

# THE BIOLOGICAL BASIS OF ECONOMICS

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# 1. INTRODUCTION

**Robson, A.J. “The Biological Basis of Economic Behavior,” J. Econ. Lit. 29 (2001), 11-33.**

**Robson, A.J. and Samuelson, L. (2010) “The Evolutionary Foundations of Preferences,” at <http://www.sfu.ca/~robson/Handbook.pdf>.**

## 2. WHY DO WE HAVE (HEDONIC) UTILITY FUNCTIONS?

**Damasio, A.R. (1994).** “Descartes’ Error...” “Somatic Marker Hypothesis”

**Robson, A.J.** “Why Would Nature Give Individuals Utility Functions?,” *J. Polit. Econ.* **109 (2001), 900-914.**

One rationale for utility— $N$  consumption bundles, strictly ranked by fitness, presented in pairs. One method—specify the optimal choice for each pair. If the cost of each specification is  $d > 0$ , the total cost is  $dN(N - 1)/2$ . A second method—first assign utilities based on fitness to each bundle, where the cost of each assignment is  $c > 0$ . Suppose there is also a fixed cost  $C > 0$ , so that the total cost is  $C + cN$ . For large  $N$ , the utility method is preferred, since it takes advantage of the transitivity of the fitness function.

A deeper rationale—Two-armed bandit, very little riding on each of a large number of repetitions. If you have the appropriate (fitness) utility over known outcomes, you can adapt optimally to initially unknown distributions. If you adapt optimally, the appropriate utility is implicit. A utility function permits decentralization of control; from Nature to the Individual.

Why could an individual not learn to make fitness-maximizing choices? If a human relied on observing the *actual* number of offspring, the small numbers would inevitably lead to a significant probability of error. Utility functions embody the collective experience of a vast number of ancestors, and thus can rely on *expected* offspring.

**Rayo, L. and Robson, A.J. “Biology and the Arguments of Utility”**

### 3. ADAPTATION OF UTILITY.

Frederick, S., and G. Loewenstein (1999). "Hedonic Adaptation." Adaptation is appropriate for biologically optimal hedonic utility; further, such utility need not have implications that differ dramatically from the conventional ones.

**Robson, A.J. "The Biological Basis of Economic Behavior," J. Econ. Lit. 29 (2001), 11-33.**

Two arms, which generate outcomes from the cts cdf,  $F$ . The individual sees only whether each realization is above or below  $c$ . If the choices both lie above or both lie below  $c$ , choice is made randomly.

What value of  $c$  minimizes the probability of error? This probability is

$$\begin{aligned}
 & (1/2) \Pr\{x_1, x_2 < c\} + (1/2) \Pr\{x_1, x_2 > c\} \\
 &= (1/2)(F(c))^2 + (1/2)(1 - F(c))^2 \\
 &= (1/2)y^2 + (1/2)(1 - y)^2
 \end{aligned}$$

Minimized with  $y = F(c) = 1/2$ ,  $c$  is the median of the distribution. It is optimal for the threshold to adapt to  $F$ .

More generally, suppose the individual has  $N$  threshold values

$$c_1 < c_2 < \dots < c_N.$$

The probability of error is now

$$\begin{aligned} PE(N) &= (1/2)(F(c_1))^2 + \dots + (1/2)(F(c_{n+1}) - F(c_n))^2 + \dots + (1/2)(1 - y_N)^2 \\ &= (1/2)(y_1)^2 + \dots + (1/2)(y_{n+1} - y_n)^2 + \dots + (1/2)(1 - y_N)^2. \end{aligned}$$

It follows, after an easy calculation, that

$$y_n = F(y_n) = n/(N + 1), \text{ for } n = 1, \dots, N.$$

If there is a cost that depends on the probability of error, and another component of cost that depends directly on  $N$ , as  $c(N)$ , the total cost is

$$\min_N PE(N) + c(N),$$

to be minimized over choice of  $N$ , at  $N^*$ . If  $c(N) \rightarrow 0$ , it follows that  $N^* \rightarrow \infty$ , and that  $PE(N^*) \rightarrow 0$ .

**Netzer, N. (2009) "Evolution of Time Preferences and Attitudes toward Risk," *American Economic Review*, 99(3), 937-55.**

A better criterion than the probability of error is maximizing the expectation of the  $x$ , where  $x$  is taken, wlog, as fitness. With one threshold, this should be chosen as the mean of  $F$  rather than the median. It is complicated with more thresholds, but Netzer solves the limiting case as  $N \rightarrow \infty$ .

**Rayo, L., and Becker, G. (2007), “Evolutionary Efficiency and Happiness,” JPE 115, 302-337**

This is, again, a metaphorical Principal-Agent problem. They show that biologically optimal hedonic utility should be adaptive, using a familiar form for the bounded rationality assumptions.

State  $s$  specifies the physical state of the world. Agent chooses  $x \in X$  as the strategy adopted.

$f(y | x, s) =$  the pdf of output  $y \in \mathfrak{R}$  given  $x, s$  known to agent

The agent here is guided by utility from output.

$V(y) =$  the hedonic utility of income  $y$ ;  $V'(y) > 0$ .

The agent then maximizes

$$E[V | x, s] = \int V(y)f(y | x, s)dy$$

over choice of  $x \in X$ .

MN chooses  $V$  to maximize  $E(y)$ .

There are bounds on  $V$  so that

$$V \in [\underline{V}, \bar{V}], \text{ so that } V \in [0, 1], \text{ wlog.}$$

Also there is limited discrimination—if

$$|E[V \mid x_1, s] - E[V \mid x_2, s]| \leq \varepsilon$$

then the individual cannot rank  $x_1$  and  $x_2$ . All choices within  $\varepsilon$  of  $\max_{x \in X} E[V \mid x, s]$  are “optimal.” Agent randomizes uniformly over this satisficing set.

Output is given by

$$\tilde{y} = m(x, s) + \tilde{z}, \text{ where } \tilde{z} \text{ has a continuous unimodal distribution, } E(\tilde{z}) = 0,$$

Let

$$\varphi(x, s) = \frac{E[y | x, s] - \min_x E[y | x, s]}{\max_x E[y | x, s] - \min_x E[y | x, s]} \in [0, 1].$$

Expected output is then

$$E[y | x, s] = \varphi \max_x E[y | x, s] + (1 - \varphi) \min_x E[y | x, s] \equiv E[y | \varphi, s]$$

where  $E[y | \cdot, s]$  is increasing. Output itself is

$$\tilde{y} = m(\varphi, s) + \tilde{z} \text{ so the pdf of } y \text{ is } f(y | \varphi, s).$$

Consider first the case that

$$E[y | \varphi, s] = E[y | \varphi]$$

and

$$f(y | \varphi, s) = f(y | \varphi)$$

so that

$$E[V | \varphi] = \int V(y) f(y | \varphi) dy.$$

The state does not affect output, but may affect the optimal  $x$ . The satisficing set is now

$$\{\varphi \mid E[V \mid \varphi] \geq E[V \mid 1] - \varepsilon\} = [\varphi_{\min}(V), 1], \text{ where} \\ E[V \mid \varphi_{\min}(V, \varepsilon)] = E[V \mid 1] - \varepsilon.$$

Now MN's problem becomes

$$\max_{V(\cdot) \in [0,1]} \varphi_{\min}(V) \equiv \varphi^* \quad (\text{I})$$

**Lemma 1.** If  $V^*$  solves (I) then  $V^*$  solves

$$\max_{V(\cdot) \in [0,1]} E[V \mid 1] - E[V \mid \varphi^*] \quad (\text{II})$$

**Proof.** Suppose not. Then  $\exists V \neq V^*$  such that

$$E[V \mid 1] - E[V \mid \varphi^*] > E[V^* \mid 1] - E[V^* \mid \varphi^*] = \varepsilon,$$

since  $\varphi^* = \varphi_{\min}(V^*)$ . But this implies that  $\varphi_{\min}(V) > \varphi^*$ , a contradiction.

**Proposition 1.** Problem (I) has as its essentially unique solution

$$V^*(y) = \begin{cases} 1 & y \geq \hat{y} \\ 0 & y < \hat{y} \end{cases}$$

where  $\hat{y}$  solves

$$f(\hat{y} | 1) = f(\hat{y} | \varphi^*).$$

**Proof.** (I) has a solution and it must also solve (II). That is:

$$\max_{V \in [0,1]} \int V(y)[f(y | 1) - f(y | \varphi^*)] dy.$$

Hence

$$V^*(y) = \begin{cases} 1 & f(y | 1) \geq f(\hat{y} | \varphi^*) \\ 0 & f(y | 1) < f(\hat{y} | \varphi^*) \end{cases}$$

The claimed solution follows from the single crossing property of the pdf's. As  $\varepsilon \rightarrow 0$ ,  $\varphi^* \rightarrow 1$  and  $\hat{y} \in [E(y | \varphi^*) - E(y | 1)] \rightarrow E(y | 1)$ .