THE BIOLOGICAL BASIS OF ECONOMICS

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Kahneman and Thaler (2007) Decision versus experienced utility
Gilbert “Stumbling on Happiness”
Gul and Pesendorfer “Mindless Economics”

- As in R&B, there are bounds on utilities (Simmons and Gallistel (1994)).
- Also as in R&B, there are limits on our ability to discriminate between different levels of utility.
- Given imperfect discrimination, a “steep” utility function is better. Given the bounds on utility, it can only be steep “where it counts.”
- This adaptation yields distinct decision and experienced utilities and hence the focussing illusion.
- The agent should be naive not sophisticated (O’Donoghue and Rabin (1999)).
- With fine discrimination, actions will approximately satisfy revealed preference, based on fitness.
\[ \tilde{z} = \tilde{z}_1 + \tilde{z}_2 \\
= [\zeta_1(x_1) + \tilde{s}_1] + [\gamma \tilde{z}_1 + \zeta_2(x_1, x_2) + \tilde{s}_2]. \]

The individual chooses \( x_1 \) and \( x_2 \). Rv’s \( \tilde{s}_1 \) and \( \tilde{s}_2 \) are unimodal, and symmetric, with pdf’s \( g_1 \) and \( g_2 \). In the second period, the individual observes the realization of \( \tilde{z}_1 \). For simplicity,

\[ \tilde{z} = \tilde{z}_1 + \tilde{z}_2 \\
= [\zeta_1(x_1) + \tilde{s}_1] + [\gamma \tilde{z}_1 + \zeta_2(x_2) + \tilde{s}_2]. \]

Nature seeks to maximize the expectation of \( \tilde{z}_1 + \tilde{z}_2 \). Nature constructs the utilities as functions of \( \tilde{z}_1 \) and \( \tilde{z}_2 \) and conditions second period utility on \( \tilde{z}_1 \). She cannot condition on \( x_1 \) or \( x_2 \).
Second period utility is $V_2(z \mid z_1)$. This is “experienced utility.” Instead of choosing the $x_2^*$ that maximizes $E_{\tilde{s}_2} V_2(z_1 + (\gamma z_1 + \tilde{\zeta}_2(x_2) + \tilde{s}_2) \mid z_1)$, the agent chooses any $x_2$ such that

$$E_{\tilde{s}_2} V_2(z_1 + (\gamma z_1 + \tilde{\zeta}_2(x_2^*) + \tilde{s}_2) \mid z_1) - E_{\tilde{s}_2} V_2(z_1 + (\gamma z_1 + \tilde{\zeta}_2(x_2) + \tilde{s}_2) \mid z_1) \leq \varepsilon_2.$$

He randomizes uniformly over $[\underline{x}_2, \overline{x}_2]$, where $\underline{x}_2 < x_2^* < \overline{x}_2$ and

$$E_{\tilde{s}_2} V_2((1 + \gamma)z_1 + \tilde{\zeta}_2(\underline{x}_2) + \tilde{s}_2 \mid z_1) = E_{\tilde{s}_2} V_2((1 + \gamma)z_1 + \tilde{\zeta}_2(\overline{x}_2) + \tilde{s}_2 \mid z_1)$$

$$= E_{\tilde{s}_2} V_2((1 + \gamma)z_1 + \tilde{\zeta}_2(x_2^*) + \tilde{s}_2 \mid z_1) - \varepsilon_2.$$
Lemma  There exists $Z_2(z_1) < \bar{Z}_2(z_1)$ such that the optimal second-period utility function satisfies

$$V_2(z|z_1) = 0, \text{ for all } z < \bar{Z}_2(z_1)$$
$$V_2(z|z_1) = 1, \text{ for all } z > \bar{Z}_2(z_1).$$

In the limit as $\varepsilon_2 \to 0$, the agent’s second-period choice $x_2 \to x_2^*$. 
First period utility is $V_1(z)$, "decision utility."
Take $\varepsilon_2 \to 0$ first. The agent now has utility $V_1(z)$ with $V_1 \in [0, 1]$. There is a satisficing set $[\underline{x}_1, \overline{x}_1]$, where

$$E_{\tilde{s}_1, \tilde{s}_2} V_1(\tilde{\zeta}_1(\underline{x}_1) + \tilde{s}_1 + [\gamma(\tilde{\zeta}_1(\underline{x}_1) + \tilde{s}_1) + \tilde{\zeta}_2(x_2^*) + \tilde{s}_2])$$

$$= E_{\tilde{s}_1, \tilde{s}_2} V_1(\tilde{\zeta}_1(\overline{x}_1) + \tilde{s}_1 + [\gamma(\tilde{\zeta}_1(\overline{x}_1) + \tilde{s}_1) + \tilde{\zeta}_2(x_2^*) + \tilde{s}_2])$$

$$= E_{\tilde{s}_1, \tilde{s}_2} V_1(\tilde{\zeta}_1(x_1^*) + \tilde{s}_1 + [\gamma\tilde{\zeta}_1(x_1^*) + \tilde{s}_1) + \tilde{\zeta}_2(x_2^*) + \tilde{s}_2]) - \varepsilon_1.$$ 

The individual randomizes uniformly over $[\underline{x}_1, \overline{x}_1]$. Evolution picks $V_1(z)$ to maximize expected fitness, subject to the above.
Lemma. There exists $\hat{Z}_1$ such that the optimal first-period utility function is

$$V_1(z) = 0, \text{ for all } z < \hat{Z}_1$$
$$V_1(z) = 1, \text{ for all } z > \hat{Z}_1.$$ 

In the limit as $\varepsilon_1 \to 0$,

$$\hat{Z}_1 = (1 + \gamma)\zeta_1(x_1^*) + \zeta_2(x_2^*),$$

and

$$Z_2(z_1) \to (1 + \gamma)z_1 + \zeta_2(x_2^*)$$
$$\bar{Z}_2(z_1) \to (1 + \gamma)z_1 + \zeta_2(x_2^*).$$
Suppose agent contemplates decision utility $V_1(z)$ as arising from a specific $z$. It follows that $E_{\tilde{s}_1, \tilde{s}_2} \{ V_2(z|\tilde{z}_1)|z \}$ is expected experienced utility, given $z$, and that

$$V_1(z) \neq E_{\tilde{s}_1, \tilde{s}_2} \{ V_2(z|\tilde{z}_1)|z \}$$
$$= \Pr \{ V_2(z|\tilde{z}_1) = 1|z \}$$
$$= \Pr \{ \tilde{s}_2 \geq 0|z \}.$$

Indeed, $E_{\tilde{s}_1, \tilde{s}_2} \{ V_2(z|\tilde{z}_1)|z \}$ increases from 0 to 1 as $z$ increases.
\[ \hat{Z}_1 = (1 + \gamma) \xi_1(x_1^*) + \xi_2(x_1^*, x_2(x_1^*)) \]
Consider $\varepsilon_1 = \varepsilon_2 = 0$. Then

$$E_{\tilde{s}_1, \tilde{s}_2} V_2(\tilde{z} | \tilde{z}_1) = \Pr[\tilde{s}_2 \geq 0] = \frac{1}{2},$$

for every $x_1$. Correctly anticipated experienced utility provides no incentives at all... A sophisticated agent would have no strict incentive to choose $x_1^*$ rather than any other $x_1$. More generally, with $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, it is strictly better for agents to be naive.
Implications

If utility has an evolutionary origin (and is hedonic), there should be a distinction between decision and experienced utility. Psychologists are prone to argue, further, that decisions would be improved if decision utility were replaced by expected experienced utility. We provide no support for this view. Decision and experienced utilities combine to produce fitness-maximizing choices. Choices based on experienced utilities can only reduce fitness. The optimal choices will nevertheless be approximately satisfy revealed-preference, with fitness as the true utility function, as long as the errors are small.
The EUT is readily derived from biological evolution, but this depends on the risk being independent across individuals, and there seems no compelling reason this should be so. Perhaps some of the risk in a hunter-gatherer society concerned the weather, which clearly has a shared component.

The opposite polar extreme to idiosyncratic risk is aggregate risk. If idiosyncratic risk means a separate personal coin is flipped for each individual, aggregate risk means one big public coin is flipped.

52 cards, 26 red and 26 black. You are risk-neutral.

1) I will pay you $(0.01)(3/2)^{52} \approx 14,000,000$ for sure OR 2) You can bet on the outcome of each card as it comes up, with red meaning I double your total so far, black meaning no change.
The payoff from the “gamble” is exactly 
\[(0.01)2^{26} = (0.01)(\sqrt{2})^{52} \approx $670,000.\] The “gamble” is worse because 
\[\sqrt{2} < \frac{3}{2}.\]
With an infinite deck, the gamble really is a gamble, and the expected amount after \(T\) draws is 
\[(0.01)(\frac{3}{2})^T.\] The two options are then equivalent.
There is a still an appealing sense in which the gamble, as in 2), is inferior to 1), when \(T \to \infty\). Consider this evolutionary contest. Type 1 has 2 offspring with probability 1/2, or 1 offspring with probability 1/2. All the risk is idiosyncratic. Type 2 also has 2 offspring with probability 1/2, or 1 with probability 1/2, but the risk is aggregate.

Given a “large” population, the number of type 1’s at date $T$ is $x(T) = (3/2)^T$, assuming $x(0) = 1$. Note that $\frac{1}{T} \ln x(T) = \ln(3/2)$. The number of type 2’s is $y(T) = 2^n(T)$, where $y(0) = 1$. Now $n(T)$ is the number of heads in a sequence of $T$ flips of a fair coin. It follows that $\frac{1}{T} \ln y(T) = \frac{n(T)}{T} \ln 2 \rightarrow \frac{1}{2} \ln 2 = \ln \sqrt{2}$, w.p. 1, by the strong law of large numbers. With an infinite deck, there are exactly 1/2 red and 1/2 black cards. It follows that $E(y(t)) = (3/2)^T$, so that the mean of $y(t)$ grows at a faster exponential rate than does $y(t)$ itself!

It follows that 
\[
\frac{1}{T} \ln \left( \frac{x(T)}{y(T)} \right) = \frac{1}{T} \ln x(T) - \frac{1}{T} \ln y(T) \rightarrow \ln(3/2) - \ln \sqrt{2} > 0 \text{ w.p. 1.}
\]

Hence $x(T)/y(T) \rightarrow \infty$, w.p. 1.
The martingale betting strategy, as used by Casanova. Consider the possible outcomes after a maximum of $T + 1$ flips of the fair coin. One possibility is that you have lost every flip up to the maximum. That is, you might have lost $1 + 2 + \ldots + 2^T = 2^{T+1} - 1$. The probability of this loss is $(\frac{1}{2})^{T+1}$. The only other possibility is that you won $1 = 2^S - (1 + \ldots + 2^{S-1})$. The probability of winning is $1 - (\frac{1}{2})^{T+1}$. The expected change in wealth is $- (\frac{1}{2})^{T+1} (2^{T+1} - 1) + 1 - (\frac{1}{2})^{T+1} = 0$.

Martingale convergence theorem. The mean is held down by the vanishingly unlikely event of a large loss.

In the biological example, the mean of Type 2 is similarly held up by vanishingly unlikely very large populations.