Evolution of Time Preference by Natural Selection: Comment

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ABSTRACT

We reexamine Alan R. Rogers' (1994) analysis of the biological basis of the rate of time preference. Although his basic insight concerning the derivation of the utility function holds up, the functional form he uses does not generate equilibrium evolutionary behavior. Moreover, Rogers relies upon an interior solution for a particular kind of intergenerational transfer. We show such interior solutions need not generally arise. Hence Rogers most striking prediction, namely that the real interest rate should be about 2% per annum, does not follow.

Key words: Biological evolution, rate of time preference, intergenerational transfers. (*JEL* Z00)

Why do individuals have the preferences they have? Why are they as risk-averse as they are or as impatient? Although these questions have traditionally been claimed to be inappropriate to address within economics, recent work has realized some of the potential of evolutionary biology and anthropology to provide illuminating answers.

One such path-breaking contribution is due to Rogers (1994), who examines the biological underpinnings of intertemporal preferences and of the rate of time preference, in particular. Rogers' argument can be paraphrased as follows. Consider the evolution of a sexually reproducing species in which resource availability plays a key role. Each individual uses resources to increase the probability of survival until the next period, but, as a simplifying assumption, has fixed fertility at every age at which he or she is alive. Furthermore, each parent can save

resources for the ultimate benefit of an offspring. Rogers supposes that savings behavior is controlled genetically, and looks for the evolutionary equilibrium level of such savings. He considers how individuals might be motivated to choose the equilibrium level of saving by means of the appropriate interdependent utility functions. These interdependent utilities are then those that would be generated by the evolutionary process.

Rogers considers that a key option is for a mother to make a "same-age transfer," that is, to save resources to benefit her daughter in the future, at a date when her daughter will be the same age as the mother is now. Since the daughter has more remaining reproductive life than her mother will at that future date, it may be advantageous for the mother to make this transfer rather than to keep the repayment for herself. This advantage is offset by the fact that the daughter is only a half-relative of her mother and by population growth. When considering the tradeoff between resources now for the mother and resources in the future for the daughter, it is only the degree of relatedness of one half and the population growth rate that matters, since the mother's biological value now and that of her daughter later are the same. Rogers assumes there is no growth in income, and that the rate of interest then equals the pure rate of time preference. With zero population growth, the overall discount factor due to time preference must then equal a half, and this is also the overall discount factor due to interest. Given a reasonable estimate of the length of a generation, he then derives a plausible estimate of annual rate of pure time preference, and of the annual real rate of interest, in the neighborhood of 2%.

Although we believe strongly in the basic biological approach adopted here, there are serious problems with Rogers' model. Altogether, these problems make much of his argument

invalid as it stands. In particular, his prediction concerning the real rate of interest does not follow. Rogers' paper is frequently cited within the small but growing research field that attempts to provide a biological basis for economic phenomena. Since it is a keystone paper in this area, it is crucial to get the analysis correct. We illustrate the problems with Rogers' approach by means of an example that permits some insight into the true properties of such a model.

Perhaps Rogers' most important theoretical claim is that the utility function of an individual can be derived from "reproductive value." The overall biological success (or "inclusive fitness") of an individual must allow for the reproductive value of descendants, as well, so this utility function is interdependent. Rogers uses a notion of reproductive value due to Ronald A. Fisher (1930), which is the expected future offspring of an individual, discounted by the rate of growth of population, and conditioning on the age of the individual. However, we show that this particular notion is inappropriate here, since it presumes that all offspring are identical, which cannot be true when the mother makes resource transfers to her offspring that vary with her age. The preferences identified by Rogers do not generate evolutionarily appropriate behavior.

In analyzing our example, that is, we need a more general notion of reproductive value. This general notion allows for different types of individual, and reflects the ultimate representation of each type's descendants in the overall population. The type of an individual here is the age of his or her mother at birth. Such types must be distinguished since offspring with larger transfers are at a biological advantage and hence have a higher reproductive value than do their poorer siblings. The evolutionary equilibrium choices are supported by utility

functions that are the total discounted general reproductive values of all the individuals who would be directly affected by any one shot variation in the prevailing choice.

Furthermore, although Rogers requires an interior solution to a same age transfer problem in his prediction of the long term interest rate, such an interior solution need not exist. Indeed, there must be a difficulty with Rogers' argument simply because there are many possible "same-age transfers" in general. That is, a 20 year old mother may save to benefit her new born daughter 20 years later. A 30 year old mother may also save to benefit her new born daughter 30 years later. But different time horizons like this for the various same age transfers require different interest rates to have interior solutions, so corner solutions must arise for most of these.[1]

Our example illustrates this claim as well. The example is constructed so as to produce an incentive for transfers and saving in favor of offspring. However, the optimal transfer that is positive is consumed when the daughter will be younger than her mother is now. The associated MRS does not depend only the degree of relatedness and the population growth rate, as it would for a same age transfer, but it depends also in a complex way on the underlying survival functions. Even though these survival functions are assumed to have infinite derivatives at zero, a corner solution arises for the same-age transfer considered.

I. The Example

Survival Probabilities and Reproduction— Individuals are born at age 0 and live, at most, to age 3. Let $P_i(c_i)$ denote the survival probability from age i to age $i+1$, when consumption is c_i at age i, for $i = 0, 1, 2$. In particular, it is assumed that $P_0 \equiv 1$, so that individuals

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[1]We thank a referee for pointing out the force of this observation.

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surely survive until the age of one.[2] The functions P_1 and P_2 are assumed to be continuously differentiable, strictly increasing, strictly concave, and to have infinite derivatives at 0. This last condition favors transfers and savings.

At each age, $i = 1, 2, 3$, each individual is paired with another individual of the same age and the couple has two offspring.[3]

Endowments and Transfers— Individuals have zero endowment at ages $i = 0, 1, 3$ but have one unit of endowment at age 2.[4] At first, we allow only one transfer: At age 2, the individual can transfer part of her endowment to her two newborn offspring, but she can make no other transfers, and cannot save. The transfer from each parent is shared equally between each of the two offspring. The new born offspring will choose to save this transfer, since survival for one period is guaranteed. The technology for intertemporal transformation is linear, with an exogenous interest factor of R. It follows that each offspring then receives $Rs/2$ at the age of one, as the return from a particular parent who chose to save s, which return the offspring is assumed to consume at once.

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[2]This last assumption is made partly to facilitate direct comparison with Rogers, who never discusses how the possibility that an intended recipient dies before receiving a transfer must affect the analysis. In particular, then, the mortality rate is not incorporated in the rate of time preference in Rogers, in contrast to Irving Fisher (1930, II.IV.60-62). Elsewhere in the present paper, we consider the implications of a positive mortality rate and incorporate it into the rate of time preference.

[3]Fertility will be held constant across all types—non-mutant and mutant.

[4]Perhaps a more realistic pattern of fertility and income would reverse the situation of three year olds, giving these income but zero fertility. This would reduce savings by the new born offspring of two year olds, since the income of three year olds could serve instead. The survival of two year olds to age three would then be a somatic (bodily) form of saving. The present assumption produces a central need for savings, which illuminates the issues that arise in Rogers model.

The genes— The transfer decision is controlled by a single gene, with each possible choice implemented by a corresponding variety (or allele) of that gene. Therefore, each allele can be identified as $s \in [0,1]$, the amount of saving at age two. Sexual reproduction means that each offspring acquires either the paternal allele or the maternal allele, each with probability $1/2$.[5] We assume that it is not possible for a parent to recognize the genetic type, or "genotype," of her offspring. Hence, individuals cannot condition their transfers on the genotype of their offspring.

Evolutionary Equilibrium— The question is: What value of s would arise in equilibrium? Consider a population with an allele that selects savings $\bar{s}$. A small proportion of the population is then replaced by a *mutant* allele that saves s. The question becomes: For what value of $\bar{s}$ is it true that, no matter what s is, the mutant allele saving s grows no faster than the original allele that saves $\bar{s}$?

A. *Characterization of Equilibrium*

To address this question, consider a population with allele that induces savings $\bar{s}$. We shall characterize the growth rate of a small measure of mutants in this population. We denote the savings of the mutants by s. To this end, first, we compute the survival probabilities of the mutants.

The survival probability of a mutant two-year-old who saved s is given by $P_2(1-s)$. The probability of survival of a mutant (or any) one year old who had one or three year old

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[5]The assumption that individuals are characterized by a single allele is that individuals are "haploid," with a single "locus." This is for simplicity, since, in reality, humans are diploid, having two alleles at each of a large number of position or "loci." Sexual reproduction actually entails each of two individuals randomly making a selection of one allele from each locus, and then contributing that to the new individual.

parents, and hence got zero transfer is simply given by $P_1(0)$. What is the probability of survival of a mutant one year old who had two year old parents? Since the mutant gene is rare, essentially all mutant individuals have one mutant and one non-mutant parent. Recall that the total resources passed down by the two parents, $\bar{s} + s$, are shared equally between their two offspring. Hence the survival probability of the one year old mutant is $P_1(R\frac{\bar{s}+s}{2})$, because the total return on savings is $(R\frac{\bar{s}+s}{2})$.

Next, we characterize the unique steady state growth factor of the mutant population. In steady state, the mutants grow with this fixed factor, and the age distribution of the mutant population is constant.[6] Let μ_i, for $i = 1, 2, 3$, denote the steady state proportion of $i-$year-olds in the mutant gene pool, ignoring newborns, so that $\sum_{i=1}^{3} \mu_i = 1$. These proportions need not match those in the general population. Thus $F(s) = \left[(\mu_1 + \mu_3) P_1(0) + \mu_2 P_1 \left(R\frac{\bar{s}+s}{2} \right) \right]$ is the overall probability of survival of a mutant individual from age one to two. It follows that, if y_t is the number of new born mutant individuals at date t, then:

$$(1) \qquad y_t = y_{t-1} + y_{t-2}F(s) + y_{t-3}P_2(1-s)F(s).$$

Each of the terms on the right hand side of (1) is now explained. First, y_{t-1} is the number of mutant new borns at date t who have one mutant one year old parent. This is because there are y_{t-1} one year old mutants in the population at date t, and each of these is paired with a non-mutant individual, to have one mutant offspring in expectation. Secondly, $y_{t-2}F(s)$ is the number of mutant new borns at date t with one mutant two year old parent; since y_{t-2} is the number of one year old mutants at date $t-1$ and $F(s)$ of these survive from

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[6]It can be shown that, no matter what the initial age distribution of the mutants might be, the mutant population converges to this steady state. Given that there are few enough mutants to start with, this steady state growth rate determines the fate of the mutant.

date $t-1$ to t. Thirdly, $y_{t-3}P_2(1-s)F(s)$ is the number of new born mutants at date t who had one mutant three year old parent; since y_{t-3} is the number of one year old mutants at date $t-2$ and $P_2(1-s)F(s)$ of these survive from date $t-2$ to t.

Since, in steady state, the mutant population grows at a constant factor, denoted by g, (1) becomes

(2) $$g^3 = g^2 + (g + P_2(1-s))F(s).$$

It follows that[7]

$$\mu_1 = \frac{1}{g}, \ \mu_3 = \mu_2 \frac{P_2(1-s)}{g}, \ \text{and} \ \mu_1 + \mu_2 + \mu_3 = 1,$$

so the steady state population proportions are given by

(3) $$\mu_1 = \frac{1}{g}, \ \mu_2 = \frac{g-1}{g+P_2}, \ \text{and} \ \mu_3 = \frac{P_2(1-s)}{g}\frac{g-1}{g+P_2}.$$

Upon substituting these values for the μ_i's into (2), we obtain the following equation implicitly determining the growth factor of the mutant population:

(4) $$g^3 = g^2 + (1 + P_2(1-s))P_1(0) + (g-1)P_1\left(R\frac{\bar{s}+s}{2}\right).$$

It follows that g as determined by this equation is maximized if and only if the right hand side is maximized over s for parametric choice of g. There must be a unique solution for this maximization problem, with first-order condition:

(5) $$P_2'(1-s)P_1(0) \geq \frac{P_1'\left(R\frac{\bar{s}+s}{2}\right)(g-1)R}{2}, \ \text{with equality if} \ s > 0.$$

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[7]Consider the second equation, for example. Suppose that the total population of ages 1, 2, and 3 is normalized to unity. It follows that $\mu_2 P_2$ is then the number of two year olds who survive to age three. Given steady state growth, this must also equal the number of three year olds from last period times the growth rate, $\mu_3 g$.

This characterizes the choice of s that yields the fastest growth rate of a small fraction of mutant alleles, where the rest of the population chooses $\bar{s}$. The right hand side of (4) is strictly concave in s. In addition, if $s = \bar{s}$, then the growth rate of the mutant alleles must match that of the original type. Hence, if the maximum of (4) occurs anywhere except at $s = \bar{s}$, the mutants can invade in the sense of initially growing faster than the original population. Conversely, if the maximum is at $s = \bar{s}$, then any nontrivial mutant with $s \neq \bar{s}$ will be strictly outdone by the original allele.

The condition $P'_1(0) = \infty$ implies that $s = 0$ is not a solution for (5) when $\bar{s} = 0$. Therefore, $\bar{s} > 0$, in equilibrium. Altogether, that is, the evolutionary equilibrium value of savings and the growth factor satisfy

(6)
$$P'_2(1 - \bar{s}) P_1(0) = \frac{P'_1(R\bar{s})(g - 1) R}{2},$$

where $g^3 = g^2 + (1 + P_2(1 - \bar{s})) P_1(0) + (g - 1) P_1(R\bar{s})$.

Note that (6) can be written in the form—

$$\frac{2P'_2(1 - \bar{s}) P_1(0)}{P'_1(R\bar{s})(g - 1)} = R,$$

where the left hand side is an appropriate marginal rate of substitution, MRS. However, this MRS depends in a complex way on the two unrelated survival functions, P_1 and P_2, and is generally not simply the factor of $2g$ that is derived by Rogers.

The factor of 2 arises here from the assumption that transfers must be made blindly and therefore equally to each of the two offspring, one of whom is a fellow mutant, in expectation, but one of whom is not. This is a particular derivation of a general result known as Hamilton's rule. (See Theodore C. Bergstrom, 1995, for a discussion of this rule and the limits to its applicability.)

B. *Reproductive Value*

Next, we argue that the equilibrium saving behavior is a solution to a problem where each two-year old individual maximizes the total discounted reproductive value of those who are directly affected by her savings. Indeed, this total reproductive value is the utility function representing equilibrium behavior.

In general, the reproductive values should be defined as the relative shares attained in the population, in the limit as the time into the future tends to infinity. (See Arthur J. Robson and Balázs Szentes, 2008.) In the present example, a heuristic approach to finding the correct reproductive values is as follows. Let $v(0_1)$ and $v(1_1)$ denote the reproductive values of individuals of ages 0 and 1, respectively, who receive no transfer; and $v(0_2)$ and $v(1_2)$ as the reproductive values of individuals of ages 0 and 1, respectively, who receive transfer $\left(\frac{\bar{s}+s}{2}\right)$. In addition, let $v(2)$ and $v(3)$ denote the reproductive values of individuals of age two and three, respectively. Considering the descendants of each type one period ahead, these values must satisfy the recursive relationships

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\begin{aligned}
(7) \qquad v(0_1) &= \frac{v(1_1)}{g}; v(0_2) = \frac{v(1_2)}{g} \\
v(1_1) &= v(0_1) + \frac{P_1(0)\,v(2)}{g}; v(1_2) = v(0_1) + \frac{P_1\left(R\frac{\bar{s}+s}{2}\right)v(2)}{g} \\
v(2) &= v(0_2) + \frac{P_2 v(3)}{g}; v(3) = v(0_1),
\end{aligned}
$$

Let us explain the third equality, yielding $v(1_1)$, the value of a one year old who had one or three year old parents, for example. Such a one year old produces an offspring with value $v(0_1)$. (Actually, each mutant parent has two offspring. What matters, however, is that, on average, only one of these would share a rare parental allele.) In addition, this one year old survives to the next period with probability $P_1(0)$, when her reproductive value is $v(2)$. This

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expected reproductive value from the next period must finally be discounted by g because population growth deflates the evolutionary importance of any given number of individuals.

We now show that the equilibrium behavior can be derived from each two-year old individual solving the following problem

$$(8) \qquad \max_{s} P_2(1-s)v(3) + \frac{P_1\left(R\frac{\bar{s}+s}{2}\right)v(2)}{g}.$$

The maximand is the relevant component of intertemporal preferences for a two year old parent. It is the expected reproductive value of the parent, as influenced by the choice of s, plus the expected discounted reproductive value of an age one recipient, also as influenced by the choice of s.[8] Thus these reproductive values generate the utility that underpins the evolutionary equilibrium. Rogers must be given credit for this basic insight. Our disagreement stems from the appropriate notion of reproductive value to be used in such an expression. We first show that our general notion of reproductive value does generate equilibrium behavior; the next subsection shows that the simple notion used by Rogers does not.

A comparison of (8) and (6) implies that what is necessary and sufficient for these problems to be identical is that

$$(9) \qquad \frac{v(3)}{v(2)} = \frac{P_1(0)}{g(g-1)}.$$

But it is a straightforward calculation to show that the equations in (7) pin down all *relative* reproductive values and that (9) holds, in particular, as required.

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[8] Although there are two offspring, each of them inherits the parent's gene with probability half. The second expression then reflects the expected number of mutant offspring.

C. *Rogers/Fisher Reproductive Value*

The notion of reproductive value used by Rogers is that due to Ronald A. Fisher (1930, pp.27-30). This notion, which is appropriate in models where offspring are identical, simply counts the future expected discounted total fertility of an individual at each age. In the present example, this would mean, using the notation w instead of v:

$$
\begin{aligned}
w(0_1) &= \frac{w(1_1)}{g} = \frac{1}{g} + \frac{P_1\left(0\right)}{g^2} + \frac{P_1\left(0\right)P_2}{g^3}; \\
w(0_2) &= \frac{w(1_2)}{g} = \frac{1}{g} + \frac{P_1\left(R^{\frac{\bar{s}+s}{2}}\right)}{g^2} + \frac{P_1\left(R^{\frac{\bar{s}+s}{2}}\right)P_2}{g^3}; \\
w(2) &= 1 + \frac{P_2}{g}; w(3) = 1.
\end{aligned}
$$

This implies that

$$
\frac{w(3)}{w(2)} = \frac{g}{g+P_2} > \frac{v(3)}{v(2)} = \frac{P_1\left(0\right)}{g(g-1)}
$$

since, using (4),

$$
g^3 - g^2 - P_1\left(0\right)g - P_1\left(0\right)P_2 = \left[P_1\left(R\frac{\bar{s}+s}{2}\right) - P_1\left(0\right)\right](g-1) > 0.
$$

Thus the evolutionary equilibrium is not the solution to

$$
\max_s P_2(1-s)w(3) + \frac{P_1\left(R^{\frac{\bar{s}+s}{2}}\right)w(2)}{2g};
$$

indeed, the solution to the problem as formulated by Rogers entails too little savings. The problem is that Fisher's notion of reproductive value is inappropriate when offspring are distinguished in any important way, perhaps by their inheritance, as here. Thus Rogers' formulation of the utility functions that support the equilibrium level of savings is incorrect on this account.

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D. *Zero Same Age Transfer*

The previous analysis assumed that the only transfer possible was from a two-year-old parent to her current newborn. Suppose now that this two-year-old parent has one more option—she can put aside resources for her current one-year-old offspring. These resources must be saved for one period, and are then given to the offspring. Thus, this offspring will be the same age when she consumes the transfer as her mother is when she gives it. This combination has then the potential to be a same-age transfer, as in Rogers. To make the correspondence between the two models exact, however, we also assume that, when a two-year-old individual determines her transfer to her current one-year old offspring, she observes neither the income derived from her parent nor her parent's age.[9]

We assume that there exists a competitive market for these savings by current one-year-old offspring. This market returns a fixed interest factor, R'. Since only a proportion of $P_1(0)$ of the intended recipients survives until the age of two, and the market is competitive, $R' = R/P_1(0)$.[10]

We now show that, in any evolutionary equilibrium, zero additional transfers like this

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[9]Suppose this individual can observe the transfer she receives, so her transfer depends on her parent's transfer. But her parent's transfer depends on her grandparent's transfer, and so on. In general, then, the donor is almost never identical, from her point of view, to the recipient. It is impossible then for the simple Rogers' first-order condition to apply. Even if the donor cannot observe the incoming transfer, there may still be an asymmetry between donor and recipient—the donor does not know her own transfer but she does know that of her offspring. However, this issue does not arise in our equilibrium because the same-age transfer is zero. Thus all two-year olds are identical in every way and we can directly relate our first-order condition to that in Rogers.

[10]Nothing crucial depends on this specific assumption. A similar conclusion is valid even if we assume instead that the saving is lost.

will be made, given that $R \leq 2g$. It can be shown that this additional transfer is zero in equilibrium if and only the following applies for each two year old parent—[11]

$$0 \in \arg\max_{s'} V(s') \equiv P_2 \left(1 - \bar{s} - s'\right) v\left(3\right) + \frac{P_1(0)P_2(1 - \bar{s} + R'\frac{s'}{2})v(3)}{g},$$

where $\bar{s}$ still denotes the equilibrium savings to newborns. The function V, from a biological viewpoint, is "inclusive fitness"; from an economic viewpoint, it is the utility function that underpins behavior. That is, it is the relevant part of intertemporal preferences for this two-year parent contemplating transfers to her current one year old offspring. The first term is the individual's own expected survival probability to age three multiplied by her age-three reproductive value. The second term is the survival probability of the current one year old offspring from age one to age three multiplied by the present value of her age-three reproductive value.

Since $R \leq 2g$, it follows that

$$\begin{aligned} V'(0) &= -P_2'(1 - \bar{s})v\left(3\right) + \frac{R'P_1(0)P_2'\left(1 - \bar{s}\right)v(3)}{2g} \\ &= \left[\frac{R}{2g} - 1\right] P_2'\left(1 - \bar{s}\right)v(3) \leq 0, \end{aligned}$$

so $s' = 0$ is an equilibrium transfer.[12]

Although Rogers does not address such issues, they necessarily arise in his model. As noted previously, mortality does not then influence the rate of time preference, in Rogers' model.

[11] A formal argument deriving this first-order condition can be made by establishing a direct connection between maximization of the growth rate and maximization of reproductive value, with no reliance then on first-order conditions or concavity. (Such a general argument can also be made for the one transfer case.)

[12] If $R > 2g$, one might imagine that two year old parents would now save as much as possible—their entire endowment—that is. Again, there would not be an interior solution for s'. However, the analysis of this case is

The reason that an interior solution does not generally exist for the same age transfer here can be explained as follows. In equilibrium, all two year olds have the same net resources. The marginal cost of a small "same age transfer" is the marginal "utility" of the age two parent at that net resource level. The marginal benefit of the transfer derives from the marginal utility of the age two offspring, *evaluated at the same level as for the parent.* In addition, the marginal benefit is inflated by the constant interest factor, R, since resources grow over time. There is also a constant factor of g that dilutes the marginal benefit because one individual is a smaller fraction of a larger population. Finally, there is a dilution factor of 2 reflecting Hamilton's rule. In the end, then, the sign of the marginal overall effect of such a same age transfer depends only on the constant factors.

Note that, although the growth factor, g, is endogenous, it must generally differ from $R/2$. For example, here it must be that $g \geq \underline{g}$ for some $\underline{g} > 1$, since all offspring survive to be one year old and have one offspring at that age, and have a positive probability of having further offspring. If the exogenous parameter $R \leq 2\underline{g}$, then $R < 2g$.

E. *Zero Population Growth and Endogenous Interest*

The Rogers approach involves assuming zero population growth, on the basis of the argument that the average growth rate for humans must have been close to zero over the two million years of our history. Our example entails a growth rate necessarily greater than zero, since all individuals survive to age one, where each couple produces two offspring, and each

complicated because there are two distinct types of two year olds—those who receive transfers like this from their parents, and those who do not. Since such complexity was not allowed for by Rogers, it will not rescue his results.

individual has a positive probability of producing more offspring after that. But nothing crucial depends on a positive growth rate.

In order to generate an example with zero population growth, all that is needed is to deflate all probabilities of survival by a factor of g, where g is the maximum growth factor as found above. That is define

$$\bar{P}_i(c_i) = P_i(c_i)/g, \text{ for } i = 0, ..., 3.$$

Suppose now that the interest factor R facing an individual is maintained at the same numerical value as above. To achieve this, the underlying technological interest factor must be scaled down by g because the uniform increase in the probability of dying would otherwise increase the actuarially fair of interest paid to surviving beneficiaries. That is, if this underlying factor is $\hat{R}$, then $R = \hat{R}/g$, so that $\hat{R} = Rg$.

The above analysis applies to this modified example with minor reinterpretation. That is, the same levels of transfers and savings yield an evolutionary equilibrium, but the equilibrium growth factor is now one. Rogers' analysis is still subject to the same criticisms, and there is no robust prediction on the real rate of interest, for example.

The discussion so far was based on a linear intertemporal technology. What would happen if the aggregate saving technology were strictly convex, so the interest factor were endogenous? Note first that a convex technology means there is implicitly a fixed factor of production, and perhaps a carrying capacity constraint. There would then be constant population in the limit, generating an interest factor that clears the market.

There still need not be an interior solution for any particular same age transfer. The basic intuition for this is as follows. One might think that a zero same-age transfer would

imply a large interest factor. This would then favor a positive same-age transfer. However, in general, the endogenous interest rate is heavily influenced by transfers that are not same-age transfers. In the example above, the interest factor is determined by the savings of new born offspring of two year old parents and not by the same-age transfer. That is, even if the interest rate is endogenous, there is no robust prediction concerning the real rate of interest, contrary to Rogers claim.

We can obtain a specific example illustrating all this as follows. Consider the example of the first three paragraphs, where the scaling down of all the probabilities of survival is fixed. Given that the technological interest factor is $\hat{R}$, the population growth rate is zero in the evolutionary equilibrium and the same-age transfer is zero. But even if the saving technology is now changed to be convex, the interest factor is forced to be $\hat{R}$, and the evolutionary equilibrium remains the same.

To see this, assume that, instead of the linear savings technology, there now exists a period-to-period technology described by the strictly increasing, differentiable, and strictly concave production function f, satisfying $f(0) = 0$, $f'(0) = \infty$ and $f(S)/S \to 0$ as $S \to \infty$. That is, if S is the total saving in the population in a certain period, then $f(S)$ is the total amount of resources available in the next period. The effective technological interest factor R is then $f(S)/S$. Such a convex technology implies a carrying capacity constraint, so that the population will reach a steady state in which it is constant.

Let $\hat{S}$ denote the unique solution of $\hat{R} = f(\hat{S})/\hat{S}$. Suppose, for the moment, that the savings behavior of individuals is unchanged. It then follows that $S = \hat{S}$ and $R = \hat{R}$ in the steady state equilibrium.[13] Furthermore, however, such a population is immune to invasion

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[13]Suppose first that $S < \hat{S}$. Then, $R = f(S)/S > \hat{R}$ so the population grows at a positive rate. This

by the mutant allele associated with any alternative savings behavior. Indeed, the fate of a mutant allele hinges on its relative growth when it is arbitrarily rare, so that the interest factor is essentially unaffected by the presence of the mutant, and the previous analysis applies. Hence, the evolutionary equilibrium of this example where the savings technology is convex coincides with the equilibrium of the example where the technology is linear, as described in the first three paragraphs. In particular, there is zero same age transfer.

F. *A Generalized Example*

One might be concerned that our conclusions hold only because we severely restricted savings and transfers. Indeed, in a generalized example, a reasonable philosophy might be that all individuals may transfer any amount contemporaneously to any living relative, and all individuals may save any amount, although they cannot borrow. This is consistent with a stylized version of a primitive agricultural society in which savings were made by means of the storage or replanting of the harvest, but where credit opportunities were limited. There were approximate precursors of such transfer and saving options in foraging societies, which helps to ensure an adequate time span for natural selection to act on preferences. In these societies, that is, a massive transfer of resources from adults to children is observed, and this transfer finances a similarly massive investment in learning how to hunt, for example.[14]

implies that S increases and R falls. Eventually, S settles at $\hat{S}$, and the interest factor becomes $\hat{R}$. Similarly, if $S > \hat{S}$ then the population grows at a negative rate, and therefore S also decreases and converges to $\hat{S}$. That is, a constant population of the appropriate size to generate total savings of $\hat{S}$ will arise in the limit.

[14]See Robson and Hillard S. Kaplan (2003). Intergenerational transfers remain massive today. Laurence J. Kotlikoff and Lawrence H. Summers (1981) present evidence suggesting that the preponderance of modern US savings is benefitting descendants rather than smoothing lifetime consumption of the same individual.

In Robson and Szentes (2008) we reanalyze the present example in this light. Although the analysis is more complex, the essential points made by the above version of the example are preserved. In addition to transferring resources to their new born offspring, two year old parents now make positive transfers to their one year offspring, who immediately consume these.[15] Two year old parents also save for one period, with the return from this being completely transferred to the new born offspring they have at age three. These offspring reinvest this transfer and finally consume it at age one. It remains inappropriate to use the simple notion of reproductive value used by Rogers. Furthermore, all of the other myriad contemporaneous transfers and savings that might be made in principle in this generalized example are chosen to be zero in equilibrium. Most significantly, that is, there are now a number of combined transfer and savings options that amount to same age transfers, and all of these are chosen to be zero.[16]

II. Conclusion

Although we are enthusiastic about Rogers' basic biological approach to deriving the rate of time preference, a number of aspects of his model do not withstand careful examination.

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[15]The complexity can be illustrated as follows. Age two parents have a strong incentive to transfer resources to their current one year old offspring. However, such transfers might well imply a proliferation of types of such offspring. There are those with no surviving parent, those with one surviving parent, and those with two. In addition, the transfers made by one parent might be contingent on whether the other parent is still alive. This complexity seems entirely tangential to the current purposes, however. Robson and Szentes (2008) are then motivated to make an alternative simplifying assumption.

[16]This extended example also illustrates the argument in the introduction that, since different same age transfers require different interest rates for interior solutions, it is, on that account alone, impossible for all such solutions to be interior simultaneously.

Relatively minor issues include his neglect of mortality when considering intertemporal choice. Mortality is not then a component of the rate of time preference. There are perhaps two key aspects of his model that do not hold up. The first of these is that the reproductive value he uses is not general enough for the present circumstances. The expression he gives for the utility function is correct *only* in its basic additive form. The second issue is that there will not generally be interior solutions for "same age transfers." Although the one half degree of relatedness between mothers and daughters does play an important role in determining the marginal rates of substitution between consumption now and consumption in the future, the theoretical situation here is more complex than described by Rogers. The most striking conclusion of Rogers—a particular real rate of interest—does not generally follow.

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