Modelling Forest Fires with Percolation

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Suppose we have a lattice (eg., $\mathbb{Z}^n$ or some subset thereof) where each lattice site is either occupied or not depending on a probability p. A *cluster* is a group of neighboring occupying sites. Percolation theory deals with the number and properties of these clusters.

Now suppose the occupied sites evolve (in discrete time steps) according to certain rules which depend (among other things) on the properties of the neighboring sites, ambient properties of the lattice at that location, and perhaps time-dependent effects. The evolution of the system will vary according to the initial distribution of occupied sites.

Examples of phenomenon that can be modelled by this method are forest fires and epidemics. In the case of forest fires each site corresponds to a location of a possible tree, and in the case of epidemics each site corresponds to a person or community of people. After the sites are populated, a fire is initiated. A critical probability p_c exists at which percolation is possible for $p \geq p_c$; the entire forest is burned or the entire population becomes infected. Another way to say this is that as p increases, the maximum cluster size (connected occupied sites) increases and becomes infinite at p_c. The duration of the forest fire (or epidemic) peaks at the critical probability, and is more pronounced as the lattice size increases. This effect is referred to as a phase transition in physics.

Our task will be to develop a model for forest fires that includes features such as topography, wind and fuel. Through simulation we will examine various evolutions and analyze how the parameters of the model affect the behaviour (such as length of duration of the fire). We will then use this model to investigate fire fighting/prevention strategies.

References:

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