Status, Intertemporal Choice and Risk-Taking

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Abstract

This paper studies endogenous risk-taking by embedding a concern for status (relative consumption) into an otherwise conventional model of economic growth. We prove that if the intertemporal production function is strictly concave, an equilibrium must converge to a unique steady state in which there is recurrent endogenous risk-taking. (The role played by concavity is clarified by considering a special case in which the production function is instead convex, in which there is no persistent risk-taking.) The steady state is fully characterized. It displays features that are consistent with the stylized facts that individuals both insure downside risk and gamble over upside risk, and it generates similar patterns of risk-taking and avoidance across environments with quite different overall wealth levels. Endogenous risk-taking here is generally Pareto-inefficient. A concern for status thus implies that persistent and inefficient risk-taking hinders the attainment of full equality.

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1. Introduction

This paper derives risk-taking behavior from the assumption that individuals derive utility from status. In particular, we show how risk-taking behavior might coexist with risk-averse behavior. Inspired by Veblen (1899) and Duesenberry (1949), we embed a concern for relative consumption into an otherwise conventional model of economic growth.[1] That is, individuals care about relative as well as absolute consumption. We presume that all fair gambles are available: there is a competitive industry that can supply such gambles at zero profit. We study the intertemporal equilibrium of such a model. The theory generates persistent endogenous risk-taking, even when there is no intrinsic uncertainty, with minimal restrictions on the shape of the utility function.

The main idea is simple. A deterministic equilibrium — i.e., an equilibrium with no endogenous randomization — in our model induces convergence across dynasties, as in the Solow or Ramsey parables. Such convergence implies there would be large gains in *relative* consumption from small increases in absolute consumption. Hence the urge to take risks becomes irresistible, destroying the presumption that the equilibrium is deterministic to begin with. We describe the dynamic equilibrium with risk-taking. The steady state of that equilibrium generates individual insurance against substantial downside risk and gambling over an intermediate range of outcomes. These explain the stylized facts that motivated the classic contribution by Friedman and Savage (1948), to which we return below.

Our emphasis on dynamics clarifies that the origin of endogenous risk-taking is the convergence of wealth induced by a strictly concave intertemporal production function. We underline this reasoning by briefly studying the case of a *convex* production function. In this case, when utility depends on status alone, there exists an equilibrium with no risk-taking. That equilibrium arises because the convexity of the production function prevents convergence.

Status, as defined here, involves a consumption externality. If risk-taking creates a gain in status, for example, this gain must be counterbalanced by a loss to other parties. It is not surprising then that the equilibrium gambling here is Pareto-inefficient— indeed this is true even if some particular forms of gambling are Pareto-efficient.

Three strands of the literature inform our approach. Friedman and Savage (1948) reconciled the simultaneous demand for insurance and lotteries by arguing that the former alleviates downside risk and the latter exploits upside risk. They studied a von Neumann-Morgenstern utility function that is first concave, then convex, and finally again concave. This model has been criticized, both for its *ad hoc* specification of the utility function as well as for its dependence on absolute wealth alone.[2] The latter generates marked shifts in behavior with the overall growth of wealth, as individuals display an attenuated appetite for insurance

[1]Frank (1985), Easterlin (2002), Scitovsky (1976), and Sen (1973), among many others, emphasize status in a similar sense; for empirical studies., see, e.g., Clark and Oswald (1996), and Dynan and Ravina (2007). There is a small but growing literature dealing with dynamic models that studies how a concern with status influences savings; see, e.g., Corneo and Jeanne (2001), Hopkins and Kornienko (2006), Arrow and Dasgupta (2009) and Xia (2009).

[2]Markowitz (1952) was an early and trenchant critique.

against downside risk.[3] Our main result delivers the Friedman-Savage findings with no assumption at all on the curvature of utility in status. Moreover, the concern with *relative* consumption creates similar patterns of risk-taking and risk-avoidance across environments with varying wealth levels.

Second, we build on Robson (1992), who modifies Friedman-Savage utility to depend on both relative as well as absolute wealth.[4] If utility is concave in wealth but convex in status, it is not hard to generate a concave-convex-concave shape for utility as a function of wealth. This construction can be immune to proportional wealth scaling. There is no presumption in favor of Pareto-efficiency — there may be too much gambling, or perhaps even too little of it.[5]

A limitation of such static models is that the concave-convex-concave utility pattern arise from *assumed* underlying curvature properties of utility in absolute and relative consumption, and of the wealth distribution. In the dynamic model we consider, no corresponding properties need be exogenously specified. The results are driven by the inevitability of convergence in any deterministic equilibrium, and the resulting need for gambling in order to spread equilibrium status out within any generation.

Our model is also related to a literature that studies a breakdown of convergence induced by "symmetry-breaking": individuals taking different actions whenever society-wide distributions are highly equal. One approach in this strand emphasizes the endogenous diversity of occupational choices at identical or near-identical wealth levels, leading to inequality.[6] Another involves the use of income distributions to create endogenous "reference points" with high marginal gains and losses in the departure of wealth from the reference points. Then perfect equality will be destabilized by individuals accumulating capital to different degrees.[7] Risk-taking plays an analogous role here.

Section 2 describes the basic setup. Section 3 studies the central model. Propositions 1 and 2 characterize a unique steady state, in which all dynasties bequeath the same amount, and start each generation with identical wealth. However, endogenous risk-taking induces a nondegenerate distribution of lifetime consumption. Proposition 3 shows that an intertemporal equilibrium must exist, and that any equilibrium path from a positive initial wealth distribution must converge to the unique steady state. Section 4 throws light on the

[3]Moreover, as discussed by Friedman (1953), there should be a distinct tendency for all individuals in the convex region and beyond to gamble their way to more extreme final wealth levels.

[4]Friedman and Savage actually proposed a rationale of concave-convex-concave utility that involves relative concerns. They sketched a model with two classes where changes in wealth within either class led to decreasing marginal utility, but changes that promoted an individual from the lower class to the upper class led to increasing marginal utility.

[5]In contrast, the Pareto-efficiency of the Friedman-Savage model is the main point in Friedman (1953). In this context, we note that Becker, Murphy and Werning (2005) also consider the incentive to gamble in a static model with rank-dependent status. They extend Robson (1992) in a number of ways, perhaps most significantly to a case in which status is a separate good that can be bought and sold. This assumption restores Pareto-efficiency. On a different note, Hopkins (2010) reexamines the consequences of greater inequality. If there is greater inequality in the exogenous way the status good is distributed, this may lead to more gambling, in contrast to the effect of greater inequality of the initial wealth distribution.

[6]See, e.g., Freeman (1996), Mookherjee and Ray (2003), Matsuyama (2004) and Ray (2006).

[7]Genicot and Ray (2010) develop this idea.

role of the assumption of a strictly concave production function, by considering a special case in which the production function is instead convex. Proposition 4 shows that a simple deterministic equilibrium exists, and it is unique in a broad class of strategies. Section 5 studies some properties of the equilibrium of the central model, among them, its inefficiency (Proposition 5). Section 6 sketches two extensions of the central model. First, in line with Veblen, we observe that the equilibrium in this model of consumption-based status can be reinterpreted as a separating equilibrium in which consumption signals unobservable wealth. Secondly, we consider how the introduction of uninsurable productive risk might improve the realism of the model, by creating dispersion in wealth and consumption. Section 7 summarizes and concludes.

2. The Basic Setup

2.1. **Feasible Set.** There is a continuum of dynasties of measure one. Each dynasty has initial wealth w, distributed according to the cumulative distribution function (henceforth, cdf) G.

Individuals consume and invest (or bequeath) in every period; each date represents a lifetime. An individual can transform part or all of starting wealth w_t into any gamble with that mean. The realized outcome is then divided between consumption c_t and k_t. Capital produces fresh wealth for the next generation according to the production function

$$w_{t+1} = f(k_t), \tag{1}$$

We assume throughout that f is strictly increasing and continuously differentiable (C^1) in k, with $f(0) \geq 0$. For the main analysis in Section 3, we will suppose that f is strictly concave. However, we entertain the possibility of the opposite curvature in Section 4. Indeed, under our intergenerational interpretation, the "production function" f may well be nonlinear. For instance, capital may have a human component — new generations acquire education and make occupational decisions — and the rate of return to human capital will generally vary with the level of such capital; see, e.g., Becker and Tomes (1979) and Loury (1981). This interpretation presumes, in addition, that there are imperfect credit markets for education.[8]

2.2. **Utility and Status.** Each individual has a utility function $u(c, s)$, which depends on consumption and status. *Status* at a particular consumption level is the fraction of the population who are consuming strictly less, plus a fraction of those who have exactly the same level of consumption. That is, if F_t is the cdf of consumption in society at date t, then, for any $c \geq 0$, status s is given by

$$s = \bar{F}_t(c) = \eta F_t^-(c) + (1 - \eta)F_t(c) \tag{2}$$

where η is some number strictly between 0 and 1 and $F_t^-(c)$ is the left hand limit of F_t at c.[9]

[8]If there were a complete competitive market for borrowing and lending, individuals would face the linear intertemporal budget constraint arising from the competitive interest rate, with an intercept term that captures the additional value brought in by human capital, so f would be affine.

[9]Setting $\eta = 1/2$ is attractive since total status is always then 1/2, across all distributions, with or without atoms. But it is not needed in the formal analysis.

In the special model of Section 4, we assume that u is a function of status alone.

2.3. **Risk.** We suppose that all fair gambles are freely available.[10] Each individual can subject part or all of that wealth to randomization, dividing the proceeds between consumption and investment. Such randomization might involve participation in the state lottery, in stock markets, or real-estate speculation. Under our interpretation, "consumption" is *lifetime* consumption, so that within-period risk-taking will include occupational choice or entrepreneurial ventures in addition to the assumption of short-term risk.

2.4. **Dynastic Objective.** Given initial wealth w, and a sequence of consumption distributions F_t, a typical dynasty maximizes the present discounted value of expected payoffs:

$$\sum_{t=0}^{\infty} \delta^t \mathbb{E} u\left(c_t, \bar{F}_t(c_t)\right) \tag{3}$$

where $\delta \in (0,1)$ is the discount factor, and expectations are taken with respect to any endogenous randomization. Of course, the constraint (1) must be respected at every date.

2.5. **Equilibrium.** Each dynasty pursues a *policy*, which involves a fair randomization of wealth (including the possibility of no randomization at all), and a split of the resulting proceeds between consumption and investment, all possibly conditioned on economy-wide and private histories. The recursive application of these policies, aggregated over all dynasties, yields a sequence of joint distributions for consumption, investment and wealth. In particular, there is a sequence $\mathbf{F} \equiv \{F_t\}$ for consumption at each date, and a corresponding sequence $\mathbf{G} \equiv \{G_t\}$ for wealth.

We assume anonymity, so only aggregates can be observed, and so single dynasty's actions are conditioned upon by others. Then each dynasty must take the entire sequence of consumption cdfs $\mathbf{F}$ as given. Each such dynasty settles on an optimal policy, which involves (possibly history-dependent) choices to maximize lifetime payoff as described in (3). An *equilibrium* is a collection of optimal policies relative to the sequence $\mathbf{F}$, which generates that very sequence after aggregation.

A *steady state* is an equilibrium in which the cdfs of consumption and wealth are time-stationary: there are distributions F^* and G^* such that $F_t = F^*$ and $G_t = G^*$ for all t.

3. Endogenous Risk-Taking

We consider first the central model in which the production function is strictly concave and utility depends on both absolute consumption and relative status. The strict concavity of the production function implies that there is convergence of wealth levels. With no gambling,

[10]With a finite number of individuals, there is an issue of satisfying the overall budget constraint. This issue disappears as the number of individuals tends to infinity, given the fairness of the gambles. Suppose there are N individuals, each with consumption budget $b > 0$. We wish to allow individuals to take the gamble with cdf F, say. Suppose this cdf has maximum consumption C and minimum 0. It is not hard to show that individuals $n = 1, ..., \tilde{m}$, say, can be given independent draws from F, with individuals $n = \tilde{m} + 1, ..., N$ treated as residual claimants, in such a way that $\tilde{m}/N \to 1$, with probability one, as $N \to \infty$.

consumption levels would also converge implying that a small increment in consumption would generate a "large" gain in status, creating strong incentives to deviate from the presumed equilibrium. There must therefore be endogenous and persistent risk-taking in equilibrium.

We describe the model. First, the production function is taken to be strictly concave, and a little more:

Assumption 1. *f is C^1, strictly increasing and strictly concave in k, with $f(0) \geq 0$, $\delta f'(0) > 1$, and $f(k) < k$ for all k large enough.*

Next, utility is taken to be a function both of consumption and status:

Assumption 2. *$u(c,s)$ is C^1, with $u(0,0) = 0$. It is increasing in s, with $u_s(c,s) > 0$ for all (c,s). It is strictly increasing and strictly concave in c, so that $u_c(c,s) > 0$ and $u_{cc}(c,s) < 0$. For every s, $u_c(c,s) \to 0$ as $c \to \infty$, and $u_c(c,s) \to \infty$ as $c \to 0$.*

Finally, we assume that initial wealths are uniformly positive and bounded.

Assumption 3. *The initial distribution G has compact support, bounded away from 0.*

We make two remarks. First, under Assumption 1, an individual will never randomize on k, whether or not continuation values are convex in investment. Any such randomization can be dominated by investing the expected value of the investment, and then taking a fair bet using the produced output. This domination is independent of the curvature of utility or continuation values. Without loss of any generality, then, we can work with the equation:

$$f(k_t) = b_{t+1} + k_{t+1},$$

where b_t is the *consumption budget* of an individual at date t, and k_{t+1} is deterministic.

Second, Assumption 2 does not impose any restriction on the curvature of $u(c,s)$ in s. However, we do assume that utility is strictly increasing in consumption, which formally rules out the pure status model. This assumption matters for the existence theorem rather than the convergence results. We discuss the pure status model separately in Section 4.

We first describe the equilibrium outcome in every period, and then embed this solution in reduced form into the fully dynamic model.

3.1. **Within-Period Equilibrium and Reduced Form Utility.** Consider an equilibrium. Suppose that at some date, the distribution of consumption budgets is given by H. With risk-taking, there will be a new distribution of consumption *realizations*, F. Consider the following characterization of the relationship of F to H. Since F is obtained from H by fair randomizations (some possibly degenerate),

[R1] F is a mean preserving spread of H.

The "reduced form utility" to any agent is defined to be $\mu(c) \equiv u(c, \bar{F}(c))$. If $\mu(c)$ were not concave, profitable deviations involving gambling would necessarily exist (given $H(0) = 0$). That is —-

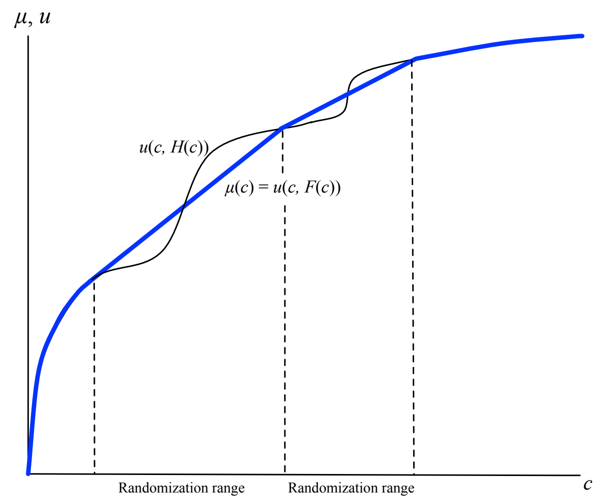

FIGURE 1. THE REDUCED FORM UTILITY.

[R2] $\mu(c) \equiv u(c, \bar{F}(c))$ is concave and continuous. It follows that F is continuous so that $\bar{F} = F$ and $\mu(c) = u(c, F(c))$.

Finally, all individuals who engage in randomization do so willingly. Hence utility is convex over the range of any randomization. [R2] calls for concavity throughout. These two restrictions imply linearity over the range of any randomization:

[R3] μ is affine over the range of any randomization used in converting H to F. Specifically, suppose that

$$\int_0^c F(x)dx > \int_0^c H(x)dx \text{ for all } c \in (\underline{c}, \bar{c}),$$

Then $\mu(c) \equiv u(c, \bar{F}(c)) = u(c, F(c))$ must be affine on $(\underline{c}, \bar{c})$.

Figure 1 illustrates a μ satisfying [R1]–[R3]. Note that there are two adjacent zones of randomization here, leading to two affine segments of the reduced form utility with different slopes. Because of this, no single equilibrium randomization has outcomes in both zones.

We can characterize the relationship of equilibrium consumption realizations and consumption budgets in Proposition 1(ii). Proposition 1(i) is an intermediate step of independent interest; it is the core observation for endogenous randomizations.

PROPOSITION **1.** (i) *Under Assumption* 2, *if* H *has compact support with* $H(0) = 0$, *there is a unique* F *associated with this* H *satisfying* [R1]—[R3].

(ii) *Under Assumptions* 2 *and* 3, *if* $\{H_t, F_t\}$ *is an equilibrium sequence of consumption budgets and realizations, then at each* t, F_t *satisfies* [R1]–[R3] *relative to* H_t.

The proofs of our results can be found in the Appendix, with technical details relegated to a self-contained online Appendix. In particular, the Appendix summarizes the proof of Proposition 1, while the online Appendix contains all details.

3.2. **Steady State.** A *steady state* is an equilibrium outcome in which the same distribution of wealth recurs period after period. We begin our discussion with the following implication of Proposition 1(i):

Corollary **1** (to Proposition 1(i))**.** *Make Assumption* 2*. For each $b > 0$, there is a unique cdf F satisfying* [R1]–R3] *with the following properties:*

(i) *The mean of F equals b:*

$$\int c dF(c) = b. \tag{4}$$

(ii) *The support of F is an interval $[a, d]$, and there exists $\alpha > 0$ such that*

$$u(c, F(c)) = u(a, 0) + \alpha(c - a) \text{ for all } c \in [a, d]. \tag{5}$$

(iii) *$a > 0$, so $\mu(c) = u(c, F(c))$ everywhere, and $\alpha = u_c(a, 0)$.*

Moreover, the slope of the affine segment α is a nonincreasing function of b.

This corollary describes how a common consumption budget must be spread out by gambling. All such gambles are fair, so we have (4). Moreover, [R3] tells us that μ must be linear over the overall domain of the gambles, which yields (5). Now we have:

Proposition **2.** *Make Assumptions* 1 *and* 2*. Then there is a steady state such that:*

(i) *Every individual (in a set of full measure) makes an identical investment k^*, given by the unique solution to $\delta f'(k^*) = 1$, and has equal starting wealth $w^* = f(k^*)$ at every date.*

(ii) *The distribution of realized consumption is as in Corollary* 1 *with $b = b^* \equiv f(k^*) - k^*$.*

Moreover:

(iii) *There is no other steady state with positive wealth for almost every individual.*

Observe that any steady state can be augmented by the addition of any mass of dynasties with zero initial wealth. Such dynasties will have zero investment and consumption. However, Proposition 2 asserts that zero wealth is the *only* impediment to uniqueness.

3.3. **Existence and Convergence.** Two crucial results are needed to justify our focus on the steady state. That steady state would be rather meaningless if equilibrium convergence to this steady state were not guaranteed from an arbitrary initial distribution of wealth. Second, we need existence of equilibrium. The following proposition resolves both these issues.

Proposition **3.** *Make Assumptions* 1, 2 *and* 3*. Then*

(i) *Under any intertemporal equilibrium, the sequence of consumption distributions must converge over time to the steady state distribution identified in Proposition* 2*.*

(ii) *An intertemporal equilibrium exists.*

It may be useful to provide an outline of the long argument leading to part (i) of Proposition 3; see the printed and online Appendices for details. Begin with any intertemporal equilibrium. By [R2], the the reduced-form utility functions $\mu_t(c) = u(c, F_t(c))$ are concave at every t. This generates an optimal growth problem with time-varying one-period utilities. By a "turnpike theorem" due to Mitra and Zilcha (1981),[11] starting from any two (positive) initial wealths, the resulting path of capital stocks must converge to *each other* over time. Therefore capital stock sequences bunch up very closely. When they do, the preservation of concavity in μ_t requires that consumption be suitably spread out using endogenous risk-taking. Further, the supports of all the gambles involved must overlap. Hence all consumption budgets ultimately fall into a range over which utility is linear (see [R3]). Thus, the marginal utility of consumption of all agents is fully equalized after some date, so not only are capital stocks are close together, they *coincide* after some finite date. The remainder of the argument consists in showing that this (common) capital stock sequence must converge.

The proof of part (ii) is entirely relegated to the online Appendix. We make two remarks. First, existence is shown in a more general setting than the model studied here, one that maintains minimal curvature restrictions on the production function and also allows for stochastic shocks to technology. Second, we establish the existence of an equilibrium in *Markovian* policies: each individual employs a policy that is independent of her own past choices and of x past distributions.

4. A Special Case With No Endogenous Risk-Taking

In this section, we illuminate the crucial role in the central model that is played by strict concavity of the production function. We do this by explicitly considering a production function that exhibits nondecreasing returns to scale. We also assume that utility depends on status alone, for expositional ease. This setting permits a remarkably simple equilibrium to exist. The convexity of the production function f implies that inequality of wealth and consumption do not fall over time, and endogenous risk-taking does not arise. In equilibrium, individual savings policies do not depend on the production or utility functions; they depend only on the discount factor.

Consider the following restrictions:

Assumption 4. *u depends on s alone and is C^1, with $u(0) = 0$ and $u'(s) > 0$ for all $s > 0$.*

Assumption 5. *f is strictly increasing, C^1, convex in k, and $f(0) = 0$.*

Assumption 6. *The initial G satisfies $G(0) = 0$, and $u(G(w))$ is concave in w.*[12]

The pure-status restriction on u and the convexity of f are implausible, but we use them here to illustrate a point. The convexity of f notwithstanding, we find an equilibrium (see

[11]In the formal proof, we use an extension of the Mitra-Zilcha theorem due to Mitra (2009).

[12]These requirements implicitly rule out the possibility of atoms—wealth levels shared by a positive measure of individuals. If $u(G(w))$ is not concave, the arguments in the online Appendix can be applied here to show that there will be an equilibrium in which individuals engage in fair bets in initial wealth, but only in the first period. If $\hat{G}$ is the post-gambling wealth distribution, then $u(\hat{G}(w))$ is concave, with linear ranges of $u(\hat{G}(w))$ associated with nontrivial gambling. The subsequent equilibrium is then as described here.

Proposition 4(i)) that generates a concave maximization problem for each individual. There is no demand for risk.

This would be less striking if there were many deterministic equilibria, with varying characteristics. However, Proposition 4(ii) shows that this equilibrium is the only "strict" and "smooth" deterministic equilibrium, for general f. A deterministic equilibrium is *strict* if at each date, individuals at all wealth levels have unique deterministic best responses.[13] In particular, in a strict equilibrium a consumption policy can be written as a function of individual wealth alone. A deterministic, strict equilibrium is *smooth* if every individual employs a sequence of differentiable consumption policies $\{c_t^i\}$, with $0 < dc_t^i(w)/dw < 1$ at all wealths w and dates t.[14]

Proposition **4.** (i) *Under Assumptions* 4, 5, *and* 6, *there exists an equilibrium in which almost every dynasty undertakes no endogenous risk, and has constant status over time. In this equilibrium, the policy*

$$c_t = (1-\delta)w_t, \tag{6}$$

is employed by almost every dynasty, which furthermore is independent of both the utility function and the initial distribution of wealth.

(ii) *Suppose that Assumption* 4 *holds, and* $f(0) = 0$. *Consider any strict deterministic smooth equilibrium described by a family* $\{c_t^i\}$ *of consumption functions. Then* $c_t^i(w) = (1-\delta)w$ *for all* i, *all* t *and all* w.

Several remarks apply to part (i) of this proposition. To begin with, the equilibrium has an extremely simple structure. Equilibrium policy depends neither on the initial distribution of wealth, nor the exact forms of the utility function and the production function. (The equilibrium *distributions* do, however, depend on these functional forms.) In fact, the equilibrium policy that we exhibit is the one that would be followed by an optimizing planner with logarithmic utility defined on absolute consumption given a linear production technology. What accounts for this structure is the delicate balance achieved across time periods: status matters today, which increases the need for current consumption, but it matters tomorrow as well, which increases the need for consumption tomorrow. In equilibrium — with a convex production function — the two effects nicely cancel in a way that induces a particularly simple equilibrium structure.[15]

Second, the equilibrium we obtain induces a *concave* optimization problem for each dynasty. With the linear equilibrium policy in place, all individuals would converge in wealth if the production function were strictly concave, as they do, for instance, in the Solow growth model. This would ultimately create a non-concave optimization problem for an individual, with small amounts of accumulation giving rise to large status gains. In contrast, the convexity of f ensures that wealth and consumption distributions stay dispersed at every date: the lack of bunching induces a concave optimization problem. It is worth noting how,

[13]Note that each individual faces the same optimization problem apart from initial wealth.

[14]These restrictions on $dc_t^i(w)/dw$ imply that all consumption levels are normal in current wealth.

[15]This observation may be viewed as a counterpart for rank-dependent status of the result established by Arrow and Dasgupta (2009) where status derives from the average consumption level.

in this sense, the *convexity* in the production function generates *concavity* in the individual optimization problem.

Finally, precisely because of the convexity of the production function, there is no incentive to gamble. While gambling has positive net expected value in the sense of output, the endogenous concavity of payoffs from that output more than outweighs the convexity in technology.[16]

Proposition 4(ii) states that our simple equilibrium is the only deterministic equilibrium in a broad class of policies. It is worth emphasizing that this result is independent of the curvature of f. What would this imply in the central model discussed in the previous section? Consider the following minor result:

Remark **1.** *Suppose f satisfies Assumption* 1*, and that Assumption* 4 *also holds. Then there is no strict, smooth deterministic equilibrium.*

The proof of this assertion is simple. If such a policy sequence were to be an equilibrium, then it must be of the linear form described in Proposition 4(ii). So convergence of all wealth and consumption levels would occur. Once at a point in time sufficiently close to the limit with complete equality, a deviator could then modify his strategy to increase consumption slightly for an arbitrarily large number of periods, at the price of reduced consumption thereafter. Such a deviation would generate a jump in status for this large number of periods and therefore would have to increase the deviator's total discounted payoff from that point onward, which is the desired contradiction. While this Remark leaves open the possibility of rather pathological deterministic equilibria, the obvious resolution is to allow randomization.[17]

5. Some Implications of the Central Model

5.1. **Risk-Averse and Risk-Preferring Choices.** We now return to further consideration of the central model developed in Section 3. Figure 2 provides a diagrammatic representation of the steady state. The first panel depicts the steady state cdf F^*. This panel is deliberately drawn to suggest that F^* has no particular shape, only that it "cancels" all curvature in u to create the affine segment (between a^* and d^*) in the second panel. In addition, zones $[0, a^*]$

[16]However, if one can buy fair insurance over the outcomes of investment gambles, any strictly convex section of the production function can be advantageously fully linearized, despite the concavity of utility. Such an outcome requires the individual to commit to investing the realizations from endogenous gambling. The assumption of strict convexity implicitly disallows such insurance against endogenous risk.

[17]Allowing randomization in consumption restores existence in this case, not surprisingly, since it is a limiting case of the central model for which existence is available. For simplicity, suppose that all individuals have the same initial wealth, w_0, say. Now, if the assumptions of Remark 1 hold, there is an equilibrium that can be derived simply from the standard one-agent problem of maximizing $\sum_{t=0}^{\infty} \delta^t \ln b_t$ subject to $w_0 = k_0 + b_0$ and $f(k_t) = k_{t+1} + b_t$ for $t = 0, \ldots.$ That is, if b_t^* and k_t^* solve this standard problem, then they also are the basis of an equilibrium here. In this equilibrium everyone invests k_t^* and randomizes the residual b_t^* with the cdf of consumption determined by the requirement that $u(F_t(c_t)) = c_t/2b_t^*$, for $c_t \in [0, 2b_t^*]$, for $t = 0, 1 \ldots,$ under the harmless normalization that $u(1) = 1$.

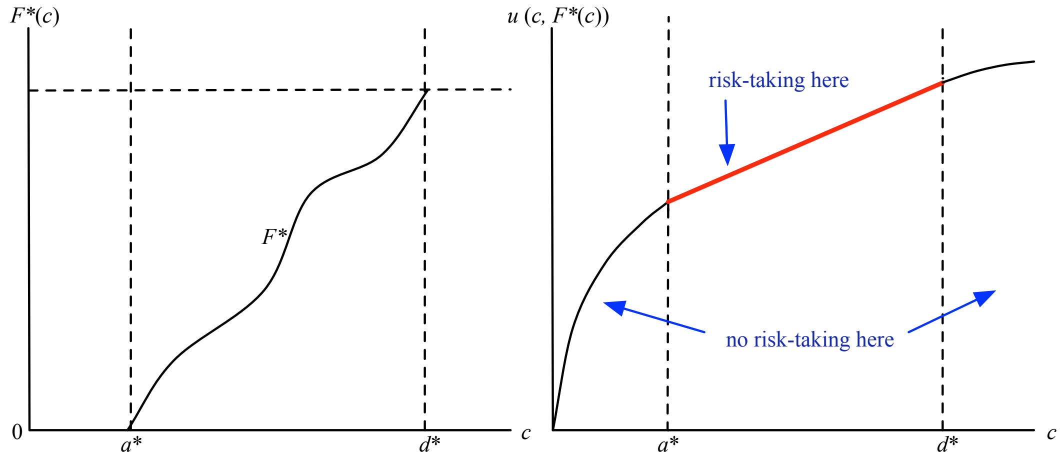

FIGURE 2. THE FRIEDMAN-SAVAGE PROPERTY

and $[d^*, \infty)$ must be present, over which no bets are taken and the utility function is strictly concave.[18]

The two regions taken together generate the phenomena that Friedman and Savage (1948) sought to explain by their postulate of an (exogenous) utility function which is alternately concave and convex. In the steady state, there is aversion to downside risk; no individual would ever take bets that would lead them into the consumption region $[0, a^*]$, or beyond d^*. Yet there *must* be risk-taking in the region $[a^*, d^*]$, as emphasized throughout the paper.

In the stark specification we study, the zones $[0, a^*]$ and $[d^*, \infty)$ are actually not inhabited in steady state. This outcome is an artificial consequence of our assumption that there is no exogenous risk. But this is easy to incorporate; see the discussion in Section 6.1 below. If this exogenous risk has realizations in the zone $[0, a^*]$ or $[d^*, \infty)$, insurance will avoid such outcomes. If some of the exogenous risk is uninsurable, all three zones will generally be actively inhabited in steady state. The central model then generates both a demand for insurance and a demand for gambling.

It is of interest is that this phenomenon — risk-aversion at the extreme ends of the distribution coupled with risk-taking elsewhere — arises "naturally" in an environment where utility depends on status. There is no need to depend on an *ad hoc* exogenous description of preferences and distributions for an explanation.[19]

5.2. **Scale-Neutrality.** The use of relative consumption guarantees that the model is, in a certain sense, scale-neutral. Two insulated societies with, say, two different production technologies, will generally settle into two different steady states. *Both* the steady states will generally exhibit the requisite patterns of risk-taking and risk-avoidance, even though

[18]The former zone is nonempty because u has unbounded steepness in c at the origin, and the latter is nonempty because steady state gambles have bounded support.

[19]One might object that (unlike Friedman and Savage) our individuals do not *strictly* prefer to bear risk. In the aggregate, however, risk-taking arises as a robust equilibrium phenomenon. From a revealed preference perspective, we have accounted for the same observations as did Friedman and Savage.

they may be located at different ranges in the wealth and consumption distribution. Unless the Friedman-Savage utility function moves around to accommodate different wealths in exactly the right way, it is not possible in their approach to generate the same phenomenon at diverse aggregate wealth levels.

5.3. **Pareto Inefficiency.** Gambling in the Friedman-Savage world is *ex-ante* efficient: there is an assumed convexity in the utility function, and this convexity is well-served by risk-taking. In our model, however, there is a pervasive consumption externality. I consider the status consequences for me of my choice to gamble, but there must be consequences for others as well, and these I ignore. Equilibrium risk-taking will then generally be Pareto-inefficient.

Consider first the special case in which u is jointly strictly concave in (c, s). Assign a status rank of 1/2 to every individual that lives in a society of perfect equality. It then follows that the steady state identified in Proposition 2 must be Pareto-inefficient. To prove this, simply ban all gambling at the steady state. All individuals continue to invest k^*, and utility in each period is $u(b^*, 1/2)$. In contrast, in the steady state with gambling, utility is given by

$$\int u(c, F^*(c))dF^*(c) < u\left(\int cdF^*(c), \int F^*(c)dF^*(c)\right) = u(b^*, 1/2),$$

where we use the strict concavity of u, and Jensen's inequality.

But inefficiency is more pervasive, and it does not require the concavity of u in status. In such cases, *some* gambling may well be Pareto efficient, in line with the static model of Robson (1992) and one of the models of Becker, Murphy and Werning (2005). Nevertheless, such efficient gambling cannot be an equilibrium outcome in the steady state derived here:

Proposition **5.** *Under Assumptions* 1 *and* 2, *the steady state identified in Proposition* 2 *must be Pareto-inefficient.*

5.4. **Consumption Distribution in Steady State.** Our model predicts key properties of the equilibrium distribution of lifetime consumption. If u is jointly strictly concave in (c, s), then Jensen's inequality implies that $u(b^*, F^*(b^*)) < u(b^*, 1/2)$, so that $b^* < (F^*)^{-1}(1/2)$. (This conclusion holds whether or not $\eta = 1/2$.) That is, the mean of the distribution is less than the median, which conflicts with the stylized fact that the mean exceeds the median. Utility functions which are concave in consumption, but convex in status, on the other hand, are capable of generating the realistic prediction that the mean exceed the median.[20] Concavity in consumption and convexity in status was the central formulation of Robson (1992), although the motivation there was independent of the argument here.

6. Two Extensions of the Central Model

6.1. **Exogenous Risk.** Suppose that there are production or ability shocks, so that we write the production function as $f(k, \theta)$ where $k \geq 0$ is the bequest as before, and $\theta \in [0, 1]$ is the realization of a random variable. We can replace Assumption 1 by

[20] It is not hard to produce an example of such a utility function where the mean exceeds the median.

Assumption 7. *f is increasing in (k, θ), C^1 and strictly concave in k, with $f(0, \theta) \geq 0$ for all $\theta \in [0, 1]$. Moreover, for all $\theta \in [0, 1]$, $\delta f_k(0, \theta) > 1$ and there exists $K \geq 0$ such that $f(k, \theta) < k$ for all $k > K$.*

Suppose that this risk can be *fully* insured in actuarially fair fashion. Then, without any loss of generality, the production function can then be taken as $\mathbb{E}_\theta f(k, \theta)$ which is deterministic and satisfies Assumption 1. So insurable risk makes no difference at all to the analysis.

However, the assumption that all risk is insurable is strong. There may be ability shocks (f includes all income sources, including wage income), or part of k may be in the form of human capital bequests that are subject to moral hazard. In this case, the full generality of Assumption 7 is needed in place of Assumption 1. An equilibrium can still be shown to exist (see the online Appendix). As long as the effect of the random variable θ is small enough, moreover, we conjecture that there will be a generalized stochastic steady state (or invariant distribution) that is close to the deterministic steady state found here, with endogenous and persistent risk-taking. The noise will disperse individuals into all three regions of the utility function that arose in the case without noise — see Figure 2. Individuals who find themselves in region where the noiseless utility was strictly concave above or below the region where it was linear would then consume and invest in a way that tends to restore them to the linear region. The stochastic steady state would then essentially involve balancing the noise introduced by uninsurable risk and this restorative behavior.

6.2. **Status from Wealth.** Status in the current model derives explicitly from consumption rather than wealth. Veblen (1899) coined the phrase "conspicuous consumption" as a reflection of the capacity of observable consumption goods to signal underlying wealth and thereby generate status. Although the current model cannot do justice to Veblen, the equilibrium here can be reinterpreted as a fully separating equilibrium in which observable consumption signals underlying unobservable wealth.[21]

To see this, reconsider the steady state of our model, in which consumption generates status. At the start of any date, almost all individuals have wealth $w^* = f(k^*)$; but the after-gambling wealth distribution has continuous cdf G^*, say, which is the cdf of after-gambling wealth $k^* + c$, where the cdf of c is F^*. That is, $G^*(k^* + c) = F^*(c)$, and status is $F^*(c)$ for all $c \geq 0$. For individual optimality, an individual who has any after-gambling wealth $w \geq 0$ must solve

$$\max_{c,c'} \Big[u(c, F^*(c)) + \delta u(c', F^*(c'))\Big] \text{ subject to } f(w - c) = c' + k^*, \tag{7}$$

where everyone else behaves in accordance with the equilibrium. That is, the individual would choose to invest k^* over the range of after-gambling wealth levels generated by the consumption gamble F^* and would find it optimal to take the gamble G^* in the first place.

It is easy to interpret this as a separating equilibrium in which observable consumption signals unobservable after-gambling wealth. Suppose everyone else behaves as before, and

[21]There is yet another formulation in which wealth is *observable*, just as consumption is in our model. This is a variant which is not accommodated. It would be of interest to extend our analysis to this case. It is not obviously more difficult, but it is the task of another paper. (It is plausible that endogenous risk-taking would also arise if there is convergence, but convergence would have to be proven.

consider the two-period problem faced by any individual. This is

$$\max_{c}\Big[u(c, G^*(w(c))) + \delta u(c', G^*(w(c')))\Big] \text{ subject to } f(w - c) = c' + k^*, \tag{8}$$

where $w(x)$ is the after-gambling wealth level inferred from observing consumption level x. However, since $w(x) \equiv k^* + x$ for all $x \geq 0$, it follows that $G^*(w(x)) = F^*(x)$, our individual solves (8) exactly as she solves (7). Moreover, by the one-shot deviation principle, no profitable deviation across multiple dates can exist. It follows that the steady state equilibrium where consumption generates status directly can be reinterpreted as a separating equilibrium where consumption signals after-gambling wealth.[22]

7. Conclusion

In this paper, we embed a concern for relative consumption into an otherwise conventional model of economic growth, and examine its consequences. In our main result, obtained with conventional concavity restrictions on the utility and production functions, there must be persistent, endogenous and inefficient risk-taking in equilibrium.

More generally, there must be persistent consumption inequality. When that inequality is generated "naturally", as it is with a constant- or increasing-returns technology, behavior is simple and deterministic. On the other hand, when inequality tends to diminish, as it does under concavity, it is recreated by endogenously generated recurrent risk-taking.

What might be the real-world manifestations of such risk-taking? We take as wide a view as possible. We might emphasize state lotteries,[23] as did Friedman and Savage, but that's only one example. One can also view the choice of career in this light, such as entrepreneurship, or occupations in which the rate of return may be low on average, but risky. Consider, for instance, the decision to become a professional basketball player, a sport with a low expected rate of return. Or consider a restauranteur who invests heavily in a new eatery, despite a half-life of six months for such establishments. Alternatively, consider a low-level member of a drug gang who earns only about the minimum wage, faces the possibility of arrest and imprisonment and of being murdered, and can only on average have only a modest chance of promotion within the gang.

The framework might also apply to individual activity on financial markets, to the extent that much of the risky outcomes may well be effectively idiosyncratic, depending on the individual decision mix. There is also a large aggregate component here, which forms a substantive topic meriting further research.[24]

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[22] Out of steady state, initial wealth levels may differ across individuals. However, realized observed consumption is still strictly increasing in total after-gambling wealth, by the weak concavity of current utility and the strict concavity of the relevant continuation value, so the argument can be generalized.

[23] The salient feature of state lotteries is that they offer a very small probability of a very large gain, and a probability near one of a small loss. It is difficult to explain why these would be the *only* unfair gambles taken in an expected utility framework. See Chew and Tan (2005) for an explanation using weighted utility.

[24] What would the current approach predict for the attitude of individuals to pure aggregate risk? Suppose that a population of individuals can decide to either enter an activity with purely aggregate risk, or stay out. If they stay out, they obtain constant consumption and a fluctuating status that depends on the number of individuals

Such phenomena admit of alternative explanations — most obviously that the subjective probabilities of success in these cases are exaggerated (perhaps — as in the case of professional sports — by the media). But the current explanation is attractive in that it does not rely on such misperception.

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in the risky activity. If they enter, they obtain risky consumption and a status that is risky, but only to the extent that others stay out. It is worth studying the equilibria of such a model, both in a static and in a dynamic context.

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Appendix

This appendix outlines the proofs of Propositions 1-5 as stated in the text. However, details (including the proofs of Lemmas stated below) are relegated to a self-contained online Appendix. In addition, the online Appendix contains a proof of existence of equilibrium in a more general model, which implies Proposition 3(ii) in particular.

We maintain Assumptions 1–3 until further notice. We begin with a central lemma.

Lemma **1.** *At any date with equilibrium cdf of consumption F, $\mu(c) \equiv u(c, \bar{F}(c))$ is concave.*

Proof of Proposition 1(i). First suppose that H has finitely many mass points. For any "initial point" a such that $H(a) < 1$, and for any "terminal point" $d > a$, let $[aHd]$ be the affine segment that connects $u(a, H(a))$ to $u(d, H(d))$. Associated with $[aHd]$ is a positive slope α, given by

$$\alpha \equiv \frac{u(d, H(d)) - u(a, H(a))}{d - a},$$

Say that $[aHd]$ is *allowable* if $\alpha \geq u_c(a, H(a))$.

Lemma **2.** *If $[aHd]$ is allowable, then the following distribution function F is well-defined and strictly increasing: $F(c) = H(c)$ for all $c \notin (a, d)$, and*

$$u(c, F(c)) = u(a, H(a)) + \alpha(c - a) \tag{9}$$

for all $c \in (a, d)$.

For allowable $[aHd]$ with associated distribution function F as described in Lemma 2, define

$$I_{[aHd]}(x) \equiv \int_a^x [F(z) - H(z)]dz$$

for $x \geq a$. Say that the allowable segment $[aHd]$ is *feasible* if

$$I_{[aHd]}(x) \geq 0 \tag{10}$$

for all $x \in [a, d)$, with equality holding at $x = d$:

$$I_{[aHd]}(d) = 0 \tag{11}$$

Because H has finitely many jumps and is flat otherwise, and because u is concave in c, it is easy to see that from any a, there are at best finitely many feasible segments (there may not be any). Construct a function $d(a)$ in the following way. If, from a, there is no feasible segment with $d > a$, set $d(a) = a$. Otherwise, set $d(a)$ to be the largest value of d among all d's that attain the highest value of α.

Lemma **3.** *Let $[aHd]$ and $[aHd']$ be two feasible segments. If $\alpha' > \alpha$, then $d' > d$.*

Lemma **4.** *For every a with $H(a) < 1$, $\alpha(a)$ and $d(a)$ are well-defined.*

Lemma **5.** *Suppose that $a^n \downarrow a$ with $d(a^n) > a^n$ for all n. Then $d(a) > a$.*[25]

Lemma **6.** *Suppose that $[aHd]$ with slope α is allowable, but* (10) *fails at $x = d$. Then the maximum slope $\alpha(a)$ from a strictly exceeds α.*

Now construct a utility function μ^* on consumption alone. In the sequel this will be the unique reduced-form utility satisfying [R1]–[R3] for the distribution H. The construction is always in one of two phases: "on the curve" or "off the curve", referring informally to whether we are "currently" following the original function $u(x, H(x))$ or are changing it in some way. Start at $a = 0$, follow the original function $u(a, H(a))$ as long as $d(a) = a$ (stay "on the curve"); at the first point at which $d(a) > a$ — and Lemma 5 guarantees that if any $d(a) > a$ exists, there is a *first* such a — move along the line segment $[aHd(a)]$ (go "off the curve"). Repeat the same process once back again "on the curve" at $d(a)$.[26] The reduced-form function — call it μ^* — will be made up of affine segments in the regions in which $d(a) > a$, and when

[25] This assertion is false for arbitrary sequences a^n; consider a distribution H with a unique mass point at a. It is clear that $d(a) = 0$, while $d(a') > 0$ for all $a' < a$.

[26] It could be that $d(d(a)) > d(a)$ so that we immediately leave the curve again at $d(a)$.

$d(a) = a$, of stretches that locally coincide with $u(c, H(c))$. It is easy to see that there are at most finitely many affine segments involved in the construction of μ^*.[27]

When H has finite support, this generates a reduced-form utility [RFU]:

Lemma 7. *μ^*, as given by the construction, satisfies* [R1]–[R3].

Now we prove that a larger class of consumption budget distributions all admit reduced-form utilities satisfying [R1]–[R3]. We begin by proving the uniqueness of such functions.

Lemma 8. *For every distribution of consumption budgets H, there is at most one RFU.*

To complete the proof of the first part of the proposition, we use an extension argument. Consider the collection $\mathcal{H}$ of all cdfs H on $[0, M]$, where $M < \infty$. We seek the existence of a mapping ϕ that assigns to each $H \in \mathcal{H}$ its unique RFU μ. Let $\mathcal{H}^{\text{fin}}$ be the subspace of $\mathcal{H}$ containing all H with finite support. Then $\mathcal{H}^{\text{fin}}$ is dense in $\mathcal{H}$ in the weak topology. Lemma 7 tells us that the mapping ϕ is already well-defined on $\mathcal{H}^{\text{fin}}$. To extend it, we use the following three lemmas:

Lemma 9. *Let G^n converge weakly to G, and (a^n, b^n) to (a, b). Then*

$$\int_{a^n}^{b^n} G^n(x)dx \to \int_a^b G(x)dx \text{ as } n \to \infty.$$

Lemma 10. *Consider any sequence $H^n \in \mathcal{H}$ converging weakly to $H \in \mathcal{H}$, and suppose that there exist associated RFUs μ^n, along with distributions of realized consumption F^n. If F^n converges weakly to F, then μ given by $\mu(c) \equiv u(c, F(c))$ for all c is the RFU for H.*

Lemma 11. *Every sequence in $\phi(\mathcal{H}^{\text{fin}})$, the space of all RFUs for distributions in $\mathcal{H}^{\text{fin}}$, admits a weakly convergent subsequence.*

Now proceed as follows. Pick any distribution $H \in \mathcal{H}$. We know that there is a sequence $H^n \in \mathcal{H}^{\text{fin}}$ that converges weakly to H. Each H^n has its (unique) RFU μ^n, with associated distribution of realized consumptions F^n. By Lemma 11, $\{F^n\}$ admits a convergent subsequence that weakly converges to some distribution F. By Lemma 10, this is an RFU for H. By Lemma 8, it is the only one, so the proof of Proposition 1(i) is complete.

To prove Proposition 1(ii), let $\{H_t, F_t\}$ be a equilibrium sequence of consumption budgets and realizations. We observe that $H_t(0) = 0$ for all $t \geq 0$. This follows easily from Assumption 3 combined with the unbounded steepness of utility $u(c, \bar{F}_t(c))$ at every date (recall that $u_c(c, s) \to \infty$ as $c \to 0$ for any s); initial wealth positive implies optimal consumption is positive at all dates.[28]

It follows that $F_t(0) = 0$ for all t. For if $F_t(0) > 0$, there must be a positive measure of individuals who take gambles that have a positive probability of generating 0. All of them have strictly positive budgets, so each such person would be better off by replacing her

[27] Indeed, the number of affine segments cannot exceed the number of atoms in H.

[28] We record this observation formally in Lemma 13 below.

gamble by one that avoids 0, which yields a status payoff $\bar{F}_t(0)$ that is discontinuously lower than $F_t(0)$. This contradicts that fact that we have an equilibrium to begin with.

It is now easy to prove [R1]–[R3], noting that [R2] is a direct corollary of Lemma 1. □

Proof of Corollary 1. Simply verify that the conditions in the statement of the corollary correspond to [R1]–[R3] when the consumption budget is degenerate, and then apply Proposition 1(i) for the case of a degenerate distribution H.[29]

To establish the very last assertion in the corollary, suppose that b is increased. Then, by (4) in the main paper, the new distribution function F must have a higher mean. It is easy to conclude that a and d must both increase. By the concavity of $u_c(.,0)$, $\alpha \equiv u_c(a,0)$ cannot increase. □

Proof of Proposition 2. Let F^* be any steady state distribution of consumption. Then we know that the RFU $\mu^*(c) = u(c, F^*(c))$ is concave. By Assumption 2 and the fact that $\mu^*(c) \geq u(c,0)$ for all c, μ^* has unbounded steepness at 0. Consider the problem of choosing $\{b_t(i), k_t(i)\}$ to maximize $\sum_{t=0}^{\infty} \delta^t \mu^*(b_t(i))$, subject to $w_t(i) = b_t(i) + k_t(i)$ and $w_{t+1}(i) = f(k_t(i))$ for all t, with $w_0(i)$ given. Because μ^* is concave and f is strictly concave, there is a unique optimal investment strategy, assigning an investment k and consumption budget b for every starting wealth w.

One can check (see, e.g., Mitra and Ray (1984)) that for each individual, k_t must converge to a steady state. Because μ^* has unbounded steepness at 0, this steady state value k^* is defined by $\delta f'(k^*) = 1$, provided $w_0(i) > 0$. Finally, F^* must be the distribution associated with the degenerate consumption budget $b^* = f(k^*) - k^*$. That verifies that if there is any steady state with positive wealth for all individuals, it must be the one described in the Proposition.

We need to complete the formalities of showing that this outcome is indeed a steady state. All we need to do is exhibit an optimal consumption policy. If the consumption budget b at any date equals $b^* \equiv f(k^*) - k^*$, take a fair bet with cdf F^*, consuming the proceeds entirely.

We already know that the investment policy is optimal. So is the consumption policy, because utilities are linear in realized consumption over the support of F^*. □

Proof of Proposition 3. We assume all the conditions given in the statement of the proposition.

Part (i): Convergence. We review the main argument. The first step is Lemma 12, based on a turnpike theorem due to Mitra and Zilcha (1981) and Mitra (2009). It states that in any equilibrium, the paths followed by all agents converge *to one another*. Lemmas 13 and 14 ensure that convergence occurs to some common sequence which has a strictly positive limit point (over time). The second step is Lemma 16, which states that when all stocks cluster sufficiently close to this common limit point, a bout of endogenous risk-taking must force all consumption budgets to lie in the same affine segment of the "reduced-form" utility function μ at that date. Lemma 17 states that all individual capital stocks must fully coincide thereafter. The remainder of the proof shows that this common path must, in turn, converge

[29]For part (iii) in particular, use the fact that $u_c(c,s) \to \infty$ as $c \to 0$ to argue that $a > 0$, and the concavity of the RFU to argue that $\alpha = u_c(a,0)$.

over time to k^*, with consumption distributions converging to F^*, the unique cdf associated (as in Corollary 1) with $d^* = f(k^*) - k^*$.

Lemma **12.** *In any equilibrium,* $\sup_{i,j} |k_t(i) - k_t(j)| \to 0$ *and* $\sup_{i,j} |b_t(i) - b_t(j)| \to 0$ *as* $t \to \infty$.

Lemma **13.** *In any equilibrium, for any i with initial wealth strictly positive,* $b_t(i) > 0$ *for every t and* $\limsup_t b_t(i) > 0$.

Lemma **14.** *There exists $\sigma > 0$ so that for every $\epsilon > 0$, there is a date T*

$$b_T(i) \in [\sigma - \epsilon, \sigma + \epsilon] \tag{12}$$

for all i.

Lemma **15.** *For any $\sigma > 0$, there exists $\psi > 0$ such that for all $\epsilon < \sigma/2$,*

$$F_t(\sigma + \epsilon) - F_t(\sigma - \epsilon) \leq \psi\epsilon \tag{13}$$

independently of t.

We now combine Lemmas 14 and 15 to prove

Lemma **16.** *There exists a date T such that for every i, $b_T(i)$ belongs to the interior of the same affine segment of μ_T; in particular, $\mu'_T(b_T(i))$ is a constant independent of i.*

Lemma **17.** *For every date $t \geq T + 1$, where T is given by Lemma 16, the wealths, investments and consumption budgets of all agents must fully coincide.*

In what follows, we consider only dates $t > T$. By Lemma 17, the equilibrium program has common values at all dates thereafter: (w_t, b_t), where all these values are strictly positive. By Proposition 1 and Corollary 1, the distribution F_t is also fully pinned down at all these dates. Denote by α_t the corresponding slopes of the affine segments of μ_t, given by (5); these too are all strictly positive.

Lemma **18.** *Suppose that for some $t \geq T + 1$, $k_t \leq k_{t+1}$ and $\alpha_t \leq \alpha_{t+1}$. Then $k_s \leq k_{s+1}$ for all $s \geq t$.*

Lemma **19.** *The common sequence of investments $\{k_t\}$, defined for $t \geq T + 1$, must converge to k^*, which solves $\delta f'(k^*) = 1$.*

The proof of Proposition 3(i) then proceeds as follows. Lemma 16 assures us that there exists a date T at which consumption budgets $b_T(i)$ belong to the same affine segment of μ_T for every i. Lemma 17 states that for every date $t \geq T + 1$, the wealths, investments and consumption budgets of all agents must fully coincide. Lemma 19 states that the common sequence of investments $\{k_t\}$, defined for $t \geq T + 1$, must converge to k^*, which solves $\delta f'(k^*) = 1$.

At the same time, Corollary 1 asserts that for all $t \geq T + 1$, the equilibrium distribution of consumptions must be the unique cdf associated with the common consumption budget b_t, where "association" is defined (and uniqueness established) in Proposition 1. Therefore the sequence of consumption distributions must converge to the unique cdf associated with $b^* = f(k^*) - k^*$. This is the unique steady state of Proposition 2, so the proof is complete.

Part (ii): Existence will follow as a corollary of Proposition 6 in the online appendix. This proves existence for a more general model than the one in the paper, in which the production function can have convex segments and there are possibly stochastic shocks to production. □

Proof of Proposition 4. For this proof, Assumptions 4–6 replace Assumptions 1–3.

Part (i). Suppose that all individuals in a set of unit measure use the policy function (6). Let $\mathbf{G} = \{G_t\}$ be the resulting sequence of wealth distributions. Clearly, for every date t and for every w in the support of G_{t+1}, $G_{t+1}(w) = G_t\left(f^{-1}(w)/\delta\right)$. We have:

LEMMA **20.** *$u(G_t(w))$ is concave for all dates t on the support of G_t.*

Fix a date t. Suppose that a particular individual employs the policy (6) for all dates $s \geq t+1$, and that every other individual employs the policy (6) at all dates. Define $V_{t+1}(w')$ to be the discounted value to our individual under these conditions, starting from wealth w' and date $t+1$. Then status at every $s \geq t+1$ is simply $\bar{F}_s(c_s) = G_{t+1}(w')$, so that

$$V_{t+1}(w') = (1-\delta)^{-1} u\left(G_{t+1}(w')\right). \tag{14}$$

Now suppose that at date t, our individual has starting wealth w, does not randomize, and chooses $k \in [0, w]$. Then her lifetime payoff at that date is given by

$$\begin{aligned} u\left(\bar{F}_t(w-k)\right) + \delta V_{t+1}\left(f(k)\right) &= u\left(\bar{F}_t(w-k)\right) + \delta(1-\delta)^{-1} u\left(G_{t+1}\left(f(k)\right)\right) \\ &= u\left(G_t\left([w-k]/(1-\delta)\right)\right) + \delta(1-\delta)^{-1} u\left(G_{t+1}\left(f(k)\right)\right) \\ &= u\left(G_t\left([w-k]/(1-\delta)\right)\right) + \delta(1-\delta)^{-1} u\left(G_t\left(k/\delta\right)\right), \end{aligned}$$

where the first equality uses (14), the second uses the fact that $\bar{F}_t(c) = G_t\left(c/(1-\delta)\right)$ for every $c \geq 0$, and the last uses $G_{t+1}(w) = G_t\left(f^{-1}(w)/\delta\right)$.

By Lemma 20, this expression is concave in both w and k so no randomization is necessary (assuming, as we do, that the stochastic outputs of investment randomizations cannot be insured). Moreover, given the concavity of $u(G_t(w))$ and the assumption that u is C^1, G_t must have left-hand and right-hand derivatives everywhere ($G_t^-(w)$ and $G_t^+(w)$ respectively), with

$$G_t^-(w) \geq G_t^+(w) \tag{15}$$

for all w. So a solution to the first-order condition

$$\begin{aligned} & -u'(r_t)G_t^+\left([w-k]/(1-\delta)\right)(1-\delta)^{-1} + \delta(1-\delta)^{-1}u'(r_{t+1})G_t^-\left(k/\delta\right)\delta^{-1} \geq 0 \\ \geq\; & -u'(r_t)G_t^-\left([w-k]/(1-\delta)\right)(1-\delta)^{-1} + \delta(1-\delta)^{-1}u'(r_{t+1})G_t^+\left(k/\delta\right)\delta^{-1} \end{aligned} \tag{16}$$

(where r_s is the resulting status in date s, for $s = t, t+1$) is an optimum. Using (15), we see that $k = \delta w$ is indeed a solution to (16), so that by the one-shot deviation principle and the fact that t and w are arbitrary, (6) is an equilibrium policy.

Part (ii).[30] Notice that each individual is atomless and therefore has the same intertemporal utility criterion as any other. Because the equilibrium is regular, we see that at any date, the solution to the optimization problem is unique except at countably many wealth levels. But it is easy to see that such a solution cannot admit more than one differentiable selection. Therefore all individuals must use the same savings policy, which we denote by $\{c_t\}$. Given

[30] We are indebted to a referee for suggesting this line of proof, which is simpler than the one we had.

this environment, let $V_t(w)$ be the (lifetime) value to a person with wealth w at date t. By using exactly the same steps as in Part (i), we see that for every w, $c = c_t(w)$ must maximize

$$u\left(G_t\left(c_t^{-1}(c)\right)\right) + \delta(1-\delta)^{-1} u\left(G_t\left(s_t^{-1}(w-c)\right)\right), \tag{17}$$

where $s_t(w) \equiv w - c_t(w)$ is also strictly increasing and differentiable, by regularity. By Lemma 1, $u(F_t(c))$ is concave in c, and so F_t is differentiable almost everywhere. Consequently, because $G_t(w)) = F_t(c_t(w))$ and c_t is differentiable and strictly increasing, G_t is also differentiable at a.e. w. Using the fact that optimal c and $w - c$ are both strictly increasing in w, we may therefore differentiate the expression (17) with respect to c at almost every w, set the resulting expression equal to zero (it is the first-order condition) and cancel common terms all evaluated at the same rank or same wealth to obtain

$$\frac{1}{c_t'(w)} = \frac{\delta}{1-\delta}\frac{1}{s_t'(w)} = \frac{\delta}{1-\delta}\frac{1}{1-c_t'(w)},$$

or $c_t'(w) = (1-\delta)$ for every t and for a.e. w. This completes the proof of the proposition. □

Proof of Proposition 5. We now revert to Assumptions 1–3 instead of Assumptions 4–6. Consider the steady state F^*; we may equivalently express it as a mapping from realized status $s \in [0,1]$ to realized consumption $c^*(s)$ at status s, given by $c^*(s) = (F^*)^{-1}(s)$. If the outcome is Pareto-efficient, that mapping must maximize the integral

$$\int u(c(s), s) ds$$

over all continuous and increasing functions c on $[0,1]$ with $\int c(s)ds = b$. But it is easy to see that a necessary condition for such maximization is that $u_c(c^*(s), s)$ is constant as s varies over $[0,1]$, or equivalently, that

$$u_c(c, F^*(c)) = \lambda \text{ for some } \lambda > 0, \tag{18}$$

for all $c \in [a, d]$. Now, recall from (5) that

$$u(c, F^*(c)) = u(a, 0) + \alpha[c - a]$$

for all $c \in [a, d]$. Because $\alpha = u_c(c, 0)$, it follows that $F^{*\prime}(a) = 0$. Consequently,

$$\frac{du_c(c, F^*(c))}{dc}|_{c=a} = u_{cc}(a, F^*(a)) + u_{cs}(a, F^*(a))F^{*\prime}(a) = u_{cc}(a, F^*(a)) < 0,$$

which contradicts (18). □