THE BIOLOGICAL BASIS OF ECONOMICS

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(*a*)

kilocalories per day

6000
5000
4000
3000
2000
1000
0

(*b*)

kilocalories per day

6000
5000
4000
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2000
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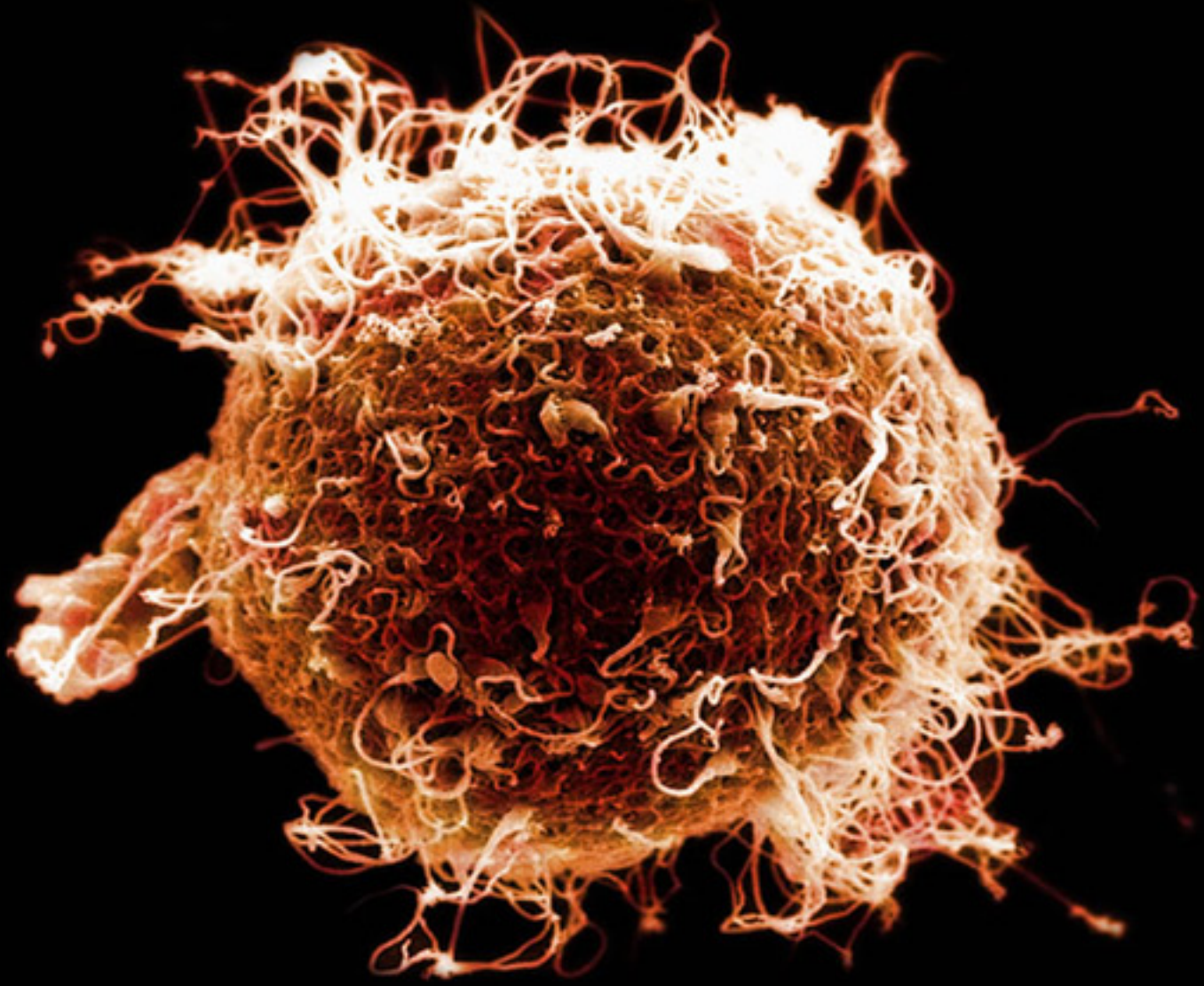
0

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75

age



Rogers, A. "Evolution of Time Preference by Natural Selection," Amer. Econ. Rev. 1994, 84, 460-481.

Reproductive value as in RA Fisher, is

$$v(x) = \frac{\sum_{y=x}^{\infty} e^{-\rho y} l(y) m(y)}{e^{-\rho x} l(x)},$$

where $l(y)$ = prob. of surviving until age y, $m(y)$ = expected offspring at age y, ρ = rate of population growth.

Rogers AER

An allele is selectively neutral if

$$v_D(x_1)\Delta P(x_1) + e^{-\rho\tau} r v_R(x_2)\Delta P(x_1+\tau) = 0$$

where $P(x)dx$ is the probability of survival from x to $x+dx$ v_D and v_R are the RV's of the donor and recipient, x_1 and x_2 are their respective ages, and τ is the time lag. It follows that

$$MRS_P = -\frac{\Delta P(x_1+\tau)}{\Delta P(x_1)} = \frac{v_D(x_1)}{re^{-\rho\tau} v_R(x_2)}.$$

Rogers AER

If survival rates depend on consumption and age as $P(x, \kappa(x))$—

$$MRS_\kappa = -\frac{\Delta\kappa(x_1+\tau)}{\Delta\kappa(x_1)} = \frac{v_D(x_1)}{re^{-\rho\tau}v_R(x_2)}\frac{P_\kappa(x_1,\kappa(x_1))}{P_\kappa(x_2,\kappa(x_2))}.$$

Rogers then assumes $MRS_\kappa = e^{i\tau}$ where i is the real rate on interest. If $x_1 = x_2$, $\frac{e^{\rho\tau}}{r} = e^{i\tau}$, so that given that $r = 1/2$, $\tau = T$, the intergenerational time, and $\rho = 0$,

$$i = \frac{\ln 2}{T}$$

If $T = 28.9$, $i = 0.024$, as is perhaps reasonable.

Robson, A.J., and Szentes, B. "Evolution of Time Preference by Natural Selection: A Comment," AER 98 (2008), 1178-88

Robson, A.J., Szentes, B., and Iantchev, E. "The Evolutionary Basis of Time Preference: Intergenerational Transfers and Sex," AEJ: Microeconomics 2012, 4(4), 172-201

Those of ages $\tau = 1, ..., T$ have incomes $I_\tau > 0$. Newborns have 0. Adults $\tau = m, ..., T$ transfer $r_\tau \geq 0$ to each of $u_\tau > 0$ offspring, $s_\tau \geq 0$ for own survival, $s_\tau + u_\tau r_\tau = I_\tau$ for $\tau = 1, ..., T$. For $\tau = 1, ..., m-1$, $u_\tau = 0$ so $r_\tau = 0$ and $s_\tau = I_\tau$. Also $s_T = 0$ so $r_T = I_T / u_T$.
Survival probability of newborns is $p_0(r_\tau)$. Survival of each parent is $p_\tau(s_\tau)$. The $p_\tau(\cdot)$, $\tau = 0, ..., T$ are continuously differentiable, increasing, strictly concave, $p_\tau(0) = \infty$. Fertilities u_τ, $\tau = 1, ...T$ are fixed. Thus $n^{t+1} = n^t L$, where $n^t = (n_1^t, ...n_T^t)$, and

$$L = \begin{bmatrix} p_0(r_1)u_1 & p_1(s_1) & 0 & . & .. & 0 \\ p_0(r_2)u_2 & 0 & p_2(s_2) & 0 & .. & 0 \\ p_0(r_3)u_3 & 0 & 0 & p_3(s_3) & .. & . \\ ... & . & . & . & .. & 0 \\ p_0(r_{T-1})u_{T-1} & 0 & . & . & 0 & p_{T-1}(s_{T-1}) \\ p_0(r_T)u_T & 0 & . & . & . & 0 \end{bmatrix}$$

.

Optimal Allocation

Euler-Lotka equation—

$$1 = \frac{p_0(r_1)u_1}{\lambda} + \frac{p_0(r_2)p_1(s_1)u_2}{\lambda^2} \ldots + \frac{p_0(r_T)p_1(s_1)\ldots p_{T-1}(s_{T-1})u_T}{\lambda^T}.$$

$qL = \lambda q$, gives the limiting population proportions. $Lv^T = \lambda v^T$, gives the appropriate reproductive values, v. With $v_1 = 1$,

$$v_\tau = \frac{p_0(r_\tau)u_\tau}{\lambda} + \frac{p_\tau(s_\tau)v_{\tau+1}}{\lambda} \text{ for } \tau = 1, \ldots, T-1, \text{ with } v_T = \frac{p_0(l_T/u_T)u_T}{\lambda}.$$

Thus

$$v_\tau = \frac{1}{\lambda}\left\{p_0(r_\tau)u_\tau + \frac{p_0(r_{\tau+1})p_\tau(s_\tau)u_{\tau+1}}{\lambda} + \ldots + \frac{p_0(r_T)p_\tau(s_\tau)\ldots p_{T-1}(s_{T-1})}{\lambda^{T-\tau}}\right.$$

for $\tau = 1, \ldots, T$.

Theorem

The optimal allocation for $\tau = m, \ldots, T-1$ is the unique solution of

$$\max_{\substack{r_\tau, s_\tau \geq 0 \\ u_\tau r_\tau + s_\tau = l_\tau}} \frac{p_0(r_\tau)u_\tau}{\lambda} + \frac{p_\tau(s_\tau)v_{\tau+1}}{\lambda} \equiv \max_{\substack{r_\tau, s_\tau \geq 0 \\ u_\tau r_\tau + s_\tau = l_\tau}} v_\tau(r_\tau, s_\tau).$$

Optimal Impatience

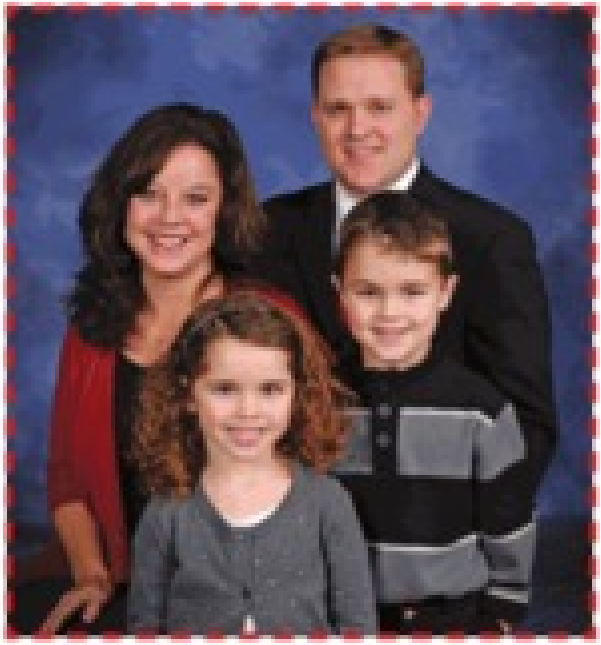
Interior solution, with FOC $p_0'(r_\tau) = p_\tau'(s_\tau)v_{\tau+1}, \tau = m, ..., T-1$. The marginal rate of substitution between l_τ and $l_{\tau+1}$ is

$$1 + \rho_\tau = \frac{\frac{\partial \lambda}{\partial l_\tau}}{\frac{\partial \lambda}{\partial l_{\tau+1}}}, \tau = 1, ..., T-1.$$

Hence, for $\tau = m, ..., T-1$,

$$1 + \rho_\tau = \frac{\lambda p_0'(r_\tau)}{p_\tau(s_\tau)p_0'(r_{\tau+1})}.$$

One component is $\frac{\lambda}{p_\tau(s_\tau)}$, which is akin to the "pure rate of time preference." The other component is $\frac{p_0'(r_\tau)}{p_0'(r_{\tau+1})}$. If the transfers r_τ increase with age, $\frac{p_0'(r_\tau)}{p_0'(r_{\tau+1})} > 1$. Once the transfers r_τ decrease with age, on the other hand, $\frac{p_0'(r_\tau)}{p_0'(r_{\tau+1})} < 1$.



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Sex and Mutants

Individuals of ages $\tau = 1, ..., T$ have incomes $I_\tau > 0$. An adult of age $\tau = m, ..., T$ transfers an amount $r_\tau/2$ to each of the $2u_\tau > 0$ joint offspring, keeping s_τ to promote her own survival to age $\tau + 1$. The budget constraint is $s_\tau + u_\tau r_\tau = I_\tau$ for $\tau = 1, ..., T$. Children have $u_\tau = 0$, so that $s_\tau = I_\tau$ for $\tau = 0, ..., m-1$. Also $s_T = 0$ so that $r_T = I_T / u_T$. Survival functions $p_\tau(\cdot)$ for $\tau = 0, ..., T-1$ are as before.
The population allocation is $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=1}^T$. Add a rare mutant with profile $\{s_\tau, r_\tau\}_{\tau=1}^T$. This mutant grows as

$$n^{t+1} = n^t L,$$

where $n^t = (n_1^t, ...n_T^t)$ and

$$L = \begin{bmatrix} p_0\left(\frac{\bar{r}_1 + r_1}{2}\right)u_1 & p_1(s_1) & 0 & . & . & 0 \\ p_0\left(\frac{\bar{r}_2 + r_2}{2}\right)u_2 & 0 & p_2(s_2) & 0 & .. & 0 \\ ... & . & . & . & . & . \\ ... & . & . & . & . & 0 \\ p_0\left(\frac{\bar{r}_{T-1} + r_{T-1}}{2}\right)u_{T-1} & 0 & . & . & 0 & p_{T-1}(s_{T-1}) \\ p_0\left(\frac{\bar{r}_T + r_T}{2}\right)u_T & 0 & . & . & . & 0 \end{bmatrix}.$$

Mutant Invasion?

The limiting growth rate λ of the mutant type satisfies $1 = \frac{p_0(\frac{\bar{r}_1+r_1}{2})u_1}{\lambda} + \frac{p_0(\frac{\bar{r}_2+r_2}{2})p_1(s_1)u_2}{\lambda^2} ... + \frac{p_0(\frac{\bar{r}_T+r_T}{2})p_1(s_1)...p_{T-1}(s_{T-1})u_T}{\lambda^T}$. Set $q = (1, \frac{p_1(s_1)}{\lambda}, ..., \frac{p_1(s_1)...p_{T-1}}{\lambda^{T-1}})$, with the normalization that $q_1 = 1$.

Reproductive values are $Lv^T = \lambda v^T$, with $v_1 = 1$. These are $v_\tau = \frac{p_0(\frac{\bar{r}_\tau+r_\tau}{2})u_\tau}{\lambda} + \frac{p_\tau(s_\tau)v_{\tau+1}}{\lambda}$, $\tau = 1, ..., T-1$, with $v_T = \frac{p_0(\frac{\bar{r}_T+r_T}{2})u_T}{\lambda}$, so

$$v_\tau = \frac{1}{\lambda}\left\{p_0\left(\frac{\bar{r}_\tau+r_\tau}{2}\right)u_\tau + ... + \frac{p_0(\frac{\bar{r}_T+r_T}{2})p_\tau(s_\tau)...p_{T-1}(s_{T-1})u_T}{\lambda^{T-\tau}}\right\}.$$

Theorem

The unique nontrivial allocations $\{s_\tau, r_\tau\}_{\tau=m}^{T-1}$ *that satisfy*

$$\max_{\substack{r_\tau, s_\tau \geq 0 \\ u_\tau r_\tau + s_\tau = l_\tau}} \frac{u_\tau p_0\left(\frac{\bar{r}_\tau+r_\tau}{2}\right)}{\lambda} + \frac{p_\tau(s_\tau)v_{\tau+1}}{\lambda} \equiv \max_{\substack{r_\tau, s_\tau \geq 0 \\ u_\tau r_\tau + s_\tau = l_\tau}} v_\tau(r_\tau, s_\tau), \qquad (1)$$

maximize the limiting growth rate of a "small" number of mutants with allocations $\{s_\tau, r_\tau\}_{\tau=m}^{T-1}$ *in a population with allocation* $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$.

ESS

If these best reply allocations are interior, $\frac{p_0'(\frac{\bar{r}_\tau + r_\tau}{2})}{2} = p_\tau'(s_\tau)v_{\tau+1}$.
Conversely, if these FOC are satisfied by $\{s_\tau, r_\tau\}_{\tau=m}^{T-1}$, this is the mutant best reply to $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$.
For $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=1}^{T}$ to be an equilibrium, it is enough that the unique best choice $\{s_\tau, r_\tau\}_{\tau=1}^{T}$ against $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=1}^{T}$ is $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=1}^{T}$.
Consider then the allocation $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$ and reproductive values $\bar{v}_\tau$, satisfying

$$\frac{p_0'(\bar{r}_\tau)}{2} = p_\tau'(\bar{s}_\tau)\bar{v}_{\tau+1}, \tau = 1, ..., T-1. \quad (2)$$

Theorem

The nontrivial allocations $\{\bar{s}_\tau, \bar{r}_\tau\}_{\tau=m}^{T-1}$ satisfying this equation are the unique ESS.

Sex and Impatience

Theorem

$\lambda = 1$ is ensured by a muliplicative effect of total population on survival rates. Adults $\tau = m, ..., T-1$ transfer too little to offspring and keep too much for own survival.

If resource allocation with sex is $\bar{r}_\tau$ and $\bar{s}_\tau$, the rate of time preference for $\tau = m, ..., T-1$, is—

$$1 + \bar{\rho}_\tau = \frac{\frac{\partial \lambda}{\partial l_\tau}}{\frac{\partial \lambda}{\partial l_{\tau+1}}} = \frac{p_0'(\bar{r}_\tau)}{p_0'(\bar{r}_{\tau+1})\bar{p}_\tau(\bar{s}_\tau)}, \tau = 1, ..., T-1,$$

With optimal r_τ^* and s_τ^*, the rate of time preference is—

$$1 + \rho_\tau^* = \frac{\frac{\partial \lambda}{\partial l_\tau}}{\frac{\partial \lambda}{\partial l_{\tau+1}}} = \frac{p_0'(r_\tau^*)}{p_0'(r_{\tau+1}^*)p_\tau(s_\tau^*)}, \tau = m, ..., T-1.$$

Have $\frac{1}{\bar{p}_\tau(\bar{s}_\tau)} = \frac{1}{\beta p_\tau(\bar{s}_\tau)} < \frac{1}{p_\tau(s_\tau^*)}, \tau = m, ..., T-1$. If $p_0(r_\tau) = \alpha r_\tau$, $\alpha > 0$ and all resource allocations are interior, sex decreases impatience of adults, no effect on children

Robson, A.J. and Szentes, B. "A Biological Theory of Public Discounting" WP, 2013

Discrete time, continuum of individuals. Single output at $t+1$ is $G(M_t, K_t, L_t)$ where M_t, K_t and L_t are public capital, private capital, and labour, all at t, respectively. 100% depreciation.

G has CRS. Defining $m = M/L$ and $k = K/L$,

$G(M, K, L) = LG(m, k, 1) = Lg(m, k)$, say. g is three times continuously differentiable, satisfies Inada conditions in each input, is strictly concave, and has the inputs as complements.

One individual in a couple has resources w_1 and privately saves k_1, the other has w_2 and saves k_2. Consume $c_1 = w - k_1 - m$ and $c_2 = w - k_2 - m$. Expected offspring is $2f(c_1 + c_2)$. f is continuously differentiable, $f'(c) > 0$, and $f(0) = 0$. If parents invest k_1 and k_2 in private capital and the per-capita public capital is m each offspring gets

$$\frac{g(m, k_1) + g(m, k_2)}{2f(c_1 + c_2)}.$$

Social Optimum

k, m and c are constant. For feasibility, $\frac{g(m,k)}{f(2c)} = m + k + c$. Each $m, k \geq 0$ determines $c(m,k) \geq 0$, say. Growth factor is $f(2c)$, so problem is $\max_{m,k \geq 0} c(m,k)$. Differentiating yields

$$2f'(2c)\, c_m(m,k)\,(c+m+k) + f(2c)\,(c_m(m,k)+1) = g_m(m,k).$$

If (m,k) is optimal $c_m = 0$, so $f(2c) = g_m(m,k)$. Similarly $f(2c) = g_k(m,k)$. Thus:

Theorem

There is a unique pair $m, k > 0$ which maximize growth. This pair is characterized by $g_m(m,k) = g_k(m,k) = f(2c)$.

Sex

Essentially all the matches involving rare mutants have one mutant and one non-mutant. Mutant growth factor is $f(\bar{c}+c)$, since each mixed couple has $2f(\bar{c}+c)$ offspring, half mutant. Mutant budget constraint is

$$m+k+c=\frac{g(m,\bar{k})+g(m,k)}{2f(\bar{c}+c)}.$$

Mutant problem is then to max c subject the budget constraint. Unique solution $c(k)$. Differentiating yields

$$2f'(\bar{c}+c)\,c'(k)\,(m+k+c(k))+2f(\bar{c}+c(k))\,(1+c'(k))=g_k(m,k).$$

If $c'(k)=0$, $2f(c(k)+\bar{c})=g_k(m,k)$. $(\bar{k},\bar{c})$ is an ESS, if and only if $k=\bar{k}$ and $c=\bar{c}$:

Theorem

For each $m>0$, there is a unique (pure strategy) ESS $(\bar{k},\bar{c})$ which satisfies $g_k(m,\bar{k})=2f(2\bar{c})$.

Second Best Public Capital

$\bar{m}$ solves $\max_m c$ s.t. $f(2c)(m+k+c) = g(m,k)$ and $g_k(m,k) = 2f(2c)$. Differentiate the first constraint wrt m—

$$\begin{aligned} & 2f'\left(2\bar{c}\left(m\right)\right)\bar{c}'\left(m\right)\left(m+\bar{k}\left(m\right)+\bar{c}(m)\right)+f\left(2\bar{c}\left(m\right)\right)\left(1+\bar{k}'\left(m\right)+\bar{c}'\right. \\ = \quad & g_m\left(m,\bar{k}\left(m\right)\right)+g_k\left(m,\bar{k}\left(m\right)\right)\bar{k}'\left(m\right). \end{aligned}$$

If $\bar{m}$ maximizes $\bar{c}$ then $\bar{c}'\left(\bar{m}\right) = 0$. Using the second constraint,

$$f\left(2\bar{c}\left(\bar{m}\right)\right)\left(1-\bar{k}'\left(\bar{m}\right)\right) = g_m\left(\bar{m},\bar{k}\left(\bar{m}\right)\right).$$

Have $\bar{k}'\left(\bar{m}\right) > 0$, so:

Theorem

Group selection for the level of public capital, given individual selection for the level of private capital, generates a level of public capital $\bar{m} > 0$, private capital $\bar{k}\left(\bar{m}\right) > 0$ and consumption $\bar{c}(\bar{m}) > 0$ which satisfy

$$g_m(\bar{m},\bar{k}\left(\bar{m}\right)) = f(2\bar{c}(\bar{m}))\left(1-\bar{k}'\left(\bar{m}\right)\right) < 2f(2\bar{c}(\bar{m})) = g_k(\bar{m},\bar{k}\left(\bar{m}\right)).$$