MAT335S - Chaos, Fractals and Dynamics

Department of Mathematics, University of Toronto
Winter 2000

Instructor: R. Pyke (office SS4083, 978-4736, pyke@math.toronto.edu)
Text: *Chaos and Fractals: New Frontiers of Science*, H-O Peitgen, H. Jürgens, D. Saupe.
Course web page: http://www.math.toronto.edu/ pyke/mat335

Please check the course web page regularly! You will find weekly comments about the course material, the homeworks, answers to student's questions (submit to me via e-mail), resources, and more.

References (On reserve in the Gerstein Science Information Centre, Short Term Loan)

- *A First Course in Chaotic Dynamical Systems*, R. Devaney. (A standard introductory text.)
- *The Fractal Geometry of Nature*, B. Mandelbrot. (Fractals in mathematics and nature, with applications, history, anecdotes, etc, written by the man who invented the 'fractal'. You must have a look at this book!)
- *Chaos and Fractals: New Frontiers of Science*, H-O. Peitgen, H. Jürgens, D. Saupe. (A copy of this book is also on reserve in the mathematics library, SS622.)

Some popular books

For a general discussion about the history behind 'chaos theory', the scientific ideas and the people involved, I would recommend the following books.

- *Chaos: Making a New Science*, J. Gleick. (A journalist's lucid description of the main discoveries and contributors to 'chaos theory'. No mathematics here though.)
- *Does God Play Dice?*, I. Stewart. (A mathematician's (entertaining) survey of the historical development of the scientific ideas that lead to 'chaos theory', along with accessible explanations of the mathematics involved.)

Videos

The audio visual library at Gerstein (one floor down) has several videos about chaos and fractals (just type in the key words 'chaos' or 'fractal' on the on-line catalogue). At some point during the course we will watch one of these in class; "Fractals: An Animated Discussion", call #002948.

Web sites

There are many web sites devoted to chaos and fractals, so there's much to explore here (for example, fractal music; university web sites describing the research of 'dynamical systems' groups working in mathematics, physics, engineering and the life sciences; interactive sites, etc). You can start with the links listed in the course web page (and the links within these links, ...).

Prerequisites

Students should have second year calculus (can be taken concurrently) and a course in linear algebra. Differential equations and complex numbers will come up, but you are not expected to have studied these before.

Marking scheme :

Final exam (3 hrs, to be scheduled at the end of term)	50% of final mark
2, 1-hour Mid-term tests (weeks of 7 Feb and 20 Mar)	30% of final mark
Homework (approximately 5)	20% of final mark

Course Outline

- **Part I - Fractals**:
  - Self-similarity, examples of classical fractals (Ch 2,3)
  - Fractal dimension (Ch 4)
  - Drawing fractals: Iterated Function Systems (Ch 5), the Chaos Game (Ch 6)
  - Fractals in nature: plants, landscapes, random fractals (Ch 7,9)

- **Part II - Dynamics and Chaos**:
  - Discrete dynamical systems (iteration of functions): random versus deterministic systems, graphical analysis, invariant sets, attracting sets, fixed points, periodic points, analyzing deterministic chaos (Ch 10)
  - Symbolic dynamics
  - Charkovsky's Theorem, bifurcations and the route to chaos, Fiegenbaum final state diagram (Ch 11)
  - Strange Attractors, continuous dynamical systems (differential equations) (Ch 12)

- **Part III - Complex Dynamics**:
  - Complex numbers (§13.2)
  - Julia sets (Ch 13)
  - The Mandelbrot set (Ch 14)