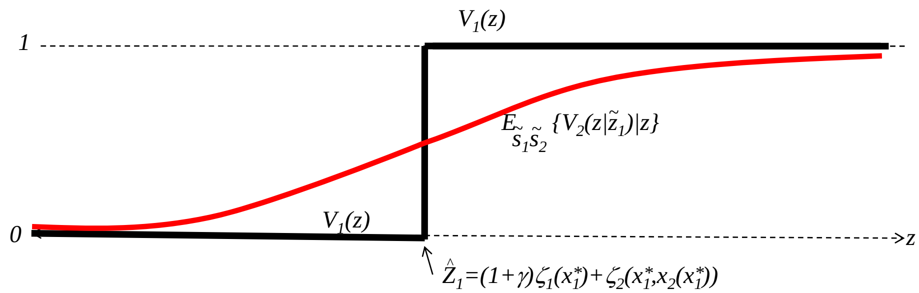

Figure 1: First-period decision utility function $V_1(z)$ and expected experienced utility $E_{\tilde{s}_1,\tilde{s}_2}\{V_2(z|\tilde{z}_1)|z\}$, as a function of z. Observations of small decision utilities will then on average give way to larger experienced utilities, while large decision utilities will on average give way to smaller experienced utilities, giving rise to a focussing illusion.

3.4 Separable Decisions

The technology $z_2 = \gamma z_1 + \zeta_2(x_1, x_2) + s_2$ encompasses two links between the first and second period, with second-period fitness being larger the better is the agent's first-period outcome, and with the optimal second-period decision depending on the first-period action. Both links are intuitive, but the first alone is capable of generating a focussing illusion. What matters is that second-period *utility* depends on the first-period outcome, not that second-period optimal *behavior* depends on the first-period outcome.

Suppose that realized fitness is given by

$$z = z_1 + z_2 \tag{15}$$

$$= \zeta_1(x_1) + s_1 + [\gamma z_1 + \zeta_2(x_2) + s_2], \tag{16}$$

where s_1 and s_2 are again realizations of random variables. Then the optimal value of x_2 is independent of x_1 and z_1. Nothing from the first period is relevant for determining the agent's optimal second-period decision. However, second-period fitness does depend on the first-period outcome.

Following the analysis of Sections 3.1–3.2, evolution's optimal design features a second-period utility function of

$$V_2(z|z_1) = 0, \text{ for all } z < \hat{Z}_2(z_1)$$
$$V_2(z|z_1) = 1, \text{ for all } z > \hat{Z}_2(z_1),$$

where, in the limit as ε_2 gets small, $\hat{Z}_2(z_1) = (1+\gamma)z_1 + \zeta_2(x_2^*)$. In the first period, we get

$$V_1(z) = 0, \text{ for all } z < \hat{Z}_1$$
$$V_1(z) = 1, \text{ for all } z > \hat{Z}_1,$$

where $\hat{Z}_1 = (1+\gamma)\zeta_1(x_1^*) + \zeta_2(x_2^*)$ (as $\varepsilon_1 \to 0$). Once again, second-period utility functions adjust in response to first-period outcomes, ensuring a focussing illusion.

3.5 Sophisticated Agents?

Evolution here has designed the agent to be naive (cf. O'Donoghue and Rabin [11]), in the sense that the first-period decision is made without anticipating the attendant second-period utility adjustment. Why not make the agent sophisticated? Why not simply let the agent make decisions on the basis of experienced utilities?

Given optimal second-period choices, and taking the limit as the utility errors tend to zero, evolution induces the agent to make an appropriate first-period choice by having the agent select x_1 to maximize

$$E_{\tilde{s}_1,\tilde{s}_2} V_1(\tilde{z}) = \Pr[(1+\gamma)\tilde{s}_1 + \tilde{s}_2 \ \geq \ ((1+\gamma)\zeta_1(x_1^*) + \zeta_2(x_1^*, x_2^*(x_1^*))) - ((1+\gamma)\zeta_1(x_1) + \zeta_2(x_1, x_2^*(x_1)))].$$

$E_{\tilde{s}_1,\tilde{s}_2} V_1(\tilde{z})$ is readily seen to be maximized at x_1^*. Suppose that instead, evolution designed the agent to maximize the expected value of the correctly anticipated, expected experienced utility, or

$$E_{\tilde{s}_1,\tilde{s}_2} V_2(\tilde{z}|\tilde{z}_1) = \Pr[\tilde{s}_2 \ \geq \ \zeta_2(x_1^*, x_2^*(x_1^*)) - \zeta_2(x_1, x_2^*(x_1))].$$

The agent's decision utility captures two effects relevant to choosing x_1, namely the effect on first-period fitness z_1, with implications that carry over to the second period, and the effect on expected second-period *incremental* fitness $\zeta_2(x_1, x_2^*(x_1))$. In contrast, the correctly anticipated experienced utility omits the first consideration. As a result, expected experienced utility falls off less rapidly than does decision utility, as the agent contemplates suboptimal choices of x_1. With positive utility errors, experienced utility must then lead to larger satisficing sets than does decision utility, and hence suboptimal choices. Making agents naive increases their fitness.

To dramatize this point, reconsider the limiting case of no utility error at all. Suppose further that $\max_{x_2} \zeta_2(x_1, x_2)$ is independent of x_1, though the maximizer may yet depend on x_1. Hence, the action the agent must take to maximize second-period incremental fitness depends on the outcome of the first period, though in each case the agent adds the same expected increment to fitness. Then we have

$$E_{\tilde{s}_1,\tilde{s}_2} V_2(\tilde{z}|\tilde{z}_1) = \Pr[\tilde{s}_2 \ \geq \ \zeta_2(x_1^*, x_2^*(x_1^*)) - \zeta_2(x_1, x_2^*(x_1))] = \frac{1}{2},$$

for *every* value of x_1. Correctly anticipated experienced utility now provides no incentives at all, while first-period decision utilities still effectively provide incentives. Why does making the agent sophisticated destroy incentives? The naive agent believes that a suboptimal choice of x_1 will decrease utility. Should such a suboptimal choice x_1 be made, however, the agent's second-period utility function will (unexpectedly) adjust to the first-period choice x_1 to still yield an expected experienced utility of $\frac{1}{2}$. From evolution's point of view, this adjustment plays the critical role of enhancing second-period incentives. Should the agent be sophisticated enough anticipate it, however, first period incentives evaporate, with expected utility now being independent of the first-period choice.

The intuition behind this result is straightforward. Evolution must create incentives in the first period, and naturally constructs decision utilities to penalize suboptimal choices. However, once a first-period alternative is chosen, evolution must now induce the best possible second-period choice. In the present model, she adjusts the agent's utility function in response to the first-period choice, causing the optimal second-period choice to induce the same expected utility, regardless of its first-period predecessor. Suboptimal first-period choices thus lead to the same experienced utility in the second period as do optimal ones. The decision-utility penalty attached to suboptimal choices in the first period is removed in the second in order to construct better second-period incentives.

4 Discussion

4.1 Extensions

We have highlighted the forces behind the focussing illusion by working with a stark model. A number of extensions would be of interest. Some of these are conceptually straightforward, even if they are analytically more tedious. For example, we would be interested in a model spanning more periods, allowing us to examine a richer collection of investment opportunities. As our model stands, a first-period investment x_1 already yields its gains in the second period. What about more prolonged investments? It may take sufficient time to acquire an education that the agent first becomes accustomed to a low consumption level, magnifying the initial utility consequences of the education's consumption-enhancing aftermath, only to have them subsequently eroded.[14] Evolution must now construct a sequence of utility functions, each serving as a decision utility for current actions and an experienced utility for past actions.

Similarly, it would be interesting to allow z_1 and z_2 (as well as x_1 and x_2) to be multidimensional. We derive utility from a variety of sources. Perhaps most importantly, we can ask not only how evolution has shaped our utility functions, given their arguments, but which arguments she has chosen to include. At first, the answer to this question seems straightforward. The currency of evolutionary success is reproduction, and evolution should simply instruct us to maximize our expected reproductive success. Even if one could solve the attendant measurement issues,[15] maximizing this goal directly is presumably beyond

[14] The relevant measure of the length of a period is determined by the how quickly evolution can induce our utility functions to adapt to our circumstances. A single fine meal is unlikely to be a preference-altering event, but it may not take long for one to feel "settled" in their circumstances, prompting drift in the "steep spot" of the utility function.

[15] For example, how do we trade off the number of children versus their "quality," presumably self-referentially defined by their reproductive success? How do we trade off children versus

our powers.[16] Instead, evolution rewards us for achieving intermediate targets, such as being well-fed and being surrounded by affectionate members of the opposite sex. But which intermediate targets should evolution reward? Clearly, our utility functions should feature arguments that, to the extent possible, are directly related to the ultimate goal of reproductive success and are sufficiently straightforward that we can perform the resulting maximization. In addition, we will suggest below that our utility functions should contain arguments that are effective at implicitly conveying information to evolution.

4.2 A More General Technology

Quasi-linearity is not needed at all for the second-period analysis. The critical step in the first-period argument arises in examining the cumulative distribution function of $(1+\gamma)\tilde{s}_1 + \tilde{s}_2$. Letting G denote this distribution, we have

$$\begin{aligned} G(z - [(1+\gamma)\zeta_1(x_1) + \zeta_2(x_1, x_2^*(x_1))]) &= \Pr[(1+\gamma)\tilde{s}_1 + \tilde{s}_2] \leq z - [(1+\gamma)\zeta_1(x_1) + \zeta_2(x_1, x_2^*(x_1))] \\ &= \Pr[\zeta_1(x_1) + \tilde{s}_1 + [\gamma(\zeta_1(x_1) + \tilde{s}_1) + \zeta_2(x_1, x_2^*(x_1)) + \tilde{s}_2] \leq z]. \end{aligned}$$

Now letting g be the density of G, we can interpret $g(z - [(1+\gamma)\zeta_1(x_1) + \zeta_2(x_1, x_2^*(x_1))])$ as the "1ikelihood" that fitness z is the result of choices $(x_1, x_2^*(x_1))$, which give rise to expected fitness $(1+\gamma)\zeta_1(x_1) + \zeta_2(x_1, x_2^*(x_1))$. Paralleling the second-period argument, it would suffice for this distribution to have the single-crossing property that $g(z - [(1+\gamma)\zeta_1(x_1^*) + \zeta_2(x_1^*, x_2^*(x_1^*))]) - g(z - [(1+\gamma)\zeta_1(\underline{x}_1) + \zeta_2(\underline{x}_1, x_2^*(\underline{x}_1))])$ is negative for small values of z (in which case $V_1(z) = 0$) and positive for large values (giving $V_1(z) = 1$), for which it suffices that g exhibit the monotone likelihood ratio property. Intuitively, higher realized fitness levels must be relatively more likely to have come from actions yielding higher expected fitness levels. Under the quasilinearity assumption (3), the cumulative distribution function of fitness in (17) is derived immediately from the cumulative distribution function G of the relatively simple linear combination $(1+\gamma)\tilde{s}_1 + \tilde{s}_2$ of the random variables $\tilde{s}_1$ and $\tilde{s}_2$. This ensures (as we show in Section 5.1) that the corresponding density g exhibits the single-crossing property.

The general technology given by (2) will give rise to an analogous utility function if the counterpart of (17) again gives rise to a single crossing property. However, now we must define the cumulative distribution function of fitness directly as

$$\hat{G}(z) = \Pr[\hat{\zeta}_1(x_1, \tilde{s}_1) + \hat{\zeta}_2(\hat{\zeta}_1(x_1, \tilde{s}_1), x_1, x_2^*(\hat{\zeta}_1(x_1, \tilde{s}_1), x_1), \tilde{s}_2) \leq z].$$

In this case, $\hat{G}$ is the cumulative distribution of a potentially complicated, nonlinear function of $\tilde{s}_1$ and $\tilde{s}_2$. We can then no longer count on $\hat{G}$ exhibiting the requisite single-crossing property. Instead, this property is now a potentially

grandchildren?

[16] Calculating the fitness implications of every action we take would be overwhelming, while feedback (such as the birth of a healthy child) is sufficiently rare as to make trial-and-error an ineffective substitute (cf. Robson [14]).

complicated joint assumption on the distributions or the random variables and the technology. Simple sufficient conditions for this property are then elusive, though we have no reason to doubt that higher realized fitnesses will again be relatively more likely to have emerged from actions yielding higher expected fitnesses.

We believe there are good reasons to expect the desired single-crossing property to hold, even if the primitive conditions leading to the requisite monotonicity property are not easily identified in the general model. Bringing us back to ideas which we opened Section 4, evolution has not only designed our utility functions, but has chosen the arguments to include in those functions. We have been chosen to have a taste for sweetness, whereas we could just as easily have been chosen to have different tastes. Among the many considerations behind what gets included in our utility functions, we expect one to be that the technology surrounding the variable in question exhibits the single-crossing properties required for simple utility functions to deliver strong incentives. We thus expect the single-crossing property to be one of the features that makes a variable a good candidate for inclusion in our utility function, and hence think it likely that the property will hold precisely *because* evolution has an incentive to attach utilities to variables with this property. Once we have that, we immediately reproduce the results of Section 3.2 in the more general setting.

4.3 Alternative Utility Functions

We have assumed that evolution writes first-period and second-period utility functions of the form $V_1(z)$ and $V_2(z|z_1)$. What if evolution could write utility functions of the form $V_1(z_1, z_2)$ and $V_2(z_1, z_2)$? Notice that the difference between the second-period utility functions $V_2(z|z_1)$ and $V_2(z_1, z_2)$ is one of notation only—allowing a function of $z = z_1 + z_2$ to be conditioned on z_1 is equivalent to allowing V_2 to depend on z_1 and z_2. In the first period, allowing z_1 and z_2 to enter the utility function separately potentially opens new possibilities. In a special case of our question, why not write the utility function as $V_1(z_1)$, i.e., why make the agent farsighted at all?

Consider first the technology given by (15)–(16). Then evolution can do no better than to give the agent the utility functions (in the limit as $\varepsilon_1 \to 0$ and $\varepsilon_2 \to 0$)

$$\begin{aligned} V_1(z_1) &= 0, \text{ for all } z_1 < \hat{z}_1 = \zeta_1(x_1^*) \\ V_1(z_1) &= 1, \text{ for all } z_1 > \hat{z}_1 = \zeta_1(x_1^*) \end{aligned}$$

$$\begin{aligned} V_2(z|z_1) &= 0, \text{ for all } z < \hat{Z}_2(z_1) = (1+\gamma)z_1 + \zeta_2(x_2^*) \\ V_2(z|z_1) &= 1, \text{ for all } z > \hat{Z}_2(z_1) = (1+\gamma)z_1 + \zeta_2(x_2^*). \end{aligned}$$

In this case, there is no need to trouble the agent with second-period implications when the agent is making his first-period choice, as the first-period action x_1 has no second-period implications. The first-period fitness z_1 still carries over into the total fitness achieved in the second period, but this does nothing but reinforce the conclusion that the agent should maximize first-period fitness.

Do we have a focussing illusion here? On the one hand, the second-period utility cutoff $\hat{Z}_2(z_1)$ adjusts in response to first-period realized fitness z_1, ensuring that the agent will often encounter second-period fitness realizations that do not match his previous expectation of second-period utility. However, only first-period outcomes and utilities shape the first-period choice. In particular, it would make no difference whether this agent were naive or sophisticated. It is as if $V_1(z_1)$ is both the decision and experienced utility for the choice of x_1. Perhaps the best description is that we have a focussing illusion, but an irrelevant one. No one in this setting would expect people to make better decisions were they better in touch with their experienced utilities.

This utility-design procedure would run into difficulties with the technology given by (1)–(3). Here, not only z_1 but the first-period choice x_1 enters the second-period fitness. It no longer suffices to simply design the agent to maximize the expected value of first-period fitness z_1, as the agent must trade off higher values of z_1 with the second-period implications of x_1. Maximizing total fitness may require settling for a lower value of expected first-period fitness, in order to invest in a level of x_1 that boosts expected second-period fitness. Of course, evolution need not design the utility function to *maximize* the expected value of z_1. It may be that the optimal value of x_1 is consistent with an expected first-period fitness value $\zeta_1(x_1^*)$, and that evolution could design a utility function that is not monotonic in z_1 and whose maximized expected value equals $\zeta_1(x_1^*)$. Even if this were feasible, however, there would still in general be more than one value of x_1 consistent with $\zeta_1(x_1^*)$, so that simply hitting the right value $\zeta_1(x_1^*)$ does not suffice to maximize fitness. This problem is likely to be all the more severe in more realistic, multidimensional models. The obvious solution to this problem is to simply write first-period utility as a function of $z = z_1 + z_2$, leading to our model.

4.4 Smooth Utility Functions

The optimal utility functions in our model assign only the utilities zero and one to realized outcomes. Can we obtain realized utilities that are not always zero or one? To see why this is important, note that the gist of a focussing illusion is that anticipations of high utilities give way to lower realizations, and anticipations of low utilities give rise to higher realizations. When anticipated utilities are always set at either their minimum or maximum value, it seems that one can hardly avoid such an outcome. Should we expect the focussing illusion to survive in a more realistic model?

To demonstrate how to do this, we begin with the model of Section 3.4. The key new feature is the addition of a shock $\tilde{r}$ that is observed by the agent before the first choice must be made but is unobservable to evolution. This shock captures the possibility that there may be characteristics of the agent's environment that affect the agent's fitness, but that fluctuate too rapidly for evolution to directly condition his behavior. The agent may know whether the most recent harvest has been good or bad, or whether the agent is in the midst of a boom or recession. Fitness thus varies with a state that is unobserved by evolution (as in Rayo and Becker [12]). Suppose that realized fitness is given by

$$\begin{aligned} z &= r + z_1 + z_2 \\ &= r + \zeta_1(x_1) + s_1 + [\gamma z_1 + \zeta_2(x_2) + s_2], \end{aligned}$$

where the associated random variables $\tilde{s}_1$, $\tilde{s}_2$ and $\tilde{r}$ are independent.

Two assumptions significantly simplify the analysis. First, $\tilde{r}$ takes only a finite number of possible outcomes $(r_1, \ldots, r_K)$. Our second assumption, made precise after acquiring the required notation, is that the dispersion in the values of $\tilde{r}$ is large relative to the supports of $\tilde{s}_1$ and $\tilde{s}_2$. Intuitively, the new information in $\tilde{r}$ the agent can observe is relatively important.

The agent is endowed with a second-period utility function $V_2(z|z_1)$. This is non-decreasing in fitness z, where $V_2(z|z_1) \in [0,1]$. Suppose that z_1 has been realized in the first-period and the agent has observed realization r_k of the random variable $\tilde{r}$. The agent then chooses from a satisficing set of the form $[\underline{x}_2^k(z_1), \overline{x}_2^k(z_1)] \ni x_2^*$ where

$$\begin{aligned} & E_{\tilde{s}_2} V_2((1+\gamma)z_1 + \zeta_2(x_2^*) + r_k + \tilde{s}_2) - E_{\tilde{s}_2} V_2((1+\gamma)z_1 + \zeta_2(\underline{x}_2^k) + r_k + \tilde{s}_2) \quad (17) \\ = \; & E_{\tilde{s}_2} V_2((1+\gamma)z_1 + \zeta_2(x_2^*) + r_k + \tilde{s}_2) - E_{\tilde{s}_2} V_2((1+\gamma)z_1 + \zeta_2(\overline{x}_2^k) + r_k + \tilde{s}_2) \quad (18) \\ = \; & \varepsilon_2, \end{aligned}$$

where $\zeta_2(\underline{x}_2^k(z_1)) = \zeta_2(\overline{x}_2^k(z_1))$.

Consider now evolution's optimal choice of $V_2(z|z_1)$. We can rewrite (17) as

$$\int V_2(z|z_1) \left[g_2(z - (1+\gamma)z_1 - \zeta_2(x_2^*) - r_k) - g_2(z - (1+\gamma)z_1 - \zeta_2(\underline{x}_2^k) - r_k) \right] dz = \varepsilon_2. \quad (19)$$

We could similarly rewrite (18), but the fact that $\zeta_2(\underline{x}_2^k(z_1)) = \zeta_2(\overline{x}_2^k(z_1))$ allows us to work with (19) alone throughout. Define $Z_2^k(z_1)$ by the requirement that

$$g_2(Z_2^k(z_1) - (1+\gamma)z_1 - \zeta_2(x_2^*) - r_k) = g_2(Z_2^k(z_1) - (1+\gamma)z_1 - \zeta_2(\underline{x}_2^k) - r_k).$$

Since g_2 is symmetric and unimodal (with nonzero derivative except at 0), there exists a unique such $Z_2^k(z_1) \in [(1+\gamma)z_1 + \zeta_2(\underline{x}_2^k) + r_k, (1+\gamma)z_1 + \zeta_2(x_2^*) + r_k]$.

If we could fix the value of r_k, we would then have precisely the problem of Section 3.4. Evolution would set $V_2(z|z_1) = 0$ for $z < Z_2^k(z_1)$ and $V_2(z|z_1) = 1$ for $z > Z_2^k(z_1)$, with $Z_2^k(z_1) \to (1+\gamma)z_1 + \zeta_2(x_2^*) + r_k$ as $\varepsilon_2 \to 0$. Now, however, we have not just one such problem, but a collection of k such problems, one corresponding to each possible value of r_k. At this point, we simplify the interaction between these problems by invoking our assumption that the successive values of r_k are sparse, relative to the support of $\tilde{s}_1$ and $\tilde{s}_2$, so that for each value of z, there is at most one value r_k that can make $g_2(z-(1+\gamma)z_1-\zeta_2(x_2^*)-r_k)$ or $g_2(z-(1+\gamma)z_1-\zeta_2(\underline{x}_2^k)-r_k)$ nonzero. Equivalently, each possible realization r_k gives rise to a set of possible realizations of $\tilde{z}$ (conditioning on z_1 throughout), each of which can arise from no other realization of $\tilde{r}_k$. On this set of values, evolution would like to set $V_2(z|z_1)$ as low as possible for $z < Z_2^k(z_1)$, and as high as possible for $z > Z_2^k(z_1)$. The implicit constraint behind the "if possible" in these statements is that $V_2(z)$ must be non-decreasing. Hence, for example, setting $V_2(z|z_1)$ relatively low for a value $z < Z_2^k(z_1)$ relevant for the realization r_k, while improving incentives conditional on realization r_k, constrains the incentives that can be provided for smaller realizations.

These observations immediately lead to the conclusion that, given z_1 and ε_2, there will be an ascending sequence of values $(V_2^0, \ldots, V_2^K)$ such that

$$\begin{aligned} V_2(z|z_1) &= V_2^0 = 0 \text{ for all } z < Z_2^1(z_1) \\ V_2(z|z_1) &= V_2^k \text{ for all } z \in [Z_2^k(z_1), Z_2^{k+1}(z_1)), \quad k = 1, ..., K-1 \\ V_2(z|z_1) &= V_2^K = 1 \text{ for all } z \geq Z_2^K(z_1). \end{aligned}$$

In the limit as $\varepsilon_2 \to 0$, we have $Z_2^k(z_1) \to (1+\gamma)z_1 + \zeta_2(x_2^*) + r_k$, and hence, a utility function given by

$$\begin{aligned} V_2(z|z_1) &= 0 \text{ for all } z < (1+\gamma)z_1 + \zeta_2(x_2^*) + r_1 \\ V_2(z|z_1) &= V_2^k \text{ for all } z \in [(1+\gamma)z_1 + \zeta_2(x_2^*) + r_k, (1+\gamma)z_1 + \zeta_2(x_2^*) + r_{k+1}), \quad k = 1, ..., K-1 \\ V_2(z|z_1) &= 1 \text{ for all } z \geq (1+\gamma)z_1 + \zeta_2(x_2^*) + r_K. \end{aligned}$$

The remaining task is then to calculate the values $V_2^1, \ldots, V_2^{K-1}$. It is straightforward to write the programming problem these values must solve and to find conditions characterizing the equilibrium. In general, however, these are quite complex. Section 5.2 presents an example in which enough structure is imposed on the problem to admit a simple closed-form solution.

The first period situation is analogous to that provided above. Evolution's criterion is then $E\left[(1+\gamma)\zeta_1(x_1) + \tilde{s}_1 + \zeta_2(x_2^*) + \tilde{s}_2 + \tilde{r}\right] = (1+\gamma)E\zeta_1(x_1) + \zeta_2(x_2^*)$, given optimal choice in the second period, but allowing for random satisficing behavior in the first. In the limit where $\varepsilon_1 \to 0$, it then follows that

$$\begin{aligned} V_1(z) &= 0 \text{ for all } z < (1+\gamma)\zeta_1(x_1^*) + \zeta_2(x_2^*) + r_1 \\ V_1(z) &= V_1^k \text{ for all } z \in [(1+\gamma)\zeta_1(x_1^*) + \zeta_2(x_2^*), (1+\gamma)\zeta_1(x_1^*) + \zeta_2(x_2^*)), \quad k = 1, \ldots, K-1 \\ V_1(z) &= 1 \text{ for all } z \geq (1+\gamma)\zeta_1(x_1^*) + \zeta_2(x_2^*) + r_K \end{aligned}$$

where the values $V_1^k, k = 0, \ldots, K$ match those of the second period.

The utility functions V_1 and V_2 now increase in K steps, becoming nearly smooth as K gets large. Once again, it is optimal to dissociate first period utility $V_1(z)$ from second period utility $V_2(z|z_1)$. Each utility function in the second period is a replica of the utility function in the first period, being a horizontal translation of the first period utility function by the random shock $(1+\gamma)\tilde{s}_1$, whose mean is zero. It can be shown that, in each neighborhood of each jump point, the first-period utility function $V_1(z)$ is more extreme than the expected second-period function $EV_2(z|\tilde{z}_1)$. Indeed, the argument is essentially identical to that used when utilities had a single jump.

4.5 Implications

Psychologists and classical economists tend to approach the concept of utility from different perspectives. Psychologists are more apt to give utility a direct hedonic interpretation, and to be comfortable with the idea of multiple forms of utility. Classical economists are more inclined to think of utility as an analytical device, and to always work with only a single notion of utility. Recent advances in behavioral economics have highlighted this apparent contradiction.

Our analysis suggests that if we interpret utility as having an evolutionary origin, in the process embracing the hedonic interpretation, then we should expect a distinction between decision and experienced utility. Psychologists are prone to go further, arguing that decisions would be improved if decision utility were replaced by expected experienced utility. Our model provides no support for this view. Decision and experienced utilities combine to produce fitness-maximizing choices. To an observer, the resulting choices will exhibit all the characteristics of rational behavior, including satisfying the revealed-preference axioms (as long as the utility errors are sufficiently small, and with fitness as the underlying utility function), despite the seeming inconsistencies between decision and experienced utilities. Replacing the resulting decisions with choices based on experienced utilities can only reduce fitness.

Of course, maximizing fitness may not be the relevant goal. There is no compelling reason why conscious beings should, as a moral imperative, strive to maximize the fitness criterion implicitly guiding their evolution. Once we abandon fitness, however, we are left with little guide as to what the appropriate welfare criterion should be, and little reason to think that emphasizing the fitness-maximizing experienced utilities should yield a welfare improvement. One might respond by arguing that experienced utility *is* the appropriate criterion, but we see little reason for singling out one particular utility function as the appropriate one.

What implications does our model have? Evolutionary explanations of behavior are intriguing, but provide their most convincing payoff when pointing to behavior implications that would hitherto have gone unnoticed. Training people to place greater emphasis on experienced utilities should perhaps paradoxically diminish the incentives to make investments in future utility. In particular, suppose we consider actions whose costs and benefits are unevenly spread over time. The action may involve costly current effort that pays off in the form of future consumption, or current consumption requiring future compensatory effort. Our comparison of naive and sophisticated agents in Section 3.5 suggests that making agents sophisticated will diminish the future utility impacts of these actions, as they realize that the utility gains or losses will be ratcheted away by future utility adjustments. Their decision making will then rely more heavily on the current implications of their choices. In essence, sophisticated agents are likely to appear to be more impatient.

5 Appendix: Proofs

5.1 Proof of Lemma 2

Let us first assume that we have already taken $\varepsilon_2 \to 0$. This ensures that, for any first-period choice x_1, the agent anticipates $x_2^*(x_1)$ as the second-period choice.

The first step of the proof now parallels that of the second period, and is to rewrite the constraints as

$$
\begin{aligned}
& \int\int V_1(\zeta_1(x_1^*) + s_1 + \gamma[\zeta_1(x_1^*) + s_1] + \zeta_2(x_1^*, x_2^*(x_1^*)) + s_2) g_1(s_1) g_2(s_2) ds_1 ds_2 \\
& \qquad - \int\int V_1(\zeta_1(\underline{x}_1) + s_1 + \gamma[\zeta_1(\underline{x}_1) + s_1] + \zeta_2(\underline{x}_1, x_2^*(\underline{x}_1)) + s_2) g_1(s_1) g_2(s_2) ds_1 ds_2 \\
= & \int\int V_1(\zeta_1(x_1^*) + s_1 + \gamma[\zeta_1(x_1^*) + s_1] + \zeta_2(x_1^*, x_2^*(x_1^*)) + s_2) g_1(s_1) g_2(s_2) ds_2 \\
& \qquad - \int\int V_1(\zeta_1(\overline{x}_1) + s_1 + \gamma[\zeta_1(\overline{x}_1) + s_1] + \zeta_2(\overline{x}_1, x_2^*(\overline{x}_1)) + s_2) g_1(s_2) g_2(s_2) ds_2 \\
= & \varepsilon_1.
\end{aligned}
$$

The next task is to execute the corresponding change of variable to rewrite these constraints as

$$
\begin{aligned}
& \int V_1(z) g(z - [(1+\gamma)\zeta_1(x_1^*) + \zeta_2(x_1^*, x_2^*(x_1^*))]) dz \qquad (20) \\
& \qquad - \int V_1(z) g(z - [(1+\gamma)\zeta_1(\underline{x}_1) + \zeta_2(\underline{x}_1, x_2^*(\underline{x}_1))]) dz \\
= & \int V_1(z) g(z - [(1+\gamma)\zeta_1(x_1^*) + \zeta_2(x_1^*, x_2^*(x_1^*))]) dz \\
& \qquad - \int V_1(z) g(z - [(1+\gamma)\zeta_1(\overline{x}_1) + \zeta_2(\overline{x}_1, \overline{x}_2^*(\overline{x}_1))]) dz \\
= & \varepsilon_1, \qquad (21)
\end{aligned}
$$

where g is the density of the random variable $(1+\gamma)\tilde{s}_1 + \tilde{s}_2$.

This ensures that there exists a $\hat{Z}_1$ with the property that $V_1(z) = 0$ for $z < \hat{Z}_1$ and $V_1(z) = 1$ for $z > \hat{Z}_1$, *if* we can show that g is symmetric and unimodal with zero derivative only at 0. In addition, as $\varepsilon_1 \to 0$, $\hat{Z}_1$ approaches $(1+\gamma)\zeta_1(x_1^*) + \zeta_2(x_1^*, x_2^*(x_1^*))$.

The next step of the proof is to establish that g indeed has the required properties. It is clear that these properties are preserved under multiplication by a nonzero scalar, so it suffices to show that if two arbitrary random variables $\tilde{s}_1$ and $\tilde{s}_2$, with densities g_1 and g_2, have these properties, then so does their sum. Let $s = s_1 + s_2$ for feasible values of s and define:

$$
\begin{aligned}
\underline{\sigma}_2(s) &= \max\{\underline{s}_2, s - \overline{s}_1\} \\
\overline{\sigma}_2(s) &= \min\{\overline{s}_2, s - \underline{s}_1\}.
\end{aligned}
$$

Notice that $\underline{\sigma}_2(s) < \overline{\sigma}_2(s)$ and that, from symmetry, $\underline{s}_1 = -\overline{s}_1$ and $\underline{s}_2 = -\overline{s}_2$. Then, letting G be the cumulative distribution of the sum s, we have

$$\begin{aligned} G(s) &= \int_{\underline{s}_2}^{\underline{\sigma}_2(s)} G_1(\overline{s}_1) g_2(s_2) ds_2 + \int_{\underline{\sigma}_2(s)}^{\overline{\sigma}_2(s)} G_1(s - s_2) g_2(s_2) ds_2 + \int_{\overline{\sigma}_2(s)}^{\overline{s}_2} G_1(\underline{s}_1) g_2(s_2) ds_2 \\ &= \int_{\underline{s}_2}^{\underline{\sigma}_2(s)} g_2(s_2) ds_2 + \int_{\underline{\sigma}_2(s)}^{\overline{\sigma}_2(s)} G_1(s - s_2) g_2(s_2) ds_2. \end{aligned}$$

We say that $\underline{\sigma}_2$ is *relevant* if $\underline{\sigma}_2 > \underline{s}_2$ (and irrelevant otherwise), and that $\overline{\sigma}_2$ is relevant if $\overline{\sigma}_2 < \overline{s}_2$. Differentiating, we have (note that $\underline{\sigma}_2 > \underline{s}_2 \Longrightarrow G_1(s - \underline{\sigma}_2) = 1$ and $\underline{\sigma}_2 = \underline{s}_2 \Longrightarrow \frac{d\underline{\sigma}_2}{ds} = 0$, which between them account for the second equality):

$$\begin{aligned} g(s) &= g_2(\underline{\sigma}_2) \frac{d\underline{\sigma}_2}{ds} - G_1(s - \underline{\sigma}_2) g_2(\underline{\sigma}_2) \frac{d\underline{\sigma}_2}{ds} + \int_{\underline{\sigma}_2(s)}^{\overline{\sigma}_2(s)} g_1(s - s_2) g_2(s_2) ds_2 \\ &= \int_{\underline{\sigma}_2(s)}^{\overline{\sigma}_2(s)} g_1(s - s_2) g_2(s_2) ds_2. \end{aligned}$$

To see that this distribution is symmetric, we note that

$$\begin{aligned} g(-s) &= = \int_{\underline{\sigma}_2(s)}^{\overline{\sigma}_2(s)} g_1(-s - s_2) g_2(s_2) ds_2 = \int_{\underline{\sigma}_2(s)}^{\overline{\sigma}_2(s)} g_1(s + s_2) g_2(s_2) ds_2 \\ &= \int_{\underline{\sigma}_2(s)}^{\overline{\sigma}_2(s)} g_1(s - s_2) g_2(-s_2) ds_2 = \int_{\underline{\sigma}_2(s)}^{\overline{\sigma}_2(s)} g_1(s - s_2) g_2(s_2) ds_2 = g(s). \end{aligned}$$

Unimodality, and the presence of a zero derivative only at zero, follow from taking another derivative to obtain:

$$\begin{aligned} G''(s) &= g_1(s - \overline{\sigma}_2) g_2(\overline{\sigma}_2) \frac{d\overline{\sigma}_2}{ds} - g_1(s - \underline{\sigma}_2) g_2(\underline{\sigma}_2) \frac{d\underline{\sigma}_2}{ds} \\ &+ \int_{\underline{\sigma}_2(s)}^{\overline{\sigma}_2(s)} g_1'(s - s_2) g_2(s_2) ds_2. \end{aligned}$$

It suffices to show that the first line is nonnegative and the second line positive when $s < 0$, with the reverse holding when $s > 0$. We present the case for $s > 0$, with the case of $s < 0$ being analogous. Consider the first line. We note that $\frac{d\overline{\sigma}_2}{ds} = \frac{d\underline{\sigma}_2}{ds} = 1$ if $\underline{\sigma}_2$ and $\overline{\sigma}_2$ are both relevant, and that an irrelevant term gives a zero derivative. Because $s > 0$, it must be that either (i) only $\underline{\sigma}_2$ is relevant (in which case the first line is nonpositive), (ii) neither $\underline{\sigma}_2$ nor $\overline{\sigma}_2$ is relevant (in which case it is zero) or (iii) both are relevant (in which case $g_1(s - \underline{\sigma}_2) = g_1(\overline{s}_1) = g_1(\underline{s}_1) = g_1(s - \overline{\sigma}_2)$ and $g_2(\overline{\sigma}_2) < g_2(\underline{\sigma}_2)$, with the first line then again being nonpositive).

Consider the second line. This expression is obviously negative if $s - \overline{\sigma}_2(s) > 0$, so assume $s - \overline{\sigma}_2(s) < 0$. Then we can write

$$\int_{\underline{\sigma}_2(s)}^{\overline{\sigma}_2(s)} g_1'(s - s_2) g_2(s_2) ds_2 = \int_{s - \overline{\sigma}_2(s)}^{-(s - \overline{\sigma}_2(s))} g_1'(s_1) g_2(s - s_1) ds_1 + \int_{-(s - \overline{\sigma}_2(s))}^{s - \overline{\sigma}_2(s)} g_1'(s_2) g_2(s - s_1) ds_1.$$

The final term on the right is clearly nonpositive, and so we concentrate on the first term on the right, for which we have

$$\begin{aligned}\int_{s-\overline{\sigma}_2(s)}^{-(s-\overline{\sigma}_2(s))} g_1'(s_1)g_2(s-s_1)ds_1 &= \int_0^{-(s-\overline{\sigma}_2(s))} g_1'(s_1)g_2(s-s_1)ds_1 + \int_0^{-(s-\overline{\sigma}_2(s))} g_1'(-s_1)g_2(s+s_1)ds_1 \\ &= \int_0^{-(s-\overline{\sigma}_2(s))} g_1'(s_1)[g_2(s-s_1)-g_2(s+s_1)]ds_1,\end{aligned}$$

which is negative since $g_1'(s_1)$ is negative for $s_1 > 0$ and $g_2(s-s_1)-g_2(s+s_1)$ is positive for $s, s_1 > 0$, completing the argument that g has the desired properties.

5.2 Calculations for Section 4.4

We assume that the functions ζ_i are given by

$$\zeta_i(x_i) = -|x_i^* - x_i|, i = 1, 2 \tag{22}$$

so that agents pay a linear penalty for straying away from the optimal choice.

Let $p_1, \ldots, p_K$ be the probabilities of $r_1, \ldots, r_K$, respectively. We can perform the integration in (19) to find that

$$G_2(\hat{Z}_2^k(z_1)-(1+\gamma)z_1-\zeta_2(\underline{x}_2^k)-r_k)-G_2(\hat{Z}_2^k(z_1)-(1+\gamma)z_1-\zeta_2(x_2^*)-r_k) = \frac{\varepsilon_2}{V_2^{k+1}-V_2^k},\ \ k = 1, ..., K-1,$$

where G_2 is the cumulative distribution function of $\tilde{s}_2$. Evolution's problem is to choose the nontrivial utilities $\{V_2^k\}_{k=1}^{K-1}$ so as to maximize

$$\sum_{k=1}^{K} p_k \Pi_k,$$

where Π_k is the expected fitness of an agent who has observed r_k and now chooses from a uniform distribution over the set $[\underline{x}_2^k, \overline{x}_2^k]$.

The first-order conditions for evolution's choice of the V_2^k are thus

$$\begin{aligned} & p_k \frac{\partial \Pi_k}{\partial V_2^k} + p_{k-1}\frac{\partial \Pi_{k-1}}{\partial V_2^k} \\ = \ & p_k \frac{\partial \Pi_k}{\partial \zeta_2(\underline{x}_2^k)}\frac{\partial \zeta_2(\underline{x}_2^k)}{\partial V_2^k} + p_{k-1}\frac{\partial \Pi_{k-1}}{\partial \zeta_2(\underline{x}_2^{k-1})}\frac{\partial \zeta_2(\underline{x}_2^{k-1})}{\partial V_2^k} \\ = \ & 0, \qquad k = 1, ..., K-1.\end{aligned}$$

Using the envelope theorem, we have

$$\begin{aligned}\frac{\partial \zeta_2(\underline{x}_2^k)}{\partial V_2^k} &= \frac{-\varepsilon_2}{g_2(\hat{Z}_2^k(z_1)-(1+\gamma)z_1-\zeta_2(\underline{x}_2^k)-r_k)(V_2^{k+1}-V_2^k)^2} \\ \frac{\partial \zeta_2(\underline{x}_2^{k-1})}{\partial V_2^k} &= \frac{\varepsilon_2}{g_2(\hat{Z}_2^{k-1}(z_1)-(1+\gamma)z_1-\zeta_2(\underline{x}_2^{k-1})-r_{k-1})(V_2^k-V_2^{k-1})^2}\end{aligned}$$

so the first-order conditions become

$$\begin{aligned} & \frac{\partial \Pi_k}{\partial \zeta_2(\underline{x}_2^k)} \frac{p_k}{g_2(\hat{Z}_2^k(z_1) - (1+\gamma)z_1 - \zeta_2(\underline{x}_2^k) - r_k)(V_2^{k+1} - V_2^k)^2} \\ = \quad & \frac{\partial \Pi_{k-1}}{\partial \zeta_2(\underline{x}_2^{k-1})} \frac{p_{k-1}}{g_2(\hat{Z}_2^{k-1}(z_1) - (1+\gamma)z_1 - \zeta_2(\underline{x}_2^{k-1}) - r_{k-1})(V_2^k - V_2^{k-1})^2}, \end{aligned}$$

for $k = 1, ..., K-1$.

Now note that (22) implies that $\Pi_k(\underline{x}_2^k) = \gamma \frac{\underline{x}_2^k - x_2^*}{2} + r_k + (1+\gamma)z_1$, so that $\frac{\partial \Pi_k}{\partial \zeta_2(\underline{x}_2^k)} = \frac{\partial \Pi_{k-1}}{\partial \zeta(\underline{x}_2^{k-1})}$. In the limit as $\varepsilon_2 \to 0$, we have $\zeta_2(\underline{x}_2^k) \to \zeta_2(x_2^*)$ and $\hat{Z}_2^k(z_1) \to (1+\gamma)z_1 + \gamma\zeta_2(x_2^*) + r_k$. In this limit, then

$$\frac{V_2^{k+1} - V_2^k}{V_2^k - V_2^{k-1}} = \sqrt{\frac{p_k}{p_{k-1}}}.$$

It follows that

$$\begin{aligned} V_2^k &= \sum_{\ell=1}^{k-1} (V_2^{\ell+1} - V_2^\ell) = K \sum_{\ell=1}^{\ell-1} \sqrt{p_m} \\ \text{where } K &= \frac{1}{\sum_{\ell=1}^K \sqrt{p_\ell}}. \end{aligned}$$

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