THE BIOLOGICAL BASIS OF ECONOMICS

Arthur J. Robson

New York University

February, 2013

Robson JET

Suppose that bundles b_1 and b_2 induce the offspring levels 1 and 2, so $\Psi(b_1) = 1$ and $\Psi(b_2) = 2$, where Ψ is the production function for expected offspring from commodities. Gamble 1 yields b_1 and b_2 each with probability $1/2$, where risk is independent. Gamble 2 also yields b_1 and b_2 each with probability $1/2$, but risk is aggregate. Individuals should prefer gamble 1 to gamble 2.

Robson JET

Given an environment s, type i faces an idiosyncratic gamble where $q_k^{i,s} =$ probability of $x_k^{i,s}$. $\Psi(x) =$ expected offspring from x. Hence $\sum_k q_k^{i,s}\Psi(x_k^{i,s}) =$ expected offspring of i in s. If state s has probability π_s, then the long run limiting growth rate of i is

$$\sum_s \pi_s \ln\left(\sum_k q_k^{i,s}\Psi(x_k^{i,s})\right).$$

In particular,
$\ln((1/2)\Psi(b_1) + (1/2)\Psi(b_2)) > (1/2)\ln\Psi(b_1) + (1/2)\ln\Psi(b_2)$,by the strict concavity of $\ln$.

Seeds in the desert. Should they germinate after one rainfall? Optimal strategy is to randomize.

Bergstrom, T.C "Storage for Good Times and Bad: Of Rats and Men," at http://www.econ.ucsb.edu/~tedb/Evolution/store.pdf
Suppose Pr{long winter} $= \alpha$, and so Pr{short winter} $= 1 - \alpha$. Type L squirrels save enough for a long winter and survive it with probability 1; type S's save only enough for a short one, and can survive this with probability 1, but die for sure in a long winter. Suppose Pr{survival of L type in the fall} $= 1/2$; Pr{survival of S type in the fall} $= 3/4$. Consider a type that saves enough for a long winter with probability π but for a short winter with probability $1 - \pi$. Growth rate is

$$r = \alpha \ln(\pi/2) + (1 - \alpha) \ln(\pi/2 + (1 - \pi)(3/4)).$$

Maximized over π at $\pi = 3\alpha$, given $\alpha \leq 1/3$

Cooper W. S., R. H. Kaplan. 1982. “Adaptive ‘coin-flipping’: a decision-theoretic examination of natural selection for random individual variation.” Journal of Theoretical Biology. 94:135–151.
The probability of a snowy winter is $p \in (0, 1/2)$; that of a clear winter is $1 - p \in (1/2, 1)$. Animals with dark coats survive clear winters for sure but die in snowy winters; those that develop white coats survive snowy winters but die in clear ones. A type that randomizes choosing a white coat with probability π and a dark coat with probability $1 - \pi$ has growth rate $r = p \ln \pi + (1 - p) \ln(1 - \pi)$. This is maximized by $\pi = p$.
C&K assert that individuals who choose white the flip are “altruistic” because the probability of such an individual dying is higher given $1 - p > 1/2 > p$.

Grafen, A. “Formal Darwinism, the individual-as-maximizing-agent analogy and bet-hedging,” Proceedings of the Royal Society B: Biological Sciences 266, 799-803.
Curry, P. “Decision Making under Uncertainty and the Evolution of Interdependent Preferences”, Journal of Economic Theory 98, 2001, 357-369.
A continuum of animals of size 1. π choose white and $1-\pi$ choose dark. Consider a small mass of size ε. If they choose white, the expected fraction of the population they will be is $\frac{p\varepsilon}{\pi} = \varepsilon$ if $p = \pi$. If they choose dark, the expected fraction of the population they will be is $\frac{1-p}{1-\pi}\varepsilon = \varepsilon$, if $p = \pi$. The type that randomizes $(\pi, 1-\pi)$ is maximizing the *expected fraction* of the population it will be.

5. INTERTEMPORAL PREFERENCES

Robson, A.J. and Samuelson, L. "The Evolutionary Basis of Intertemporal Preferences," AER P&P 97 (2007) 496-500.
Agents live ℓ periods, producing x_i offspring in $i = 1, \ldots, \ell$. Given a large population, it evolves as

$$L = \begin{bmatrix} e^{-\delta}x_1 & e^{-\delta} & 0 & \ldots & 0 & 0 \\ e^{-\delta}x_2 & 0 & e^{-\delta} & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ e^{-\delta}x_{\ell-1} & 0 & 0 & \ldots & 0 & e^{-\delta} \\ e^{-\delta}x_\ell & 0 & 0 & \ldots & 0 & 0 \end{bmatrix} \text{``}$$

$$N(t+1)^T = N(t)^T L = \ldots = N(0)^T L^{t+1}$$

R&S AER P&P

Perron-Frobenius Theorem. Growth rate is the dominant eigenvalue λ—the unique positive root of the Euler-Lotka equation—

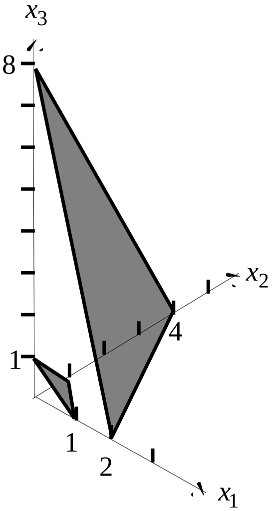
$$1 = \frac{x_1}{e^\delta \lambda} + \frac{x_2}{(e^\delta \lambda)^2} + \ldots + \frac{x_\ell}{(e^\delta \lambda)^\ell}.$$

Note that

$$\frac{1}{t} \ln \|N(t)\| \to \ln \tilde{\lambda},$$

where $\|N(t)\| = N_1(t) + ... + N_\ell(t)$. On an evolutionary indifference surface, $e^\delta \tilde{\lambda}$ is constant. These surfaces are thus hyperplanes.

R&S AER P&P



R&S AER P&P

When λ is constant,

$$0 = \frac{dx_i}{(e^\delta \lambda)^i} + \frac{dx_{i+1}}{(e^\delta \lambda)^{i+1}} \text{ so that } -\frac{dx_{i+1}}{dx_i} = e^\delta \lambda.$$

Preferences Over Consumption

Let $f_\tau(c_\tau)$ give age-τ births as a function of period-τ consumption c_τ. The f_τ are strictly increasing and concave.

For any consumption vector c,

$$1 \;=\; \frac{f_1(c_1)}{\theta} + ... + \frac{f_\tau(c_\tau)}{\theta^\tau} + \ldots + \frac{f(c_{\ell-1})}{\theta^{\ell-1}} + \frac{f_\ell(c_\ell)}{\theta^\ell},$$

where $\theta > 0$ is constant on an indifference surface. No additively separable representation, but $\theta(c_1, ..., c_\ell)$ and hence $\lambda(c_1, ..., c_\ell)$ is strictly increasing and quasi-concave.

R&S AER P&P

Geometric Discounting

Let $(f_1, \ldots, f_\ell)$ be the optimal utility profile and $\theta = e^\delta \lambda$, where λ is the Frobenius root of the Leslie matrix. Then

$$1 = \frac{f_1}{\theta} + \frac{f_2}{\theta^2} + \ldots + \frac{f_\ell}{\theta^\ell}.$$

If $(f_1', \ldots, f_\ell')$ is suboptimal then

$$\frac{f_1}{\theta} + \frac{f_2}{\theta^2} + \ldots + \frac{f_\ell}{\theta^\ell} \quad = \quad 1 \quad > \quad \frac{f_1'}{\theta} + \frac{f_2'}{\theta^2} + \ldots + \frac{f_\ell'}{\theta^\ell}.$$

Since $\ln \theta = \delta + \ln \tilde{\lambda}$, the rate of time discount is equal to the rate of population growth plus the mortality rate.

Robson, A.J. and Samuelson, L. “The Evolution of Impatience with Aggregate Uncertainty,” AER 99 (2009) 1925-53.
Aggregate risk in an age structured population means that Leslie matrix is random, i.i.d., say. Hence—

$$\tilde{N}(t+1)^T = N(0)^T \tilde{L}(1)\tilde{L}(2)...\tilde{L}(t)$$

“Sub-additive ergodic theorem” guarantees the existence of λ such that

$$\frac{1}{t}\ln\left\|\tilde{N}(t)\right\| \rightarrow \Lambda, \text{ wp } 1.$$

Aggregate uncertainty slows growth. If matrices are iid, then $\Lambda < \ln\bar{\lambda}$, where $\bar{\lambda}$ is the dominant eigenvalue of $E(\tilde{L})$.

R & S AER 2009

All survival probabilities are subject to an aggregate shock so—

$$L = \tilde{s}\bar{L}, \text{ where } \bar{L} = \begin{bmatrix} x_1 & 1 & 0 & \dots & 0 & 0 \\ x_2 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ x_{\ell-1} & 0 & 0 & \dots & 0 & 1 \\ x_\ell & 0 & 0 & \dots & 0 & 0 \end{bmatrix}$$

where $\tilde{s}$ is i.i.d. It follows that

$$\tilde{N}(t)^T = \tilde{s}(1)...\tilde{s}(t)N(0)^T L^t.$$

so that

$$\frac{1}{t}\ln\left\|\tilde{N}(t)\right\| = \frac{\sum_{\tau=1}^t \ln\tilde{s}(\tau)}{t} + \frac{1}{t}\ln\left\|N(0)^T L^t\right\| \to E\ln\tilde{s} + \ln\bar{\lambda} = E\ln\tilde{s}\bar{\lambda}$$

where $\bar{\lambda}$ is the dominant eigenvalue of the matrix $\bar{L}$.

R & S AER 2009

The growth rate is then

$$\Lambda = E \ln \tilde{s}\bar{\lambda}(x) < \ln E\tilde{s}\bar{\lambda}(x) = \ln \bar{s}\bar{\lambda}(x),$$

where $\bar{s} = E(\tilde{s})$. Then

$$-\frac{dx_{i+1}}{dx_i} = \frac{\frac{d\Lambda}{dx_i}}{\frac{d\Lambda}{dx_{i+1}}} = \frac{\frac{d\bar{\lambda}}{dx_i}}{\frac{d\bar{\lambda}}{dx_{i+1}}} = \bar{\lambda} > \frac{e^{\Lambda}}{\bar{s}}.$$

Thus the discount rate is the growth rate with no mortality. This rate exceeds the growth rate plus the mortality rate from mean survival.

R & S AER 2009

Gurven and Kaplan (2007).
Individuals start reproducing at age 15 and stop at age 45, that the probability of giving birth in a given year is 0.15. The dominant eigenvalue solves $1 = \sum_{\tau=15}^{45} \frac{(.15)}{\bar{\lambda}^t}$, so $\bar{\lambda} = 1.05675$ and $\ln \bar{\lambda} = 0.055$.
If the growth rate is zero,

$$0 \;=\; \Lambda \;=\; \ln \bar{\lambda} + E \ln \tilde{S} \;=\; 0.055 + E \ln \tilde{S},$$

With probability $1 - p$ the death rate is about two percent. With probability p a catastrophe with survival rate of $S^{\dagger}$ appears. Then $E \ln \tilde{S} = p \ln S^{\dagger} + (1-p)(-0.02) = -0.055$. Thus $p = 0.05$ if $S^{\dagger} = 0.50$, for example. Then $\ln E\tilde{S} = -0.045$, so aggregate uncertainty generates an extra 1%.

Andersen, Harrison, Lau and Rutström (2008)