Using Mathematics to Solve Real World Problems

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"Operations Research" is the profession that applies mathematical methods like this to problems arising in industry, healthcare, finance, etc.

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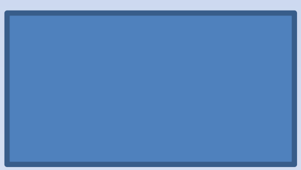
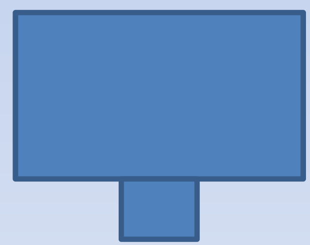
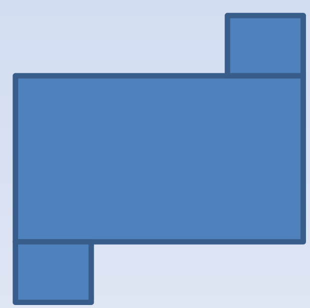


large block

small block



table

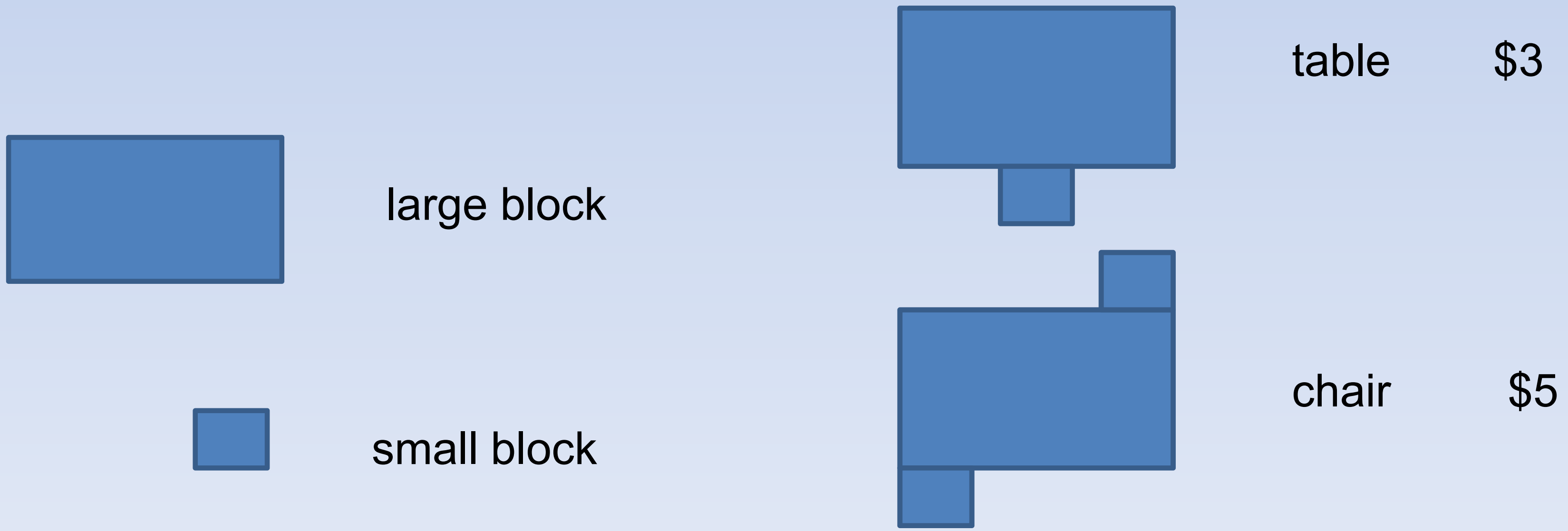


chair

A problem:

A furniture manufacturer produces two sizes of boxes (large, small) that are used to make either a table or a chair.

A table makes $3 profit and a chair makes $5 profit.

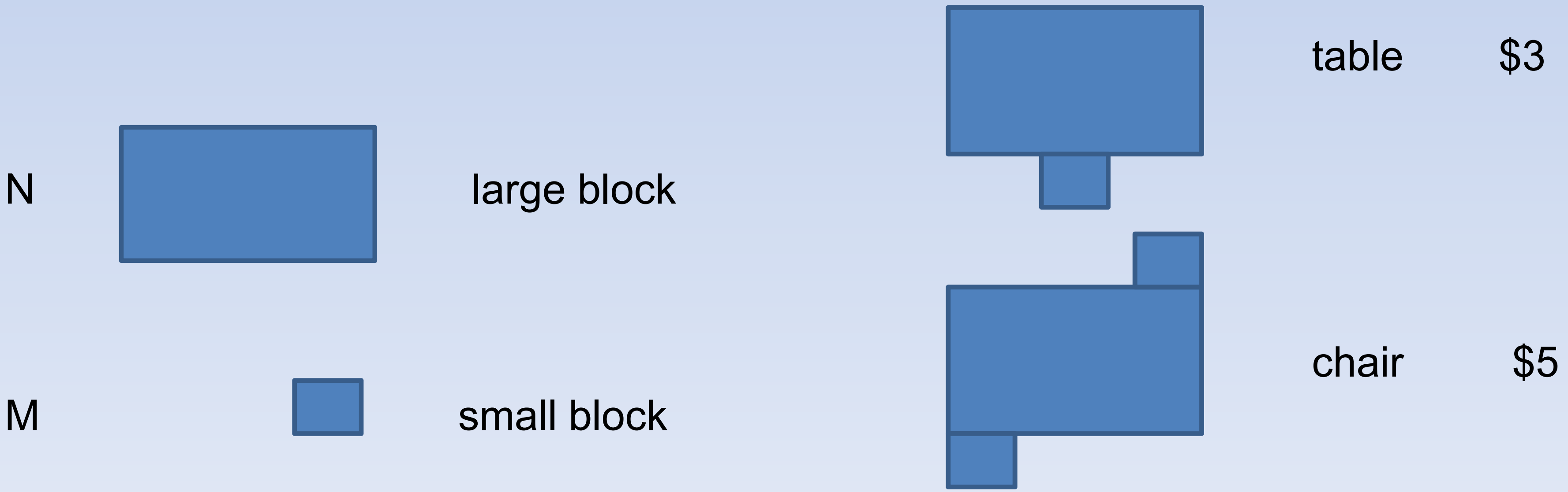


A problem:

A furniture manufacturer produces two sizes of boxes (large, small) that are used to make either a table or a chair.

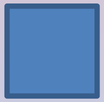
A table makes \$3 profit and a chair makes \$5 profit.

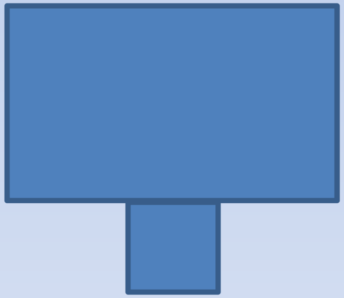
If M small blocks and N large blocks are produced, how many tables and chairs should the manufacturer make in order to obtain the greatest profit?



Large block

Small block





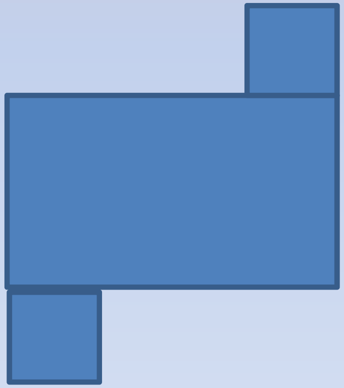
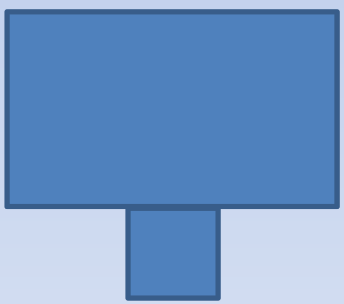


Table
1 large block
1 small block

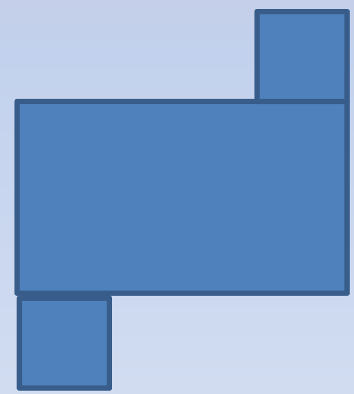
Chair
1 large block
2 small blocks

Problem: Given **M** small blocks and **N** large blocks, how many tables and chairs should we make to obtain the most profit?

Profit: **$3** **$5**



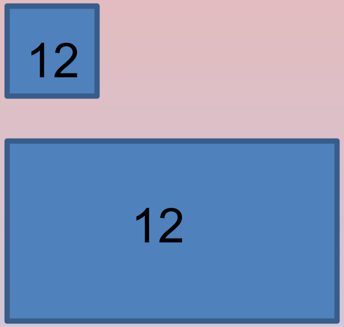
Table



Chair

Example: 12 small blocks

and 12 large blocks



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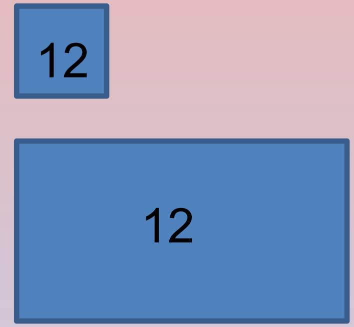
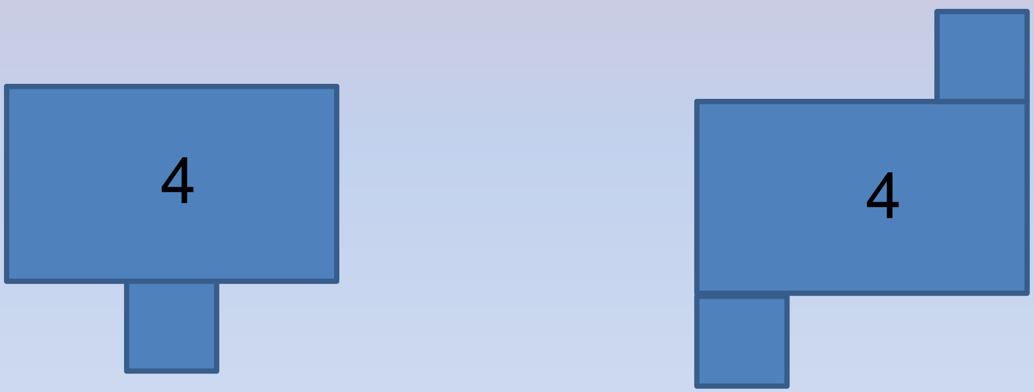


We can make 4 tables and 4 chairs:



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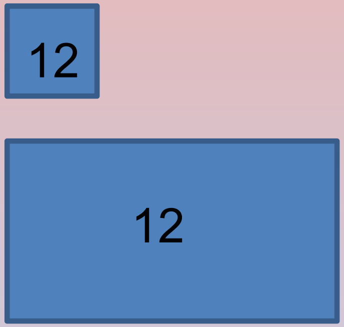

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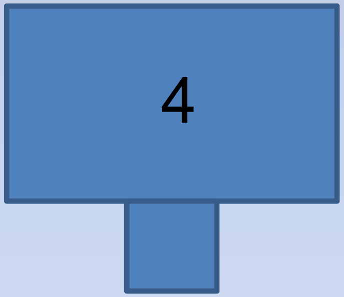
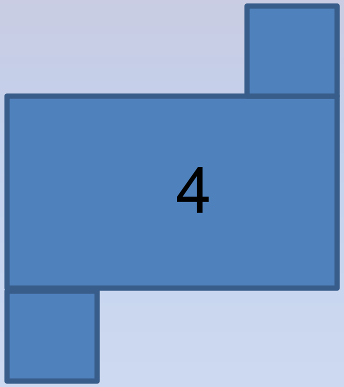




4 large
4 small

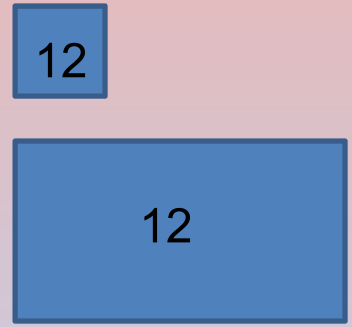
+

4 large
8 small

=

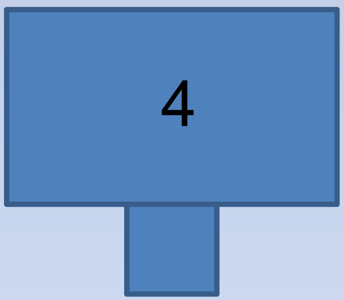
8 large
<u>12 small</u>

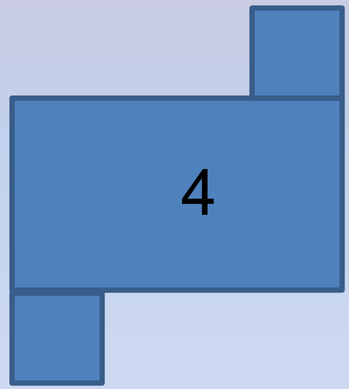
Example: 12 small blocks



and 12 large blocks

We can make 4 tables and 4 chairs:





4 large
4 small

+

4 large
8 small

=

8 large
<u>12 small</u>

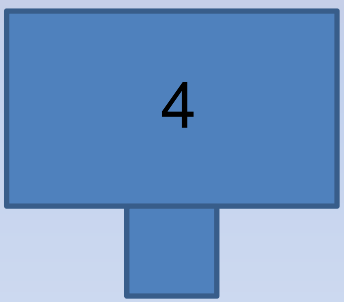
Profit = ($3) x 4 + ($5) x 4 = $32

12 small blocks

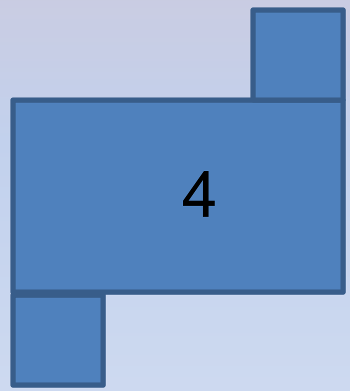
12 large blocks

4 tables and 4 chairs:

$3



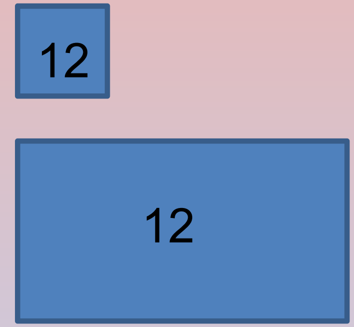
$5



Used:
8 large blocks
12 small blocks
(4 large blocks left)

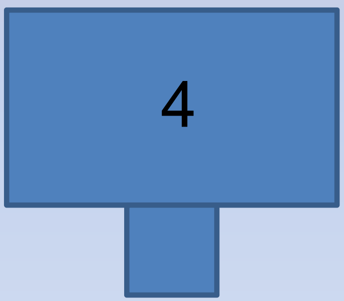
12 small blocks

12 large blocks

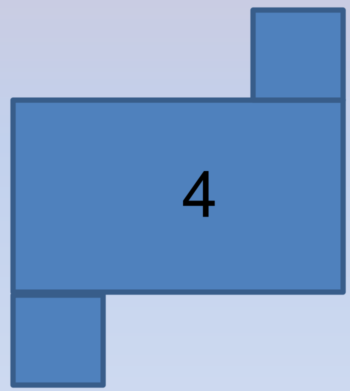


4 tables and 4 chairs:

$3



$5



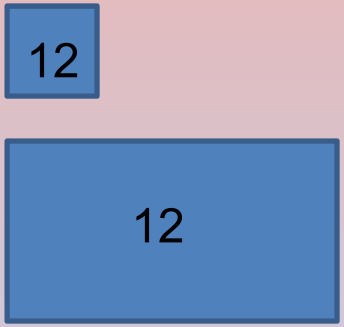
Used:
8 large blocks
<u>12</u> small blocks
(4 large blocks left)

I can make 2 more tables if I make 1 less chair; **3 chairs and 6 tables** → increase my profit! (1 chair → 2 tables, profit goes up by $1)

Profit = ($3) x 6 + ($5) x 3 = $33

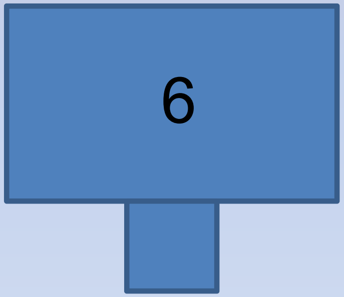
12 small blocks

12 large blocks

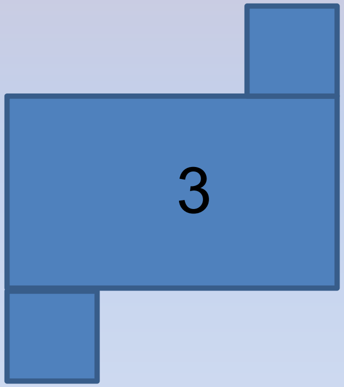


6 tables and 3 chairs:

$3



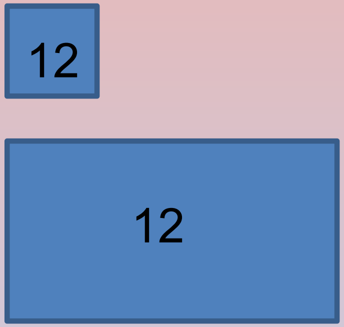
$5



Used:
9 large blocks
<u>12</u> small blocks
(3 large blocks left)

12 small blocks

12 large blocks



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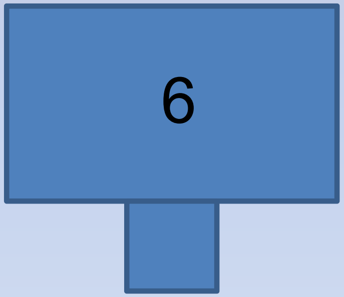

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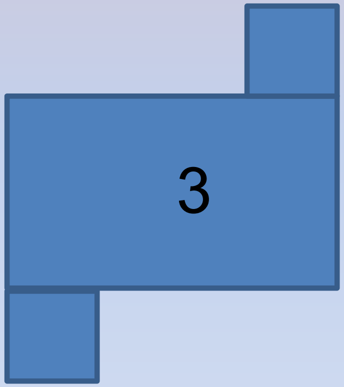


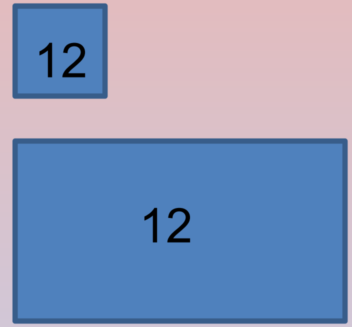
Used:
9 large blocks
12 small blocks
(3 large blocks left)

I can do it again; change one chair into 2 tables; **8 tables and 2 chairs**

Profit = ($3) x 8 + ($5) x 2 = $34

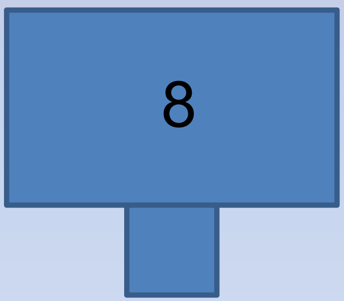
12 small blocks

12 large blocks

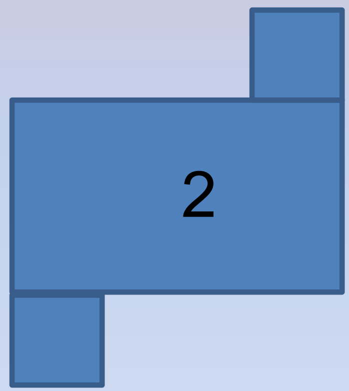


8 tables and 2 chairs:

$3



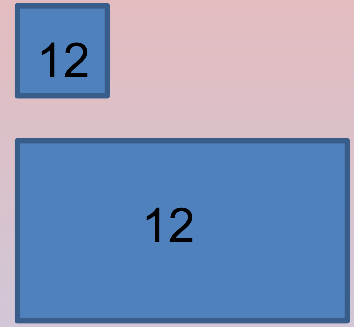
$5



Used:
10 large blocks
<u>12</u> small blocks
(2 large blocks left)

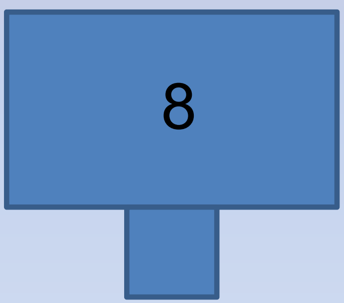
12 small blocks

12 large blocks

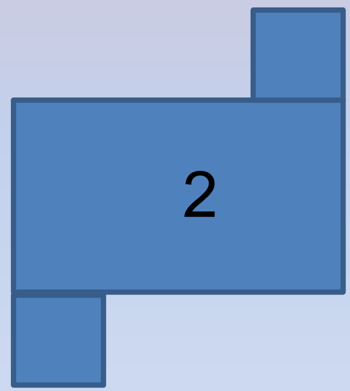


8 tables and 2 chairs:

$3



$5



Used:
10 large blocks
12 small blocks
(2 large blocks left)

I can do it again; change one chair into 2 tables; **10 tables and 1 chair**

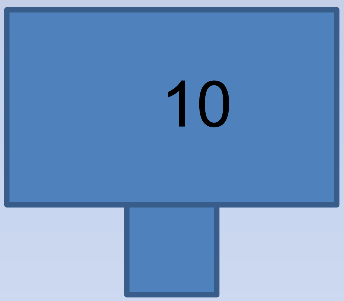
Profit = ($3) x 10 + ($5) x 1 = $35

12 small blocks

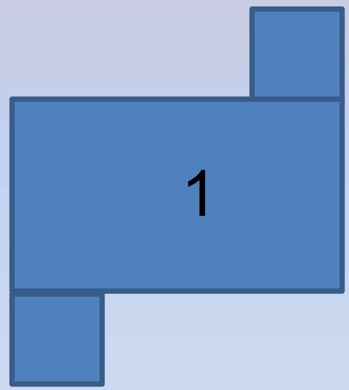
12 large blocks

10 tables and 1 chair:

$3



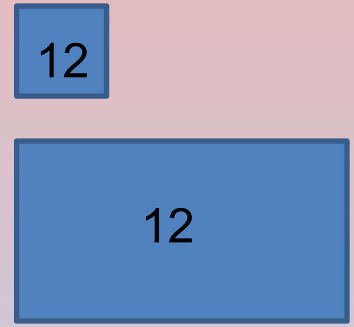
$5



Used:
11 large blocks
<u>**12**</u> small blocks
(1 large block left)

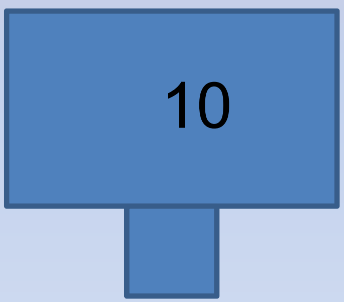
12 small blocks

12 large blocks

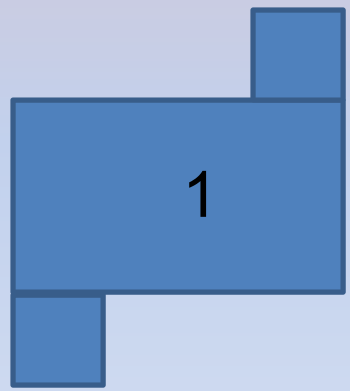


10 tables and 1 chair:

$3



$5



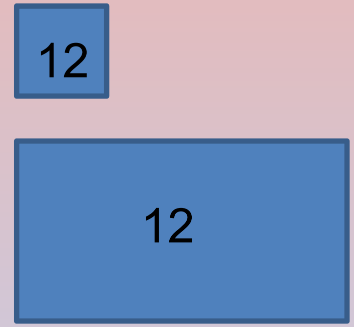
Used:
11 large blocks
<u>12 </u>small blocks
(1 large block left)

I can do it again; change one chair into 2 tables; **12 tables and 0 chairs**

Profit = ($3) x 12 + ($5) x 0 = $36

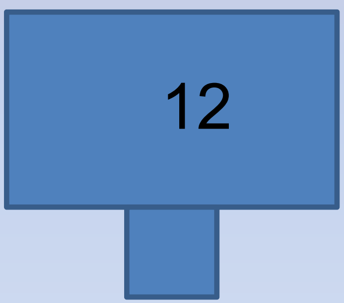
12 small blocks

12 large blocks



12 tables and 0 chairs:

$3



Profit = $36

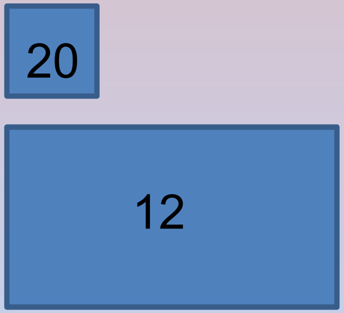
Used:
12 large blocks
12 small blocks
(no blocks left)

Is this the best?

<u>Now you try:</u>

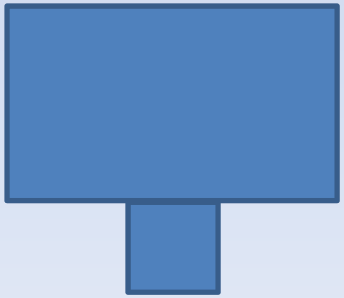
20 small blocks

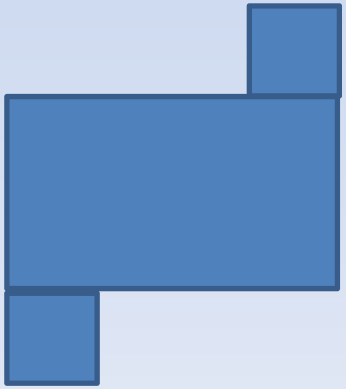
12 large blocks



How many tables and chairs?

$3



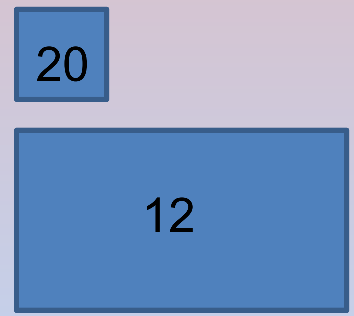


$5

Another:

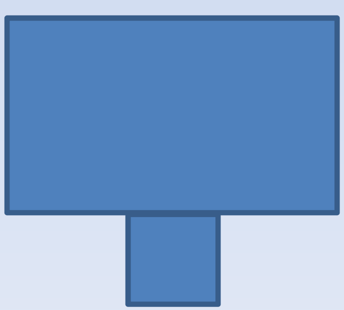
25 small blocks

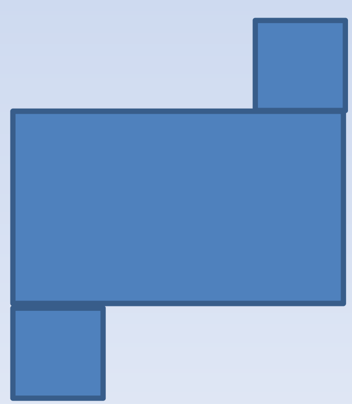
12 large blocks



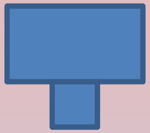
How many tables and chairs?

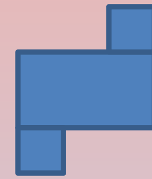
$3





$5

table  $3

chair  $5

small = 12
large = 12

tables	0	2	3	4	6	8	10	11	12
chairs	6	5	4	4	3	2	1	0	0
profit	30	31	29	32	33	34	35	33	36

Not many small blocks

small = 20
large = 12

tables	2	4	5	6	7	8	10	11	12
chairs	9	8	7	6	5	4	2	1	0
profit	51	52	50	48	46	44	40	38	36

Not many small blocks

small = 25
large = 12

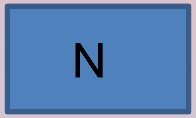
tables	0	4	5	6	7	8	10	11	12
chairs	12	8	7	6	5	4	2	1	0
profit	60	52	50	48	46	44	40	38	36

Many small blocks

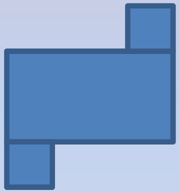
Summary

M = # small blocks

N = # large blocks



Table

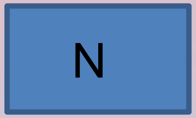


Chair

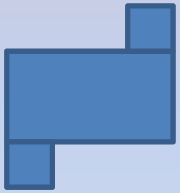
Two cases:

M = # small blocks

N = # large blocks



Table



Chair

Two cases:

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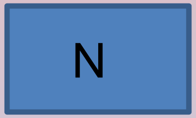
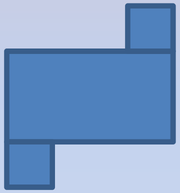
N = # large blocks



Table



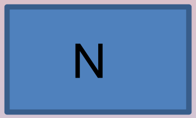
Chair

1. <u>Many small blocks</u>: **M >= 2N**

Two cases:

M = # small blocks

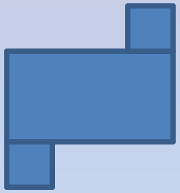
N = # large blocks



1. <u>Many small blocks</u>: **M >= 2N**

→ make N chairs, 0 tables

Table



Chair

Two cases:

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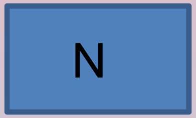

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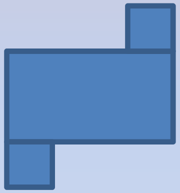


Chair

2. <u>Not many small blocks</u>: **M < 2N**

Two cases:

1. <u>Many small blocks</u>: **M >= 2N**

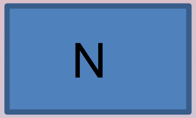
   → make N chairs, 0 tables

2. <u>Not many small blocks</u>: **M < 2N**

   → mixture of tables and chairs . . . .

M = # small blocks

N = # large blocks



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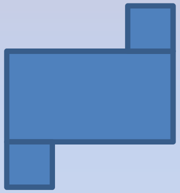

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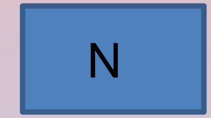

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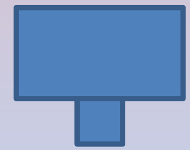

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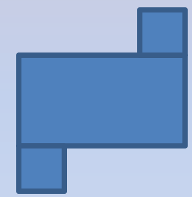

Chair

2. <u>Not many small blocks</u>: **M < 2N**

→ mixture of tables and chairs . . . .

What is the magic number of tables and chairs??

Two cases:

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   → make N chairs, 0 tables

2. <u>Not many small blocks</u>: **M < 2N**

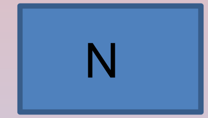
   → mixture of tables and chairs . . . .

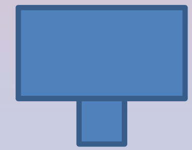
What is the magic number of tables and chairs??

Let's make a mathematical model to find out

M = # small blocks

N = # large blocks





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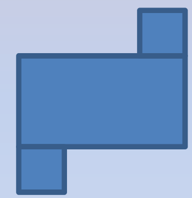

Chair

M = # small blocks

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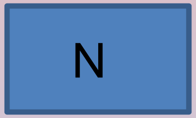

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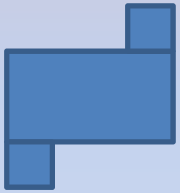


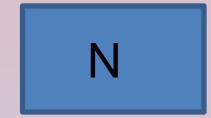
Chair

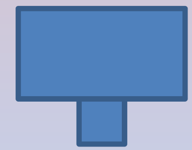
X = # tables built
Y = # chairs built

Our variables

M = # small blocks

N = # large blocks





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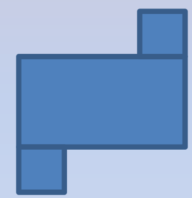


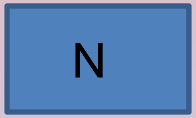
Chair

X = # tables built
Y = # chairs built

How many can I build?

M = # small blocks

N = # large blocks



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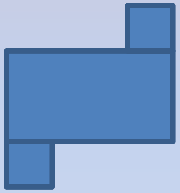


Chair

X = # tables built
Y = # chairs built

How many can I build?

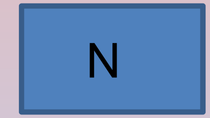
X + 2Y <= M; I have only M small blocks

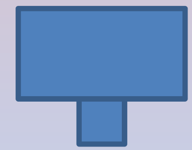
X + Y <= N; I have only N large blocks

Our equations

M = # small blocks

N = # large blocks





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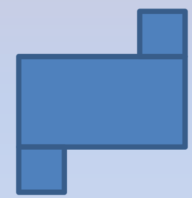

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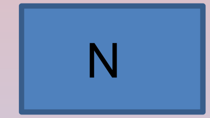

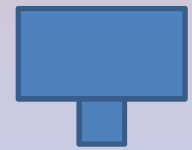

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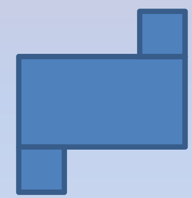


Chair

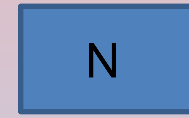
Our equations

(Note: these are inequalities, not equalities!)

X = # tables built
Y = # chairs built

M = # small blocks

N = # large blocks



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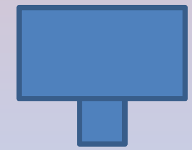

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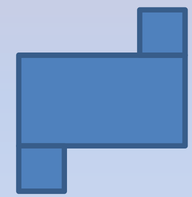

Chair

Let's plot all possible choices for X and Y for a given M and N, and then we'll pick the X,Y that gives the greatest profit.

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Y = # chairs built

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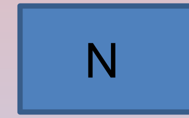

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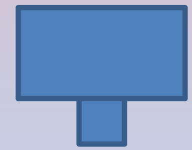

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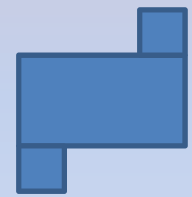

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Solving our equations . . .

Intermission: A primer on linear equations

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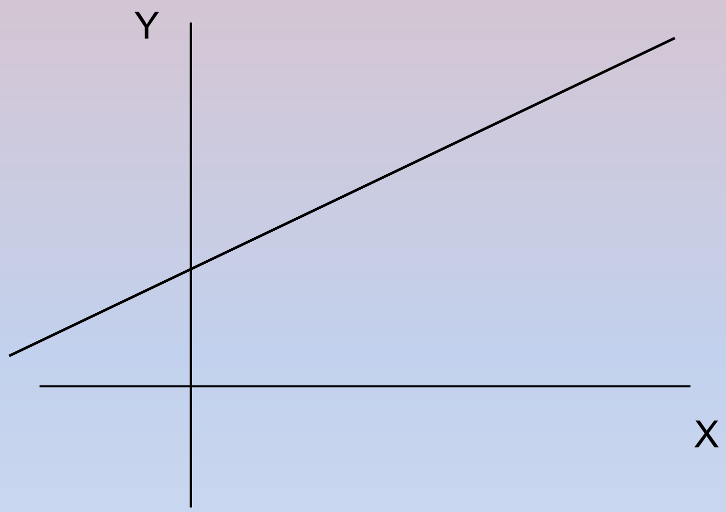
A linear equation: $\underline{a\mathbf{X} + b\mathbf{Y} = c}$, or $\underline{\mathbf{Y} = m\mathbf{X} + b}$, a,b,c,m numbers, **X,Y** variables

Intermission: A primer on linear equations

A linear equation: <u>a**X** + c**Y** = d</u>, or <u>**Y** = m**X** + b</u>, a,b,c,d,m numbers, **X,Y** variables

If we plot all the X,Y that satisfy a linear equation, it forms a line:



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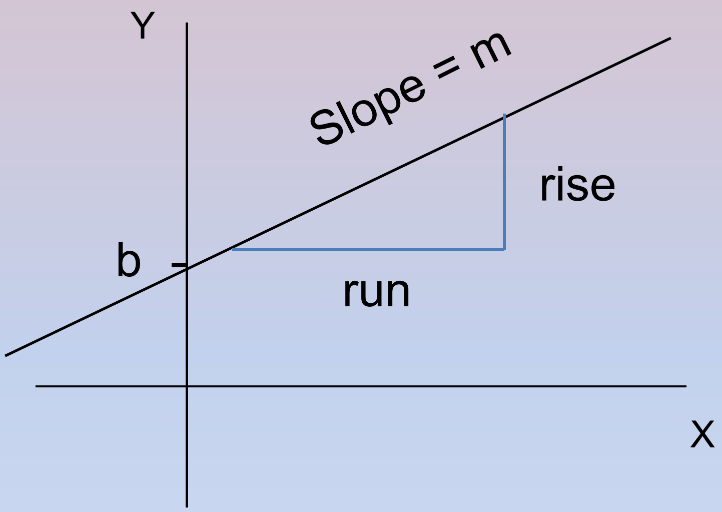

Slope of a line: $\frac{\text{rise}}{\text{run}}$ = m

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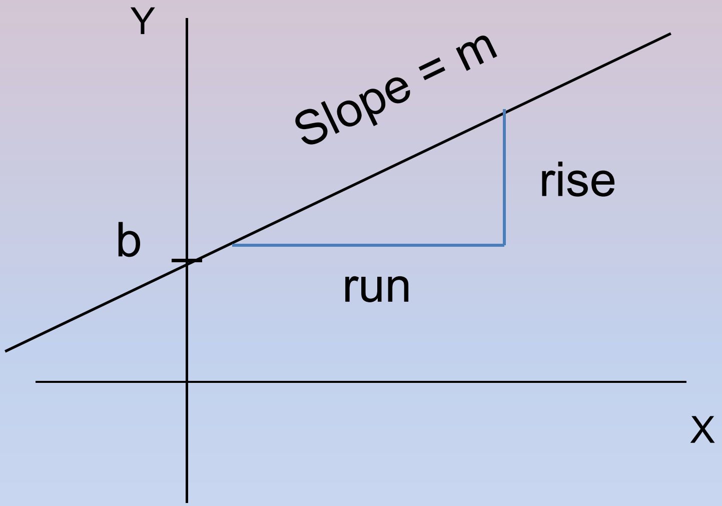

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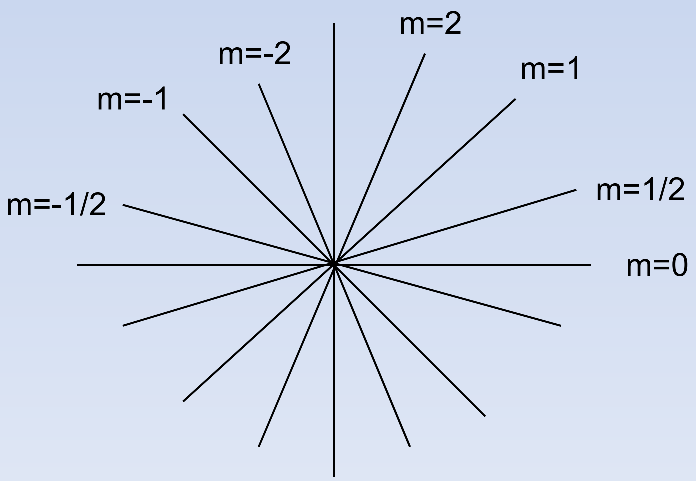

Back to our problem . . .

Let's say M = 12, N = 12

$X + 2Y <= 12$	→	$Y = -(1/2) X + 6$	slope = -1/2
$X + Y <= 12$	→	$Y = -(1) X + 12$	slope = -1

X = # tables
Y = # chairs
M = # small blocks
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We want to find all possible X and Y that satisfy these two equations.

Let's say M = 12, N = 12

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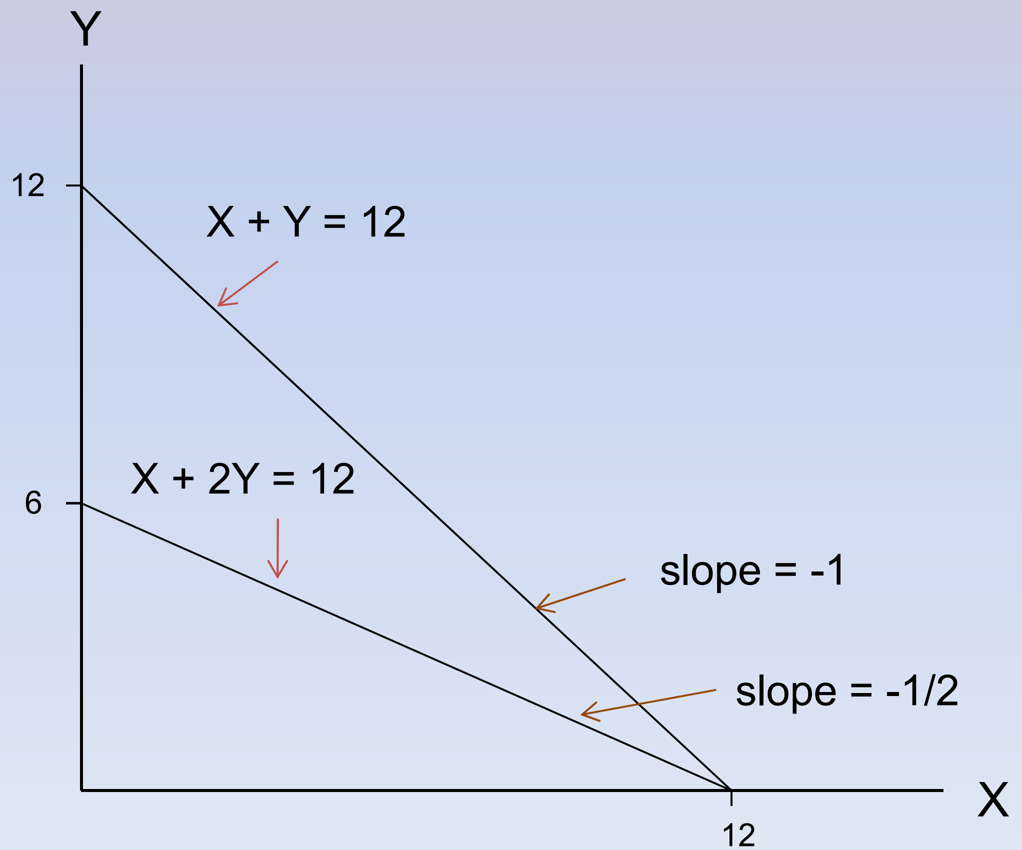
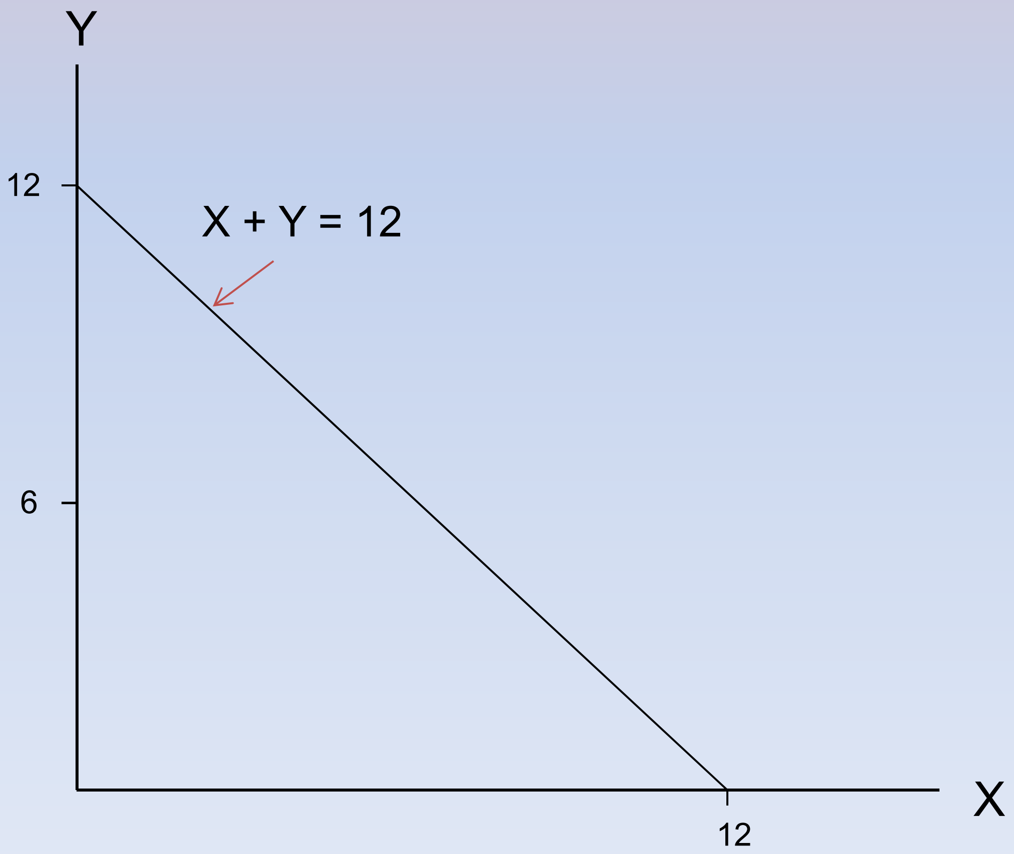
We want to find all possible X and Y that satisfy these two equations.
First draw the equality lines;



Now the inequalities:

X + Y <= 12   which side of the line is this region?

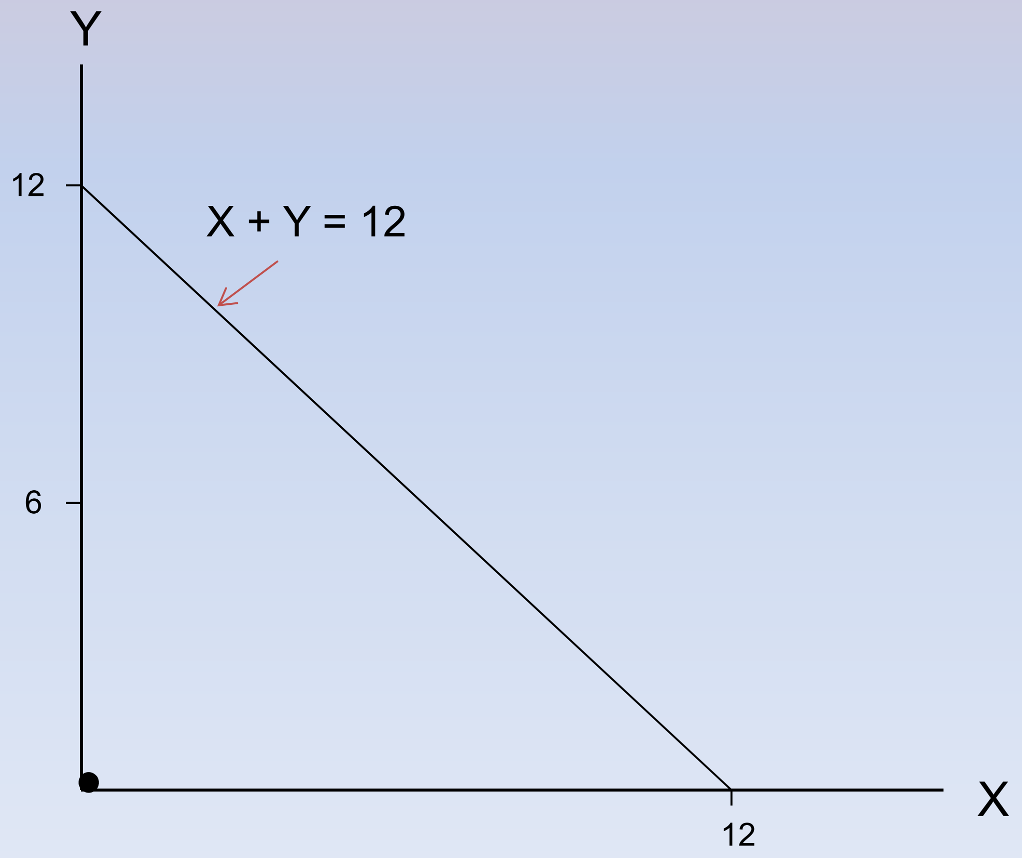


Now the inequalities:

| X + Y <= 12 |

which side of the line is this region?

Let's check one point: Is (0,0) in this region?



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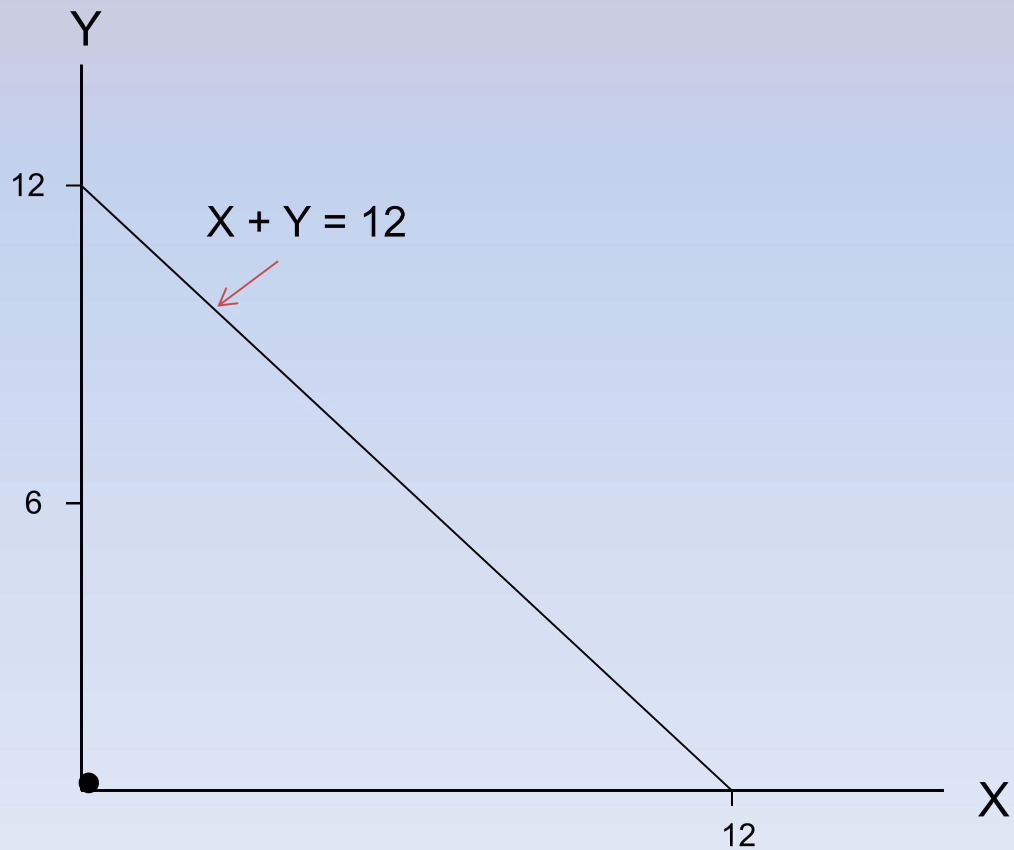

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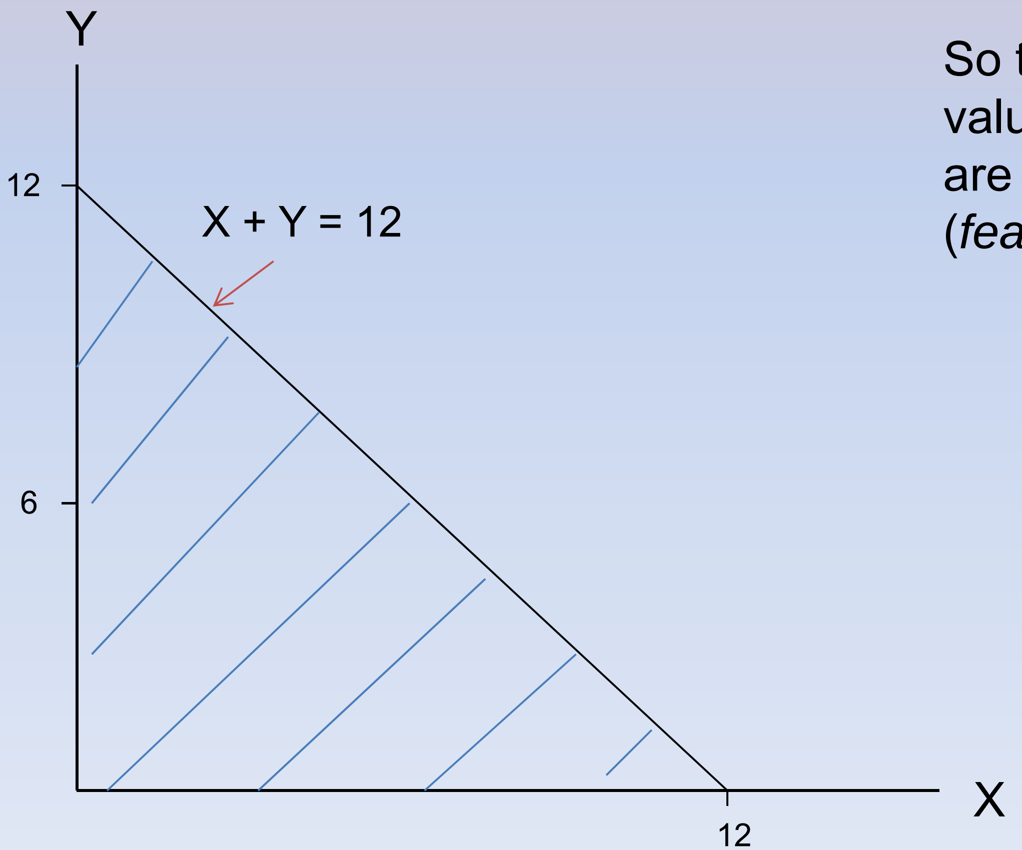
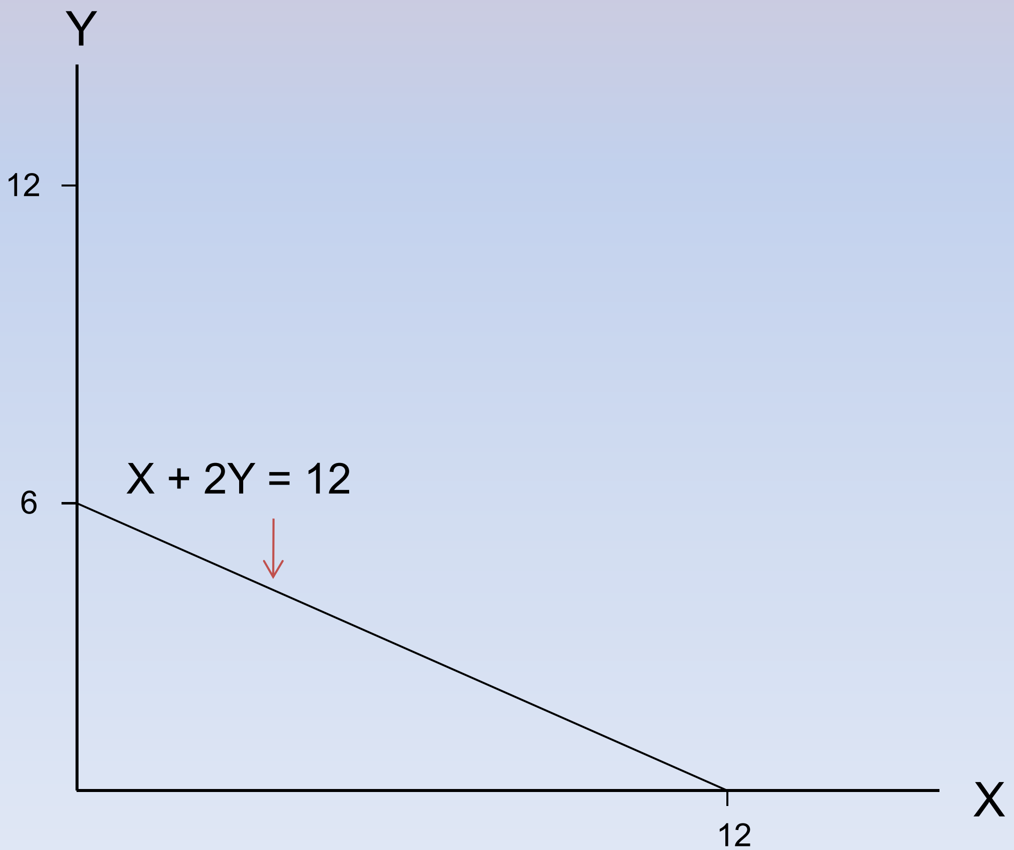
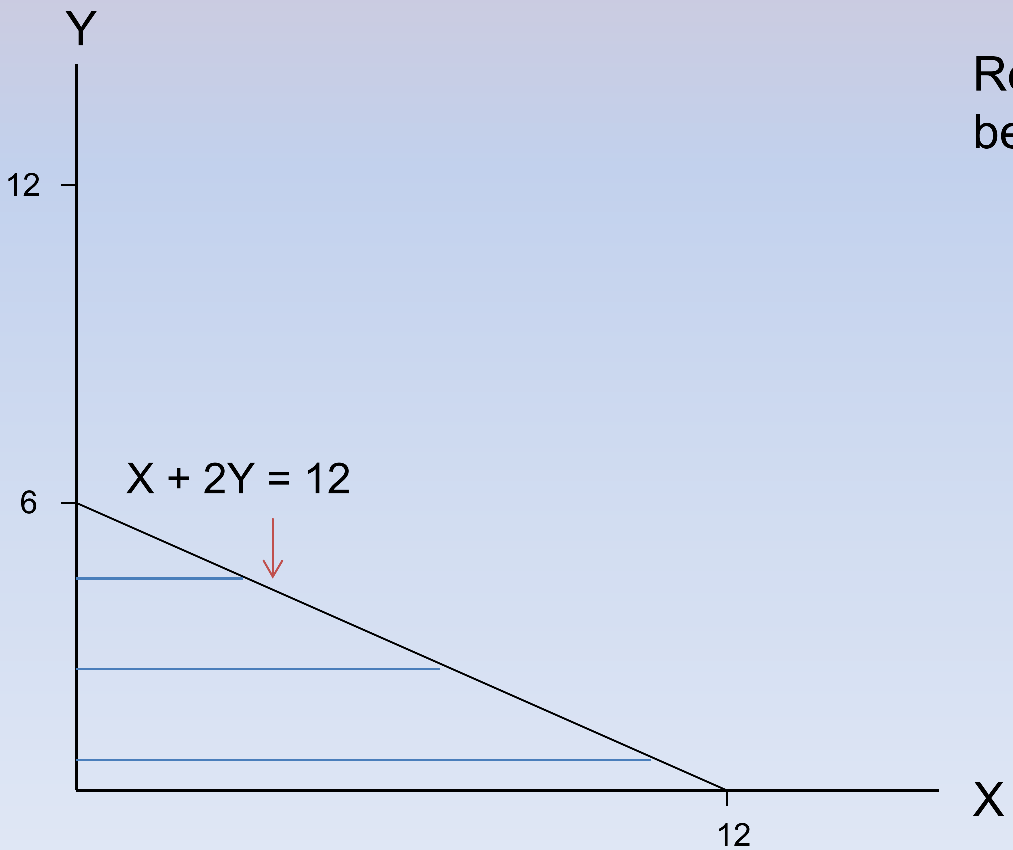


So the allowed values of X and Y are <u>below</u> this line (*feasible* points)

$X + 2Y <= 12$ which side of the line is this region?



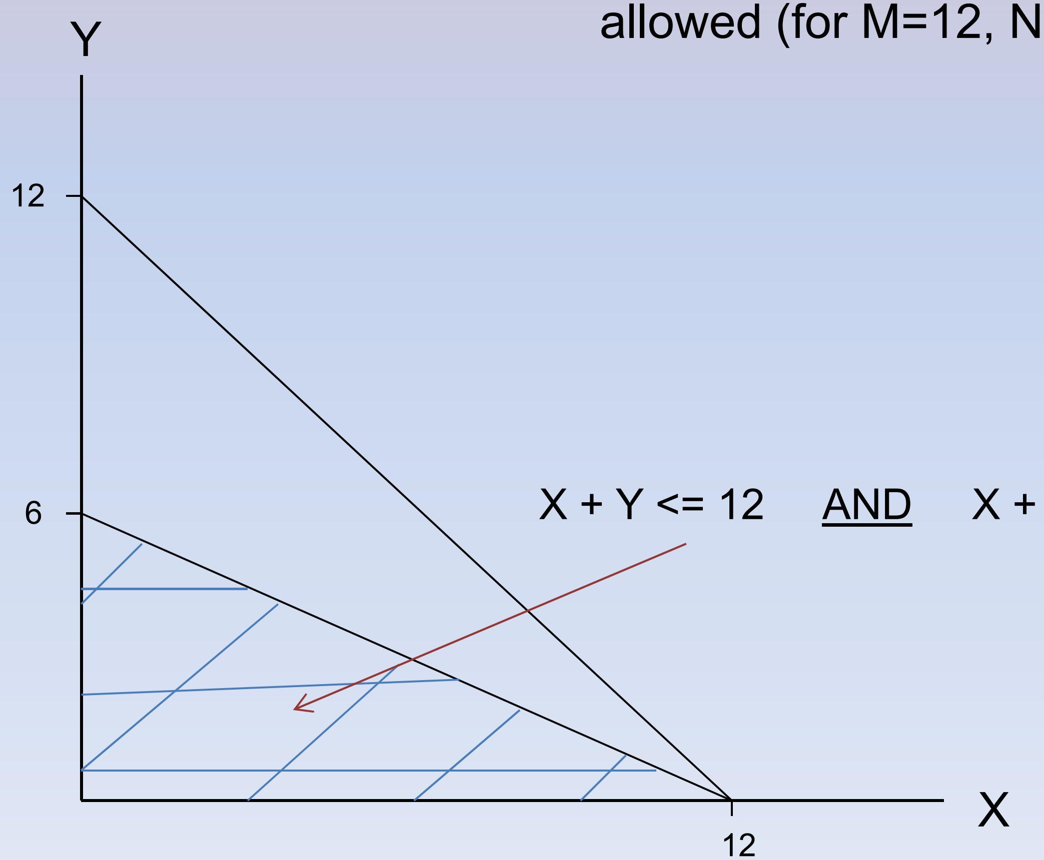
X + 2Y <= 12   which side of the line is this region?



Region is also below the line

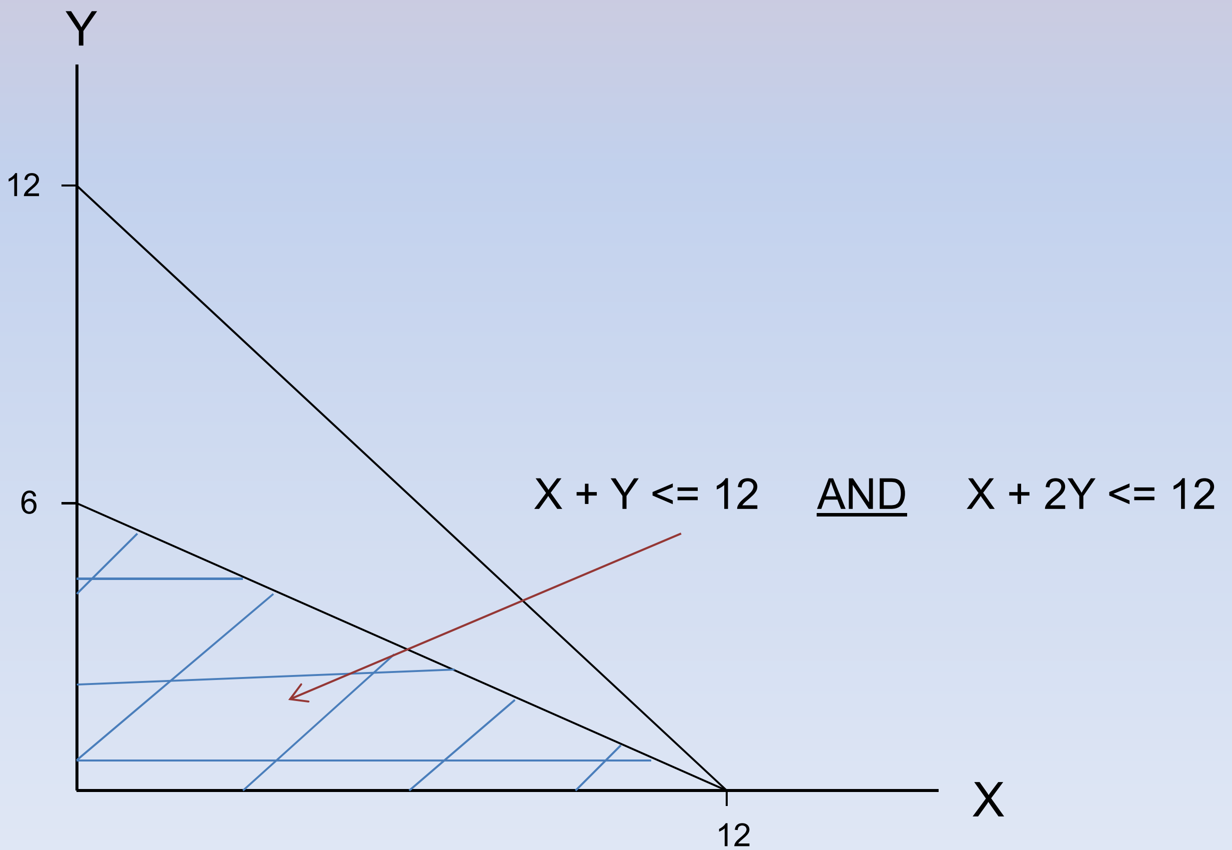
So the allowed region for both inequalities is the common region (the intersection)

Only X and Y in this region are allowed (for M=12, N=12)



Now, for X and Y in this region, which one gives the highest profit?

Profit; **P = 3X + 5Y** → Y = -(3/5)X + P/5, slope = -3/5



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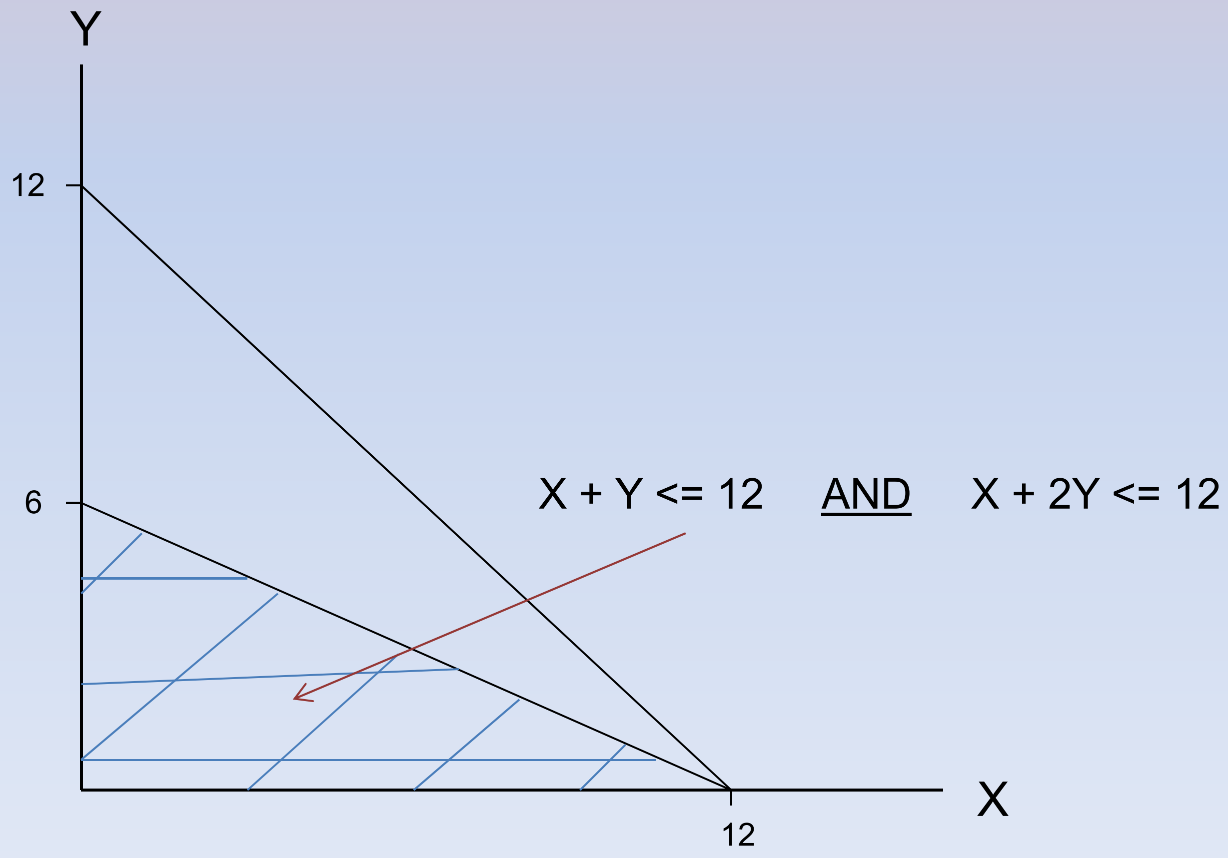
Let's plot the profit line P = 3X + 5Y



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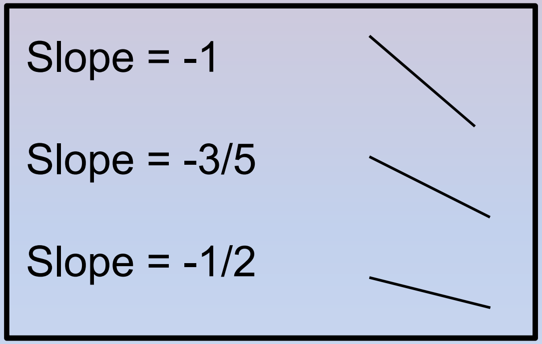
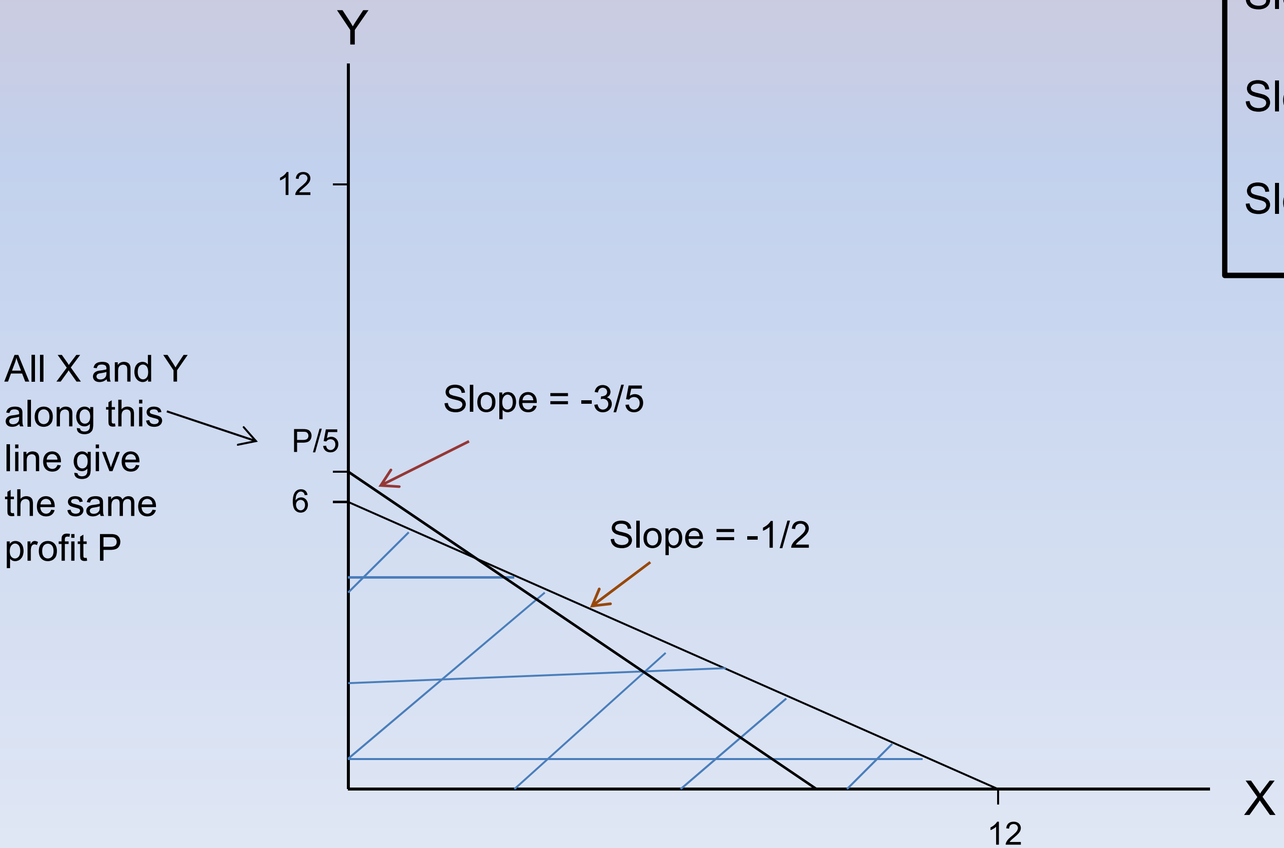
Along the profit line, all (X,Y) give the same profit P





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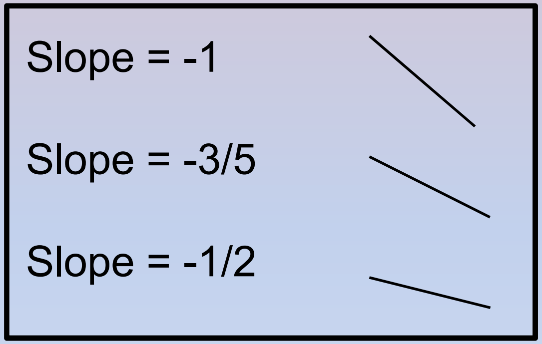

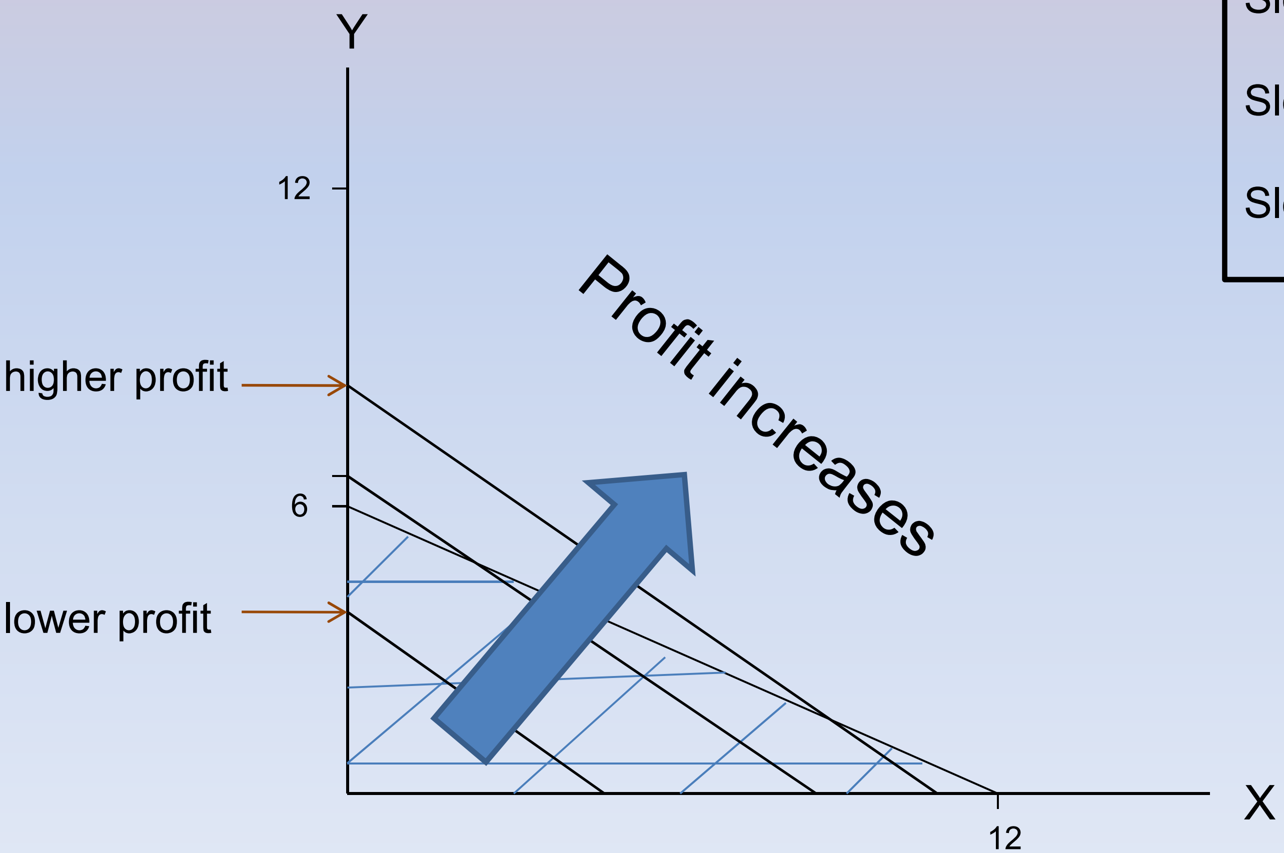

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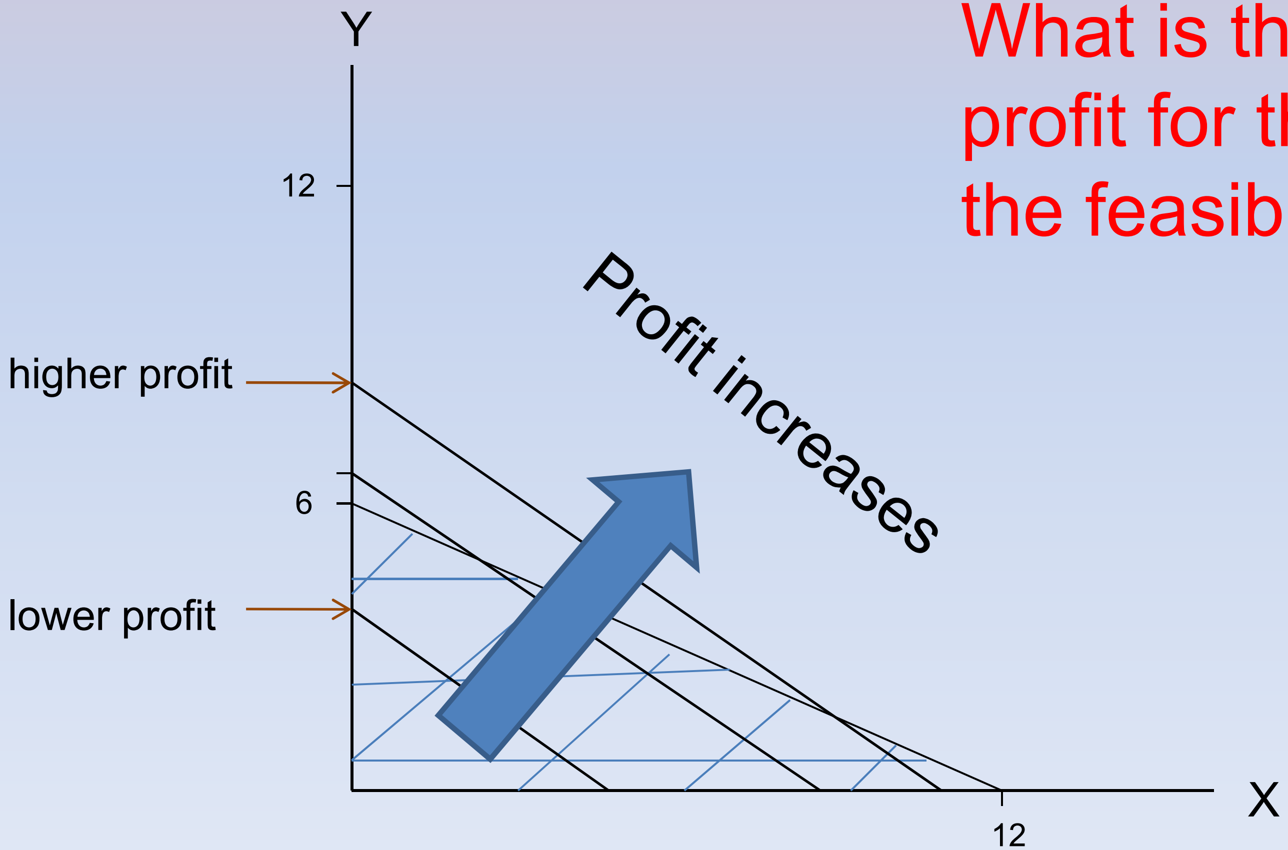
What is the highest profit for those X,Y in the feasible region?



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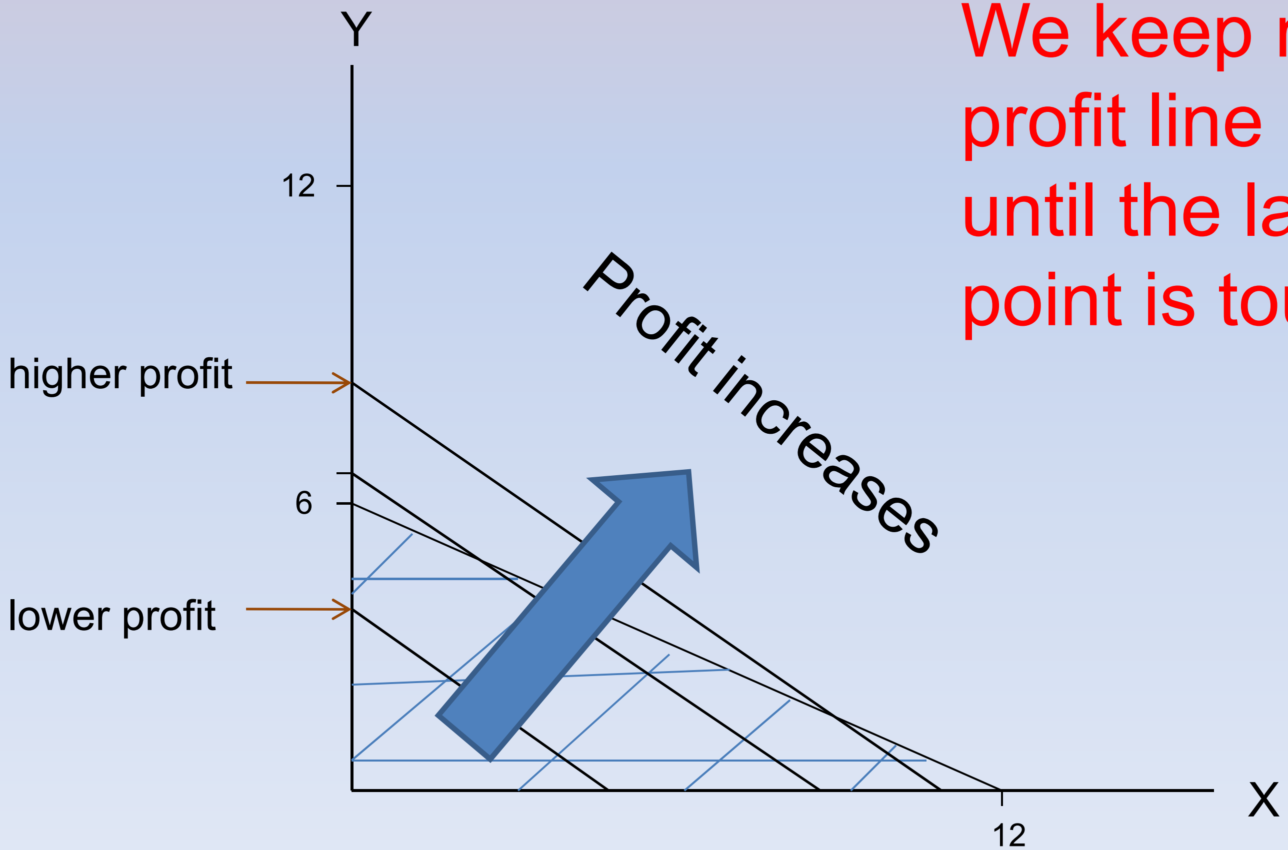

We keep moving the profit line upwards until the last feasible point is touched

Now, for X and Y in this region, which one gives the highest profit?

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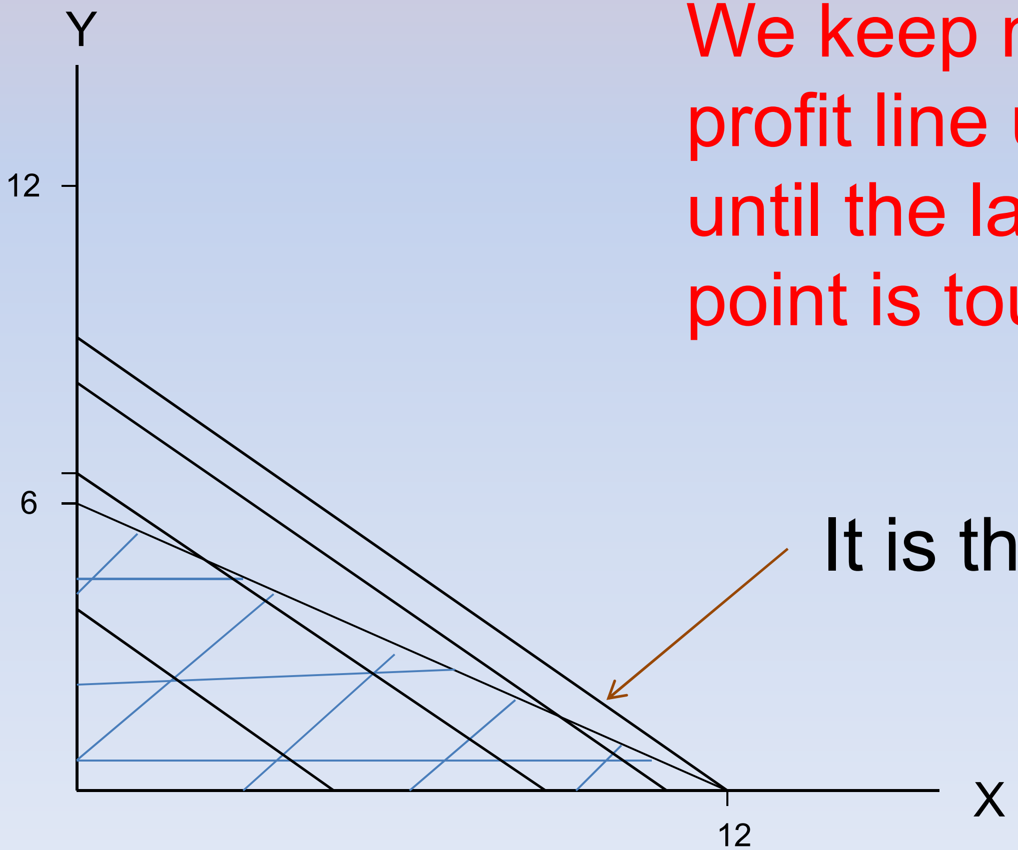

We keep moving the profit line upwards until the last feasible point is touched

It is this line!

Now, for X and Y in this region, which one gives the highest profit?

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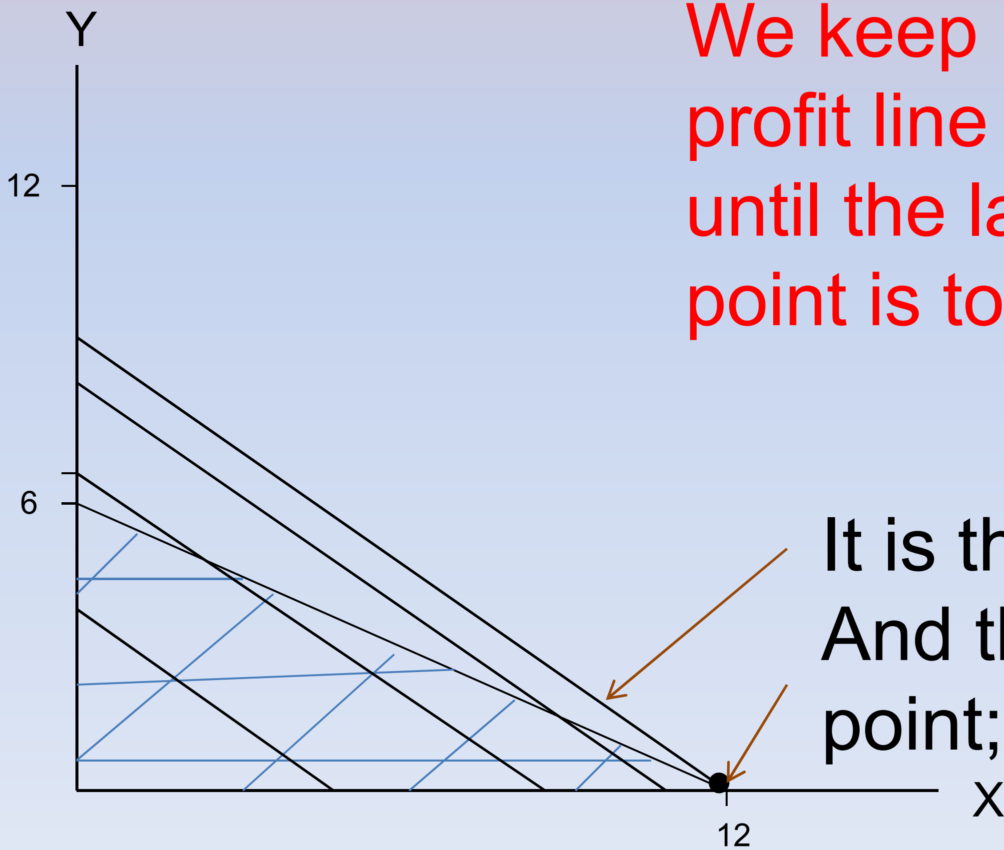
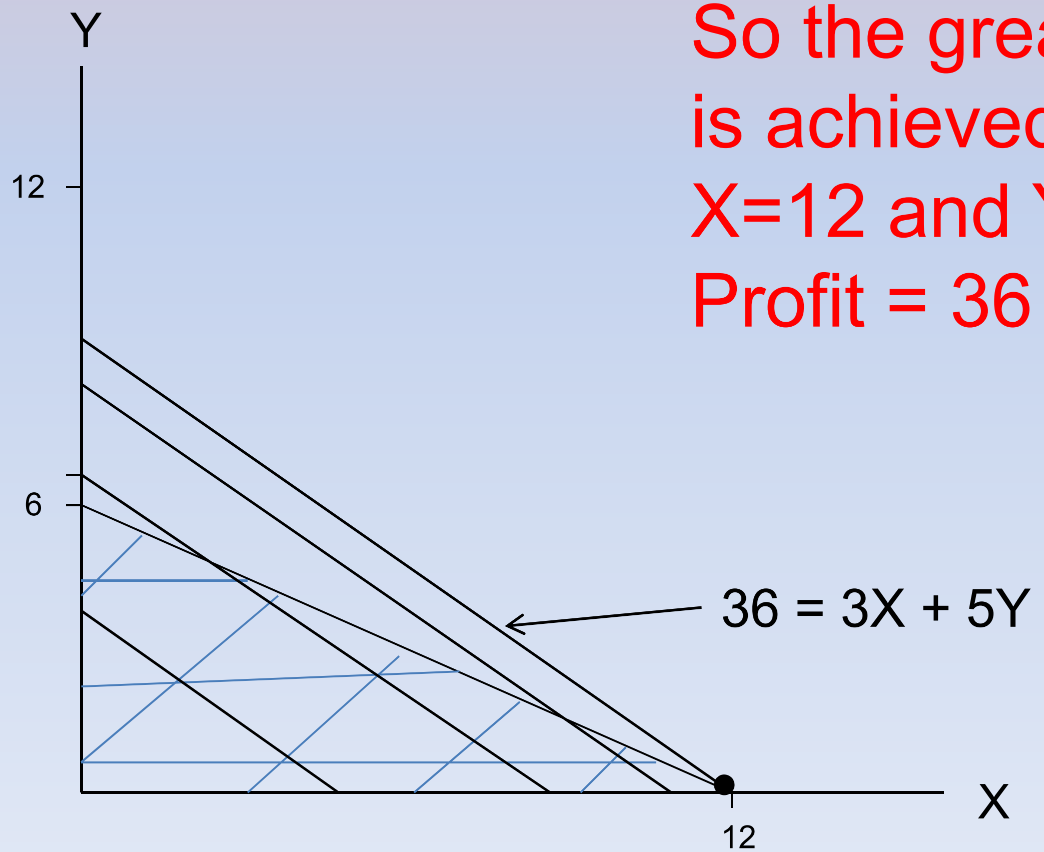


We keep moving the profit line upwards until the last feasible point is touched

It is this line!
And this is the only point; (12, 0)

Now, for X and Y in this region, which one gives the highest profit?

Profit; $P = 3X + 5Y$ → $Y = -(3/5)X + P/5$, **slope = -3/5**, Y intercept = P/5

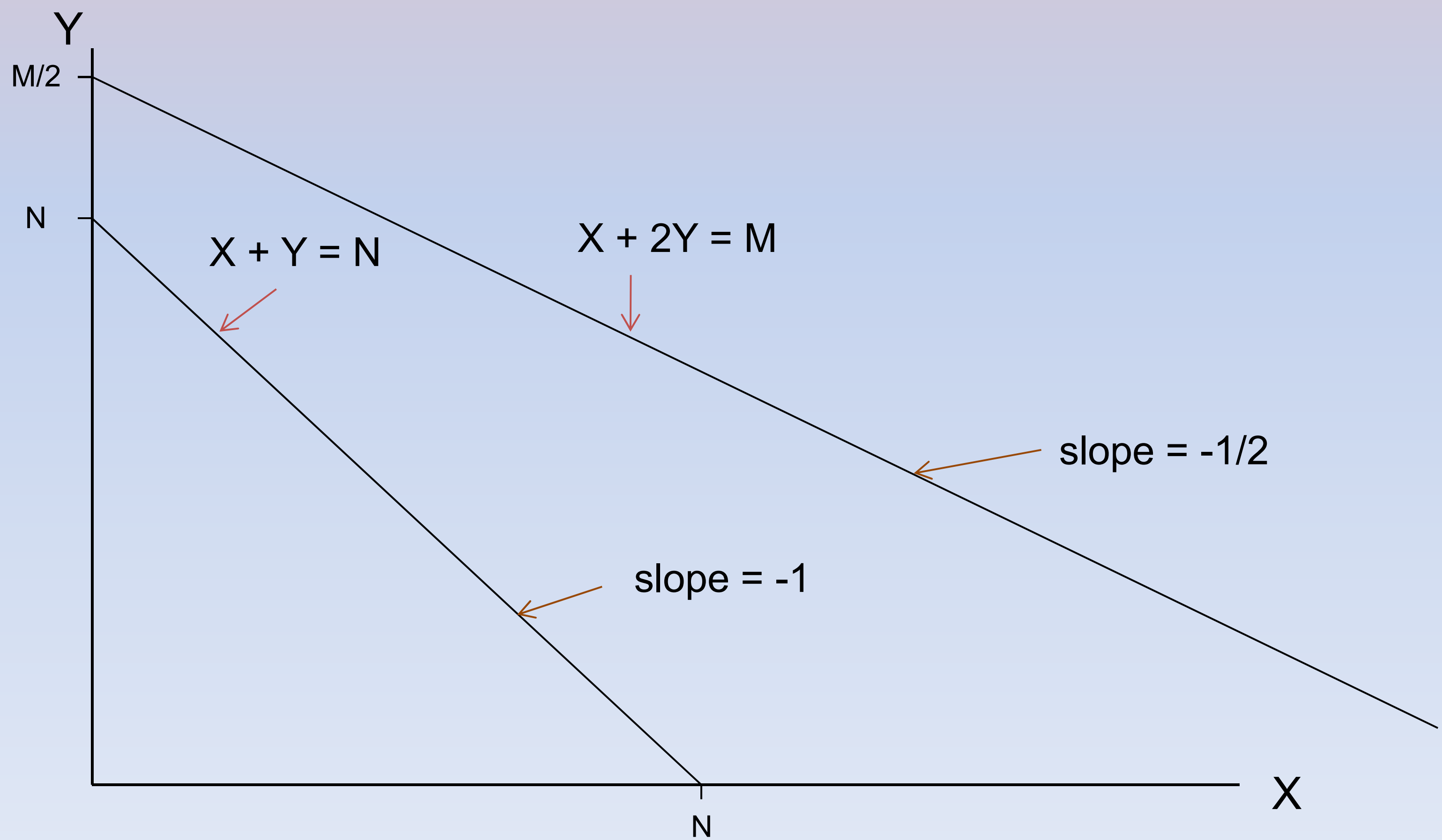


So the greatest profit is achieved when X=12 and Y=0; Profit = 36

Now let's look at Case 1: M >=2N → N< M/2

$X + 2Y <= M$	→	$Y = -(1/2) X + M/2$	slope = -1/2
$X + Y <= N$	→	$Y = -(1) X + N$	slope = -1

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

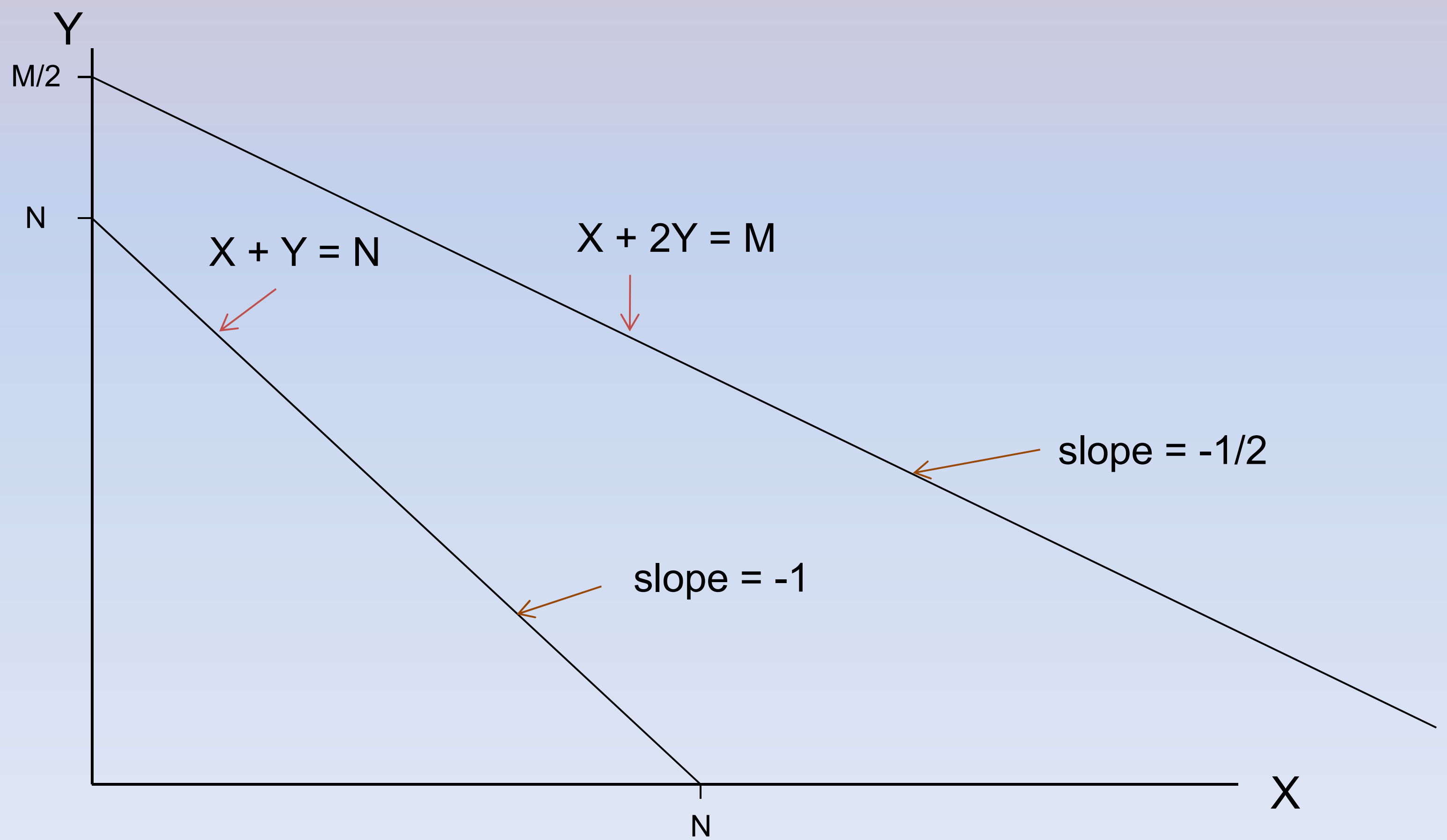


Now let's look at Case 1: M >=2N → N< M/2

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

X + 2Y <= M	→	Y = -(1/2) X + M/2	slope = -1/2
X + Y <= N	→	Y = -(1) X + N	slope = -1

We want to find all possible X and Y that satisfy these two equations.

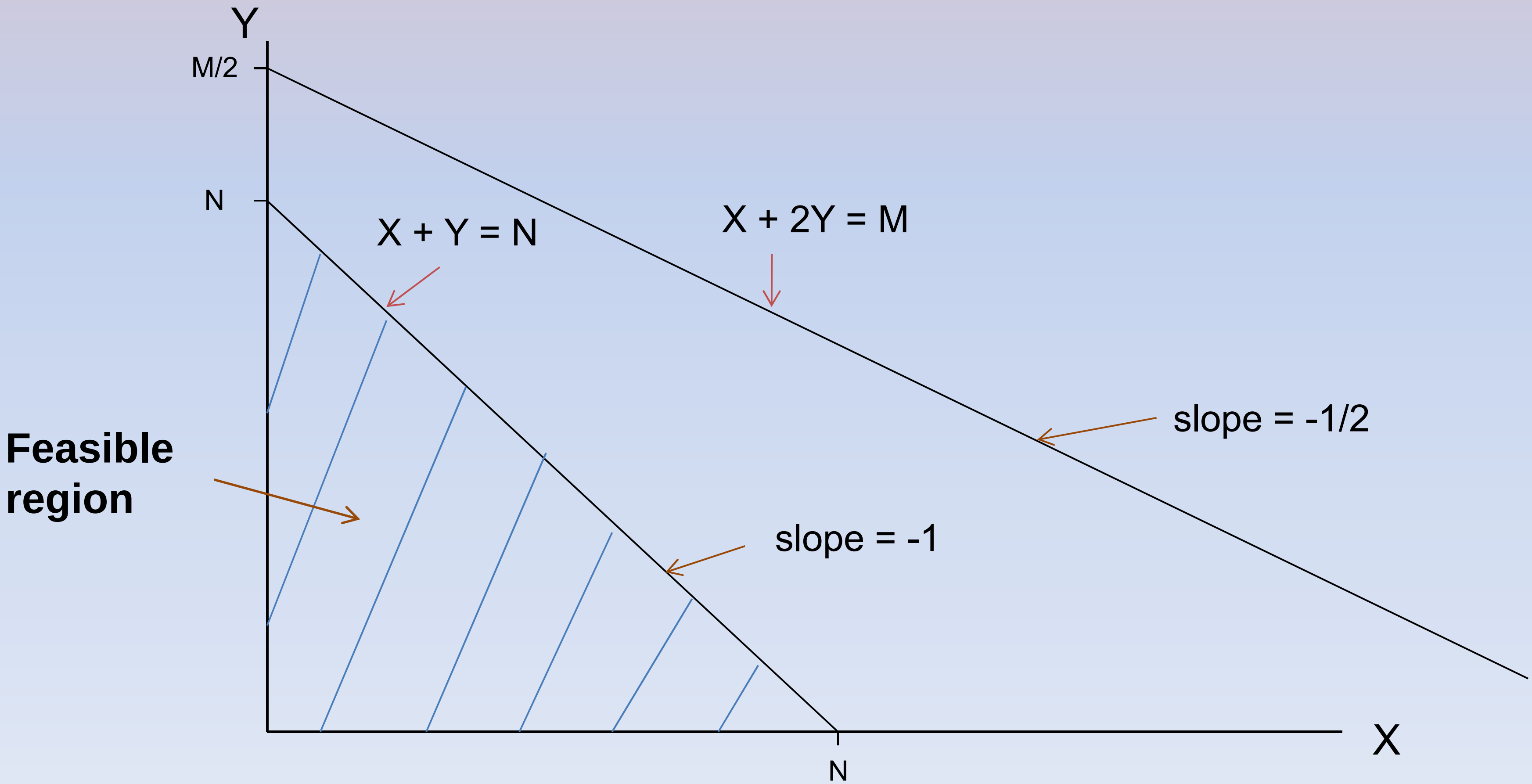


Now let's look at Case 1: M >=2N → N< M/2

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

$X + 2Y <= M$
$X + Y <= N$

We want to find all possible X and Y that satisfy these two equations.

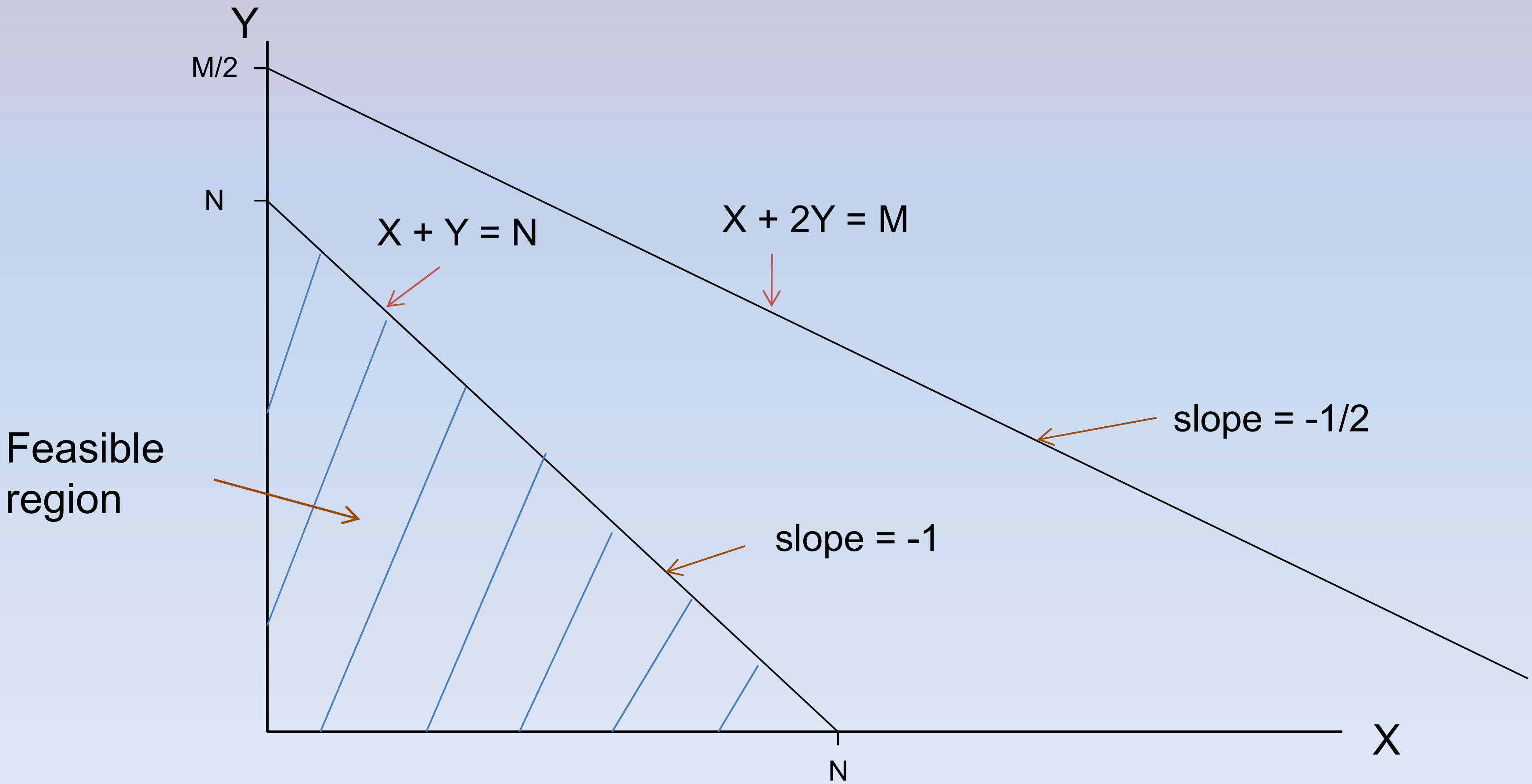


Now let's look at Case 1: M >=2N → N< M/2

X + 2Y <= M
X + Y <= N

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

Now plot the profit line: 3X + 5Y = P

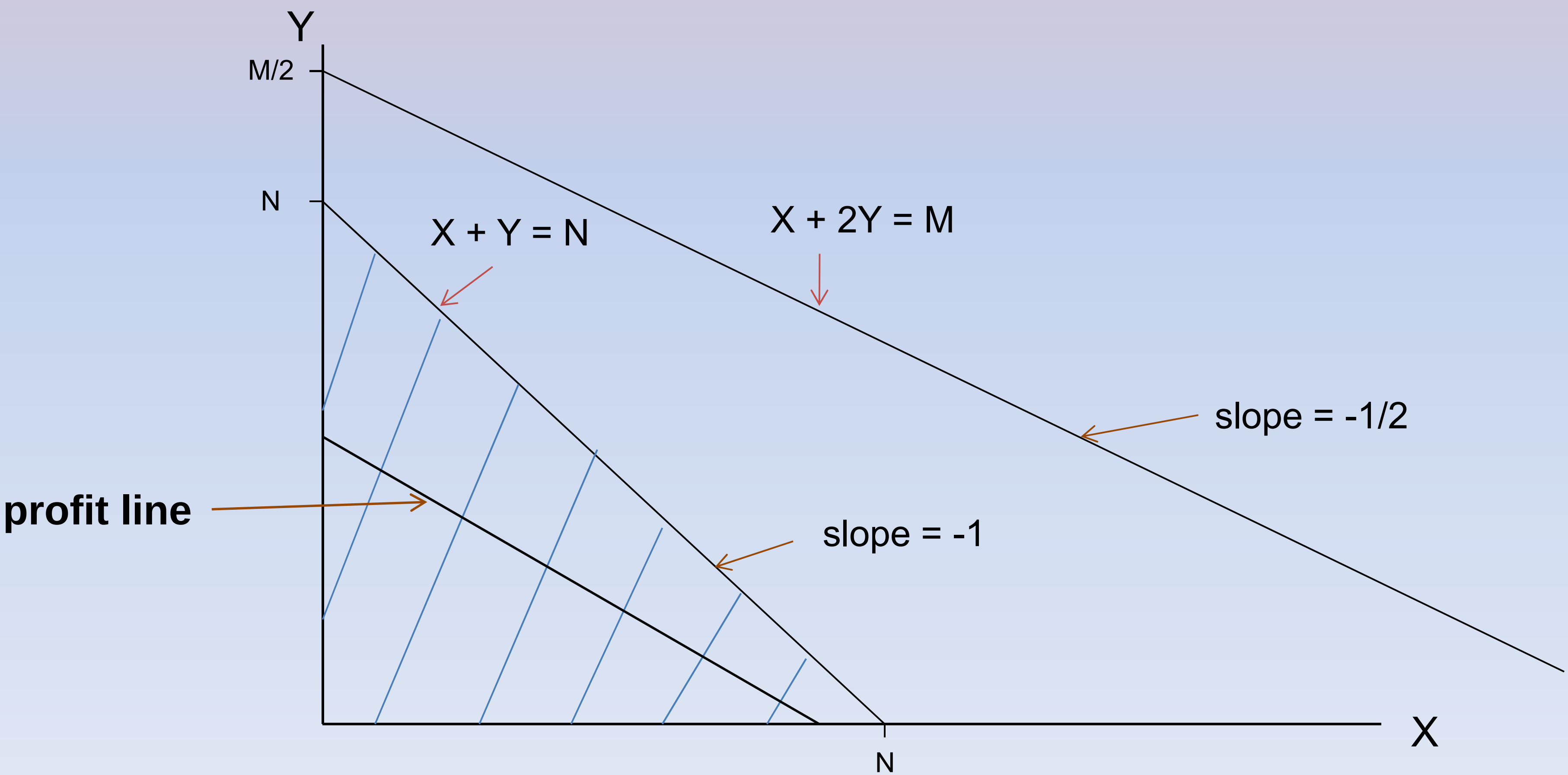


Now let's look at Case 1: M >=2N → N< M/2

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

X + 2Y <= M
X + Y <= N

Now plot the profit line: 3X + 5Y = P (slope = -3/5)



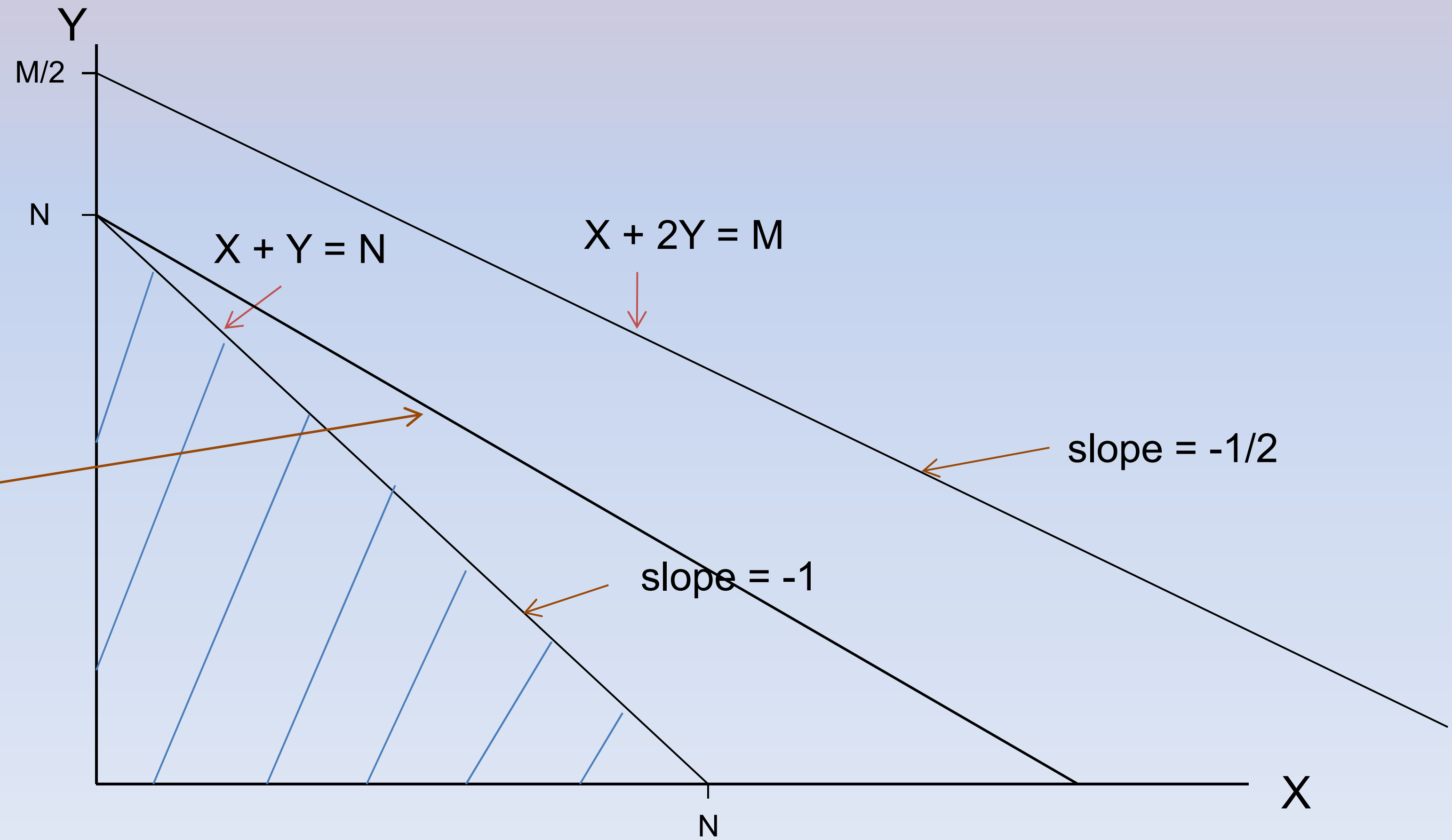
Now let's look at Case 1: M >=2N → N< M/2

$X + 2Y <= M$
$X + Y <= N$

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

Move the profit line up until it last touches the feasible region

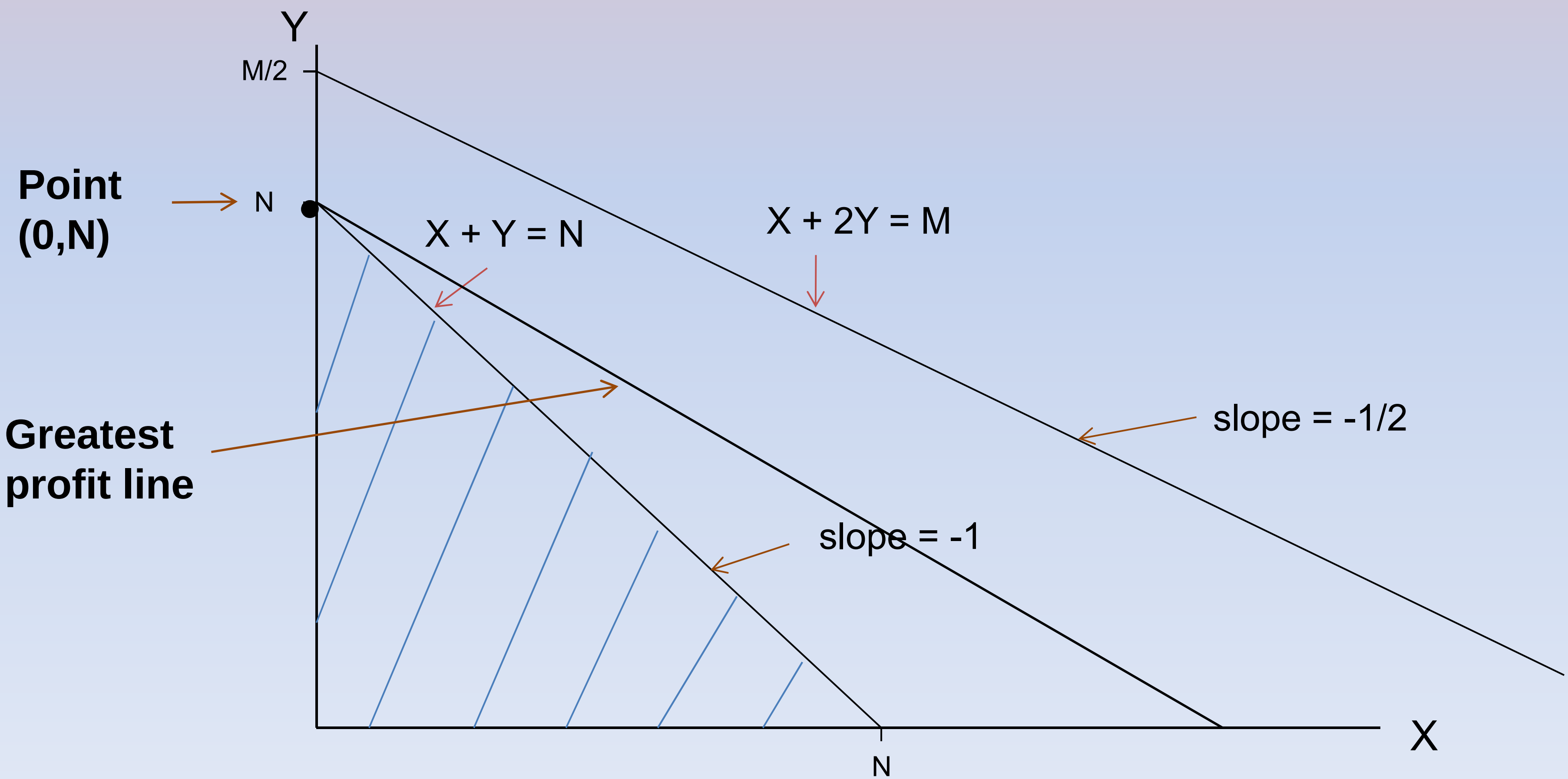
Greatest profit line



Now let's look at Case 1: M >=2N → N< M/2

X + 2Y <= M
X + Y <= N

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

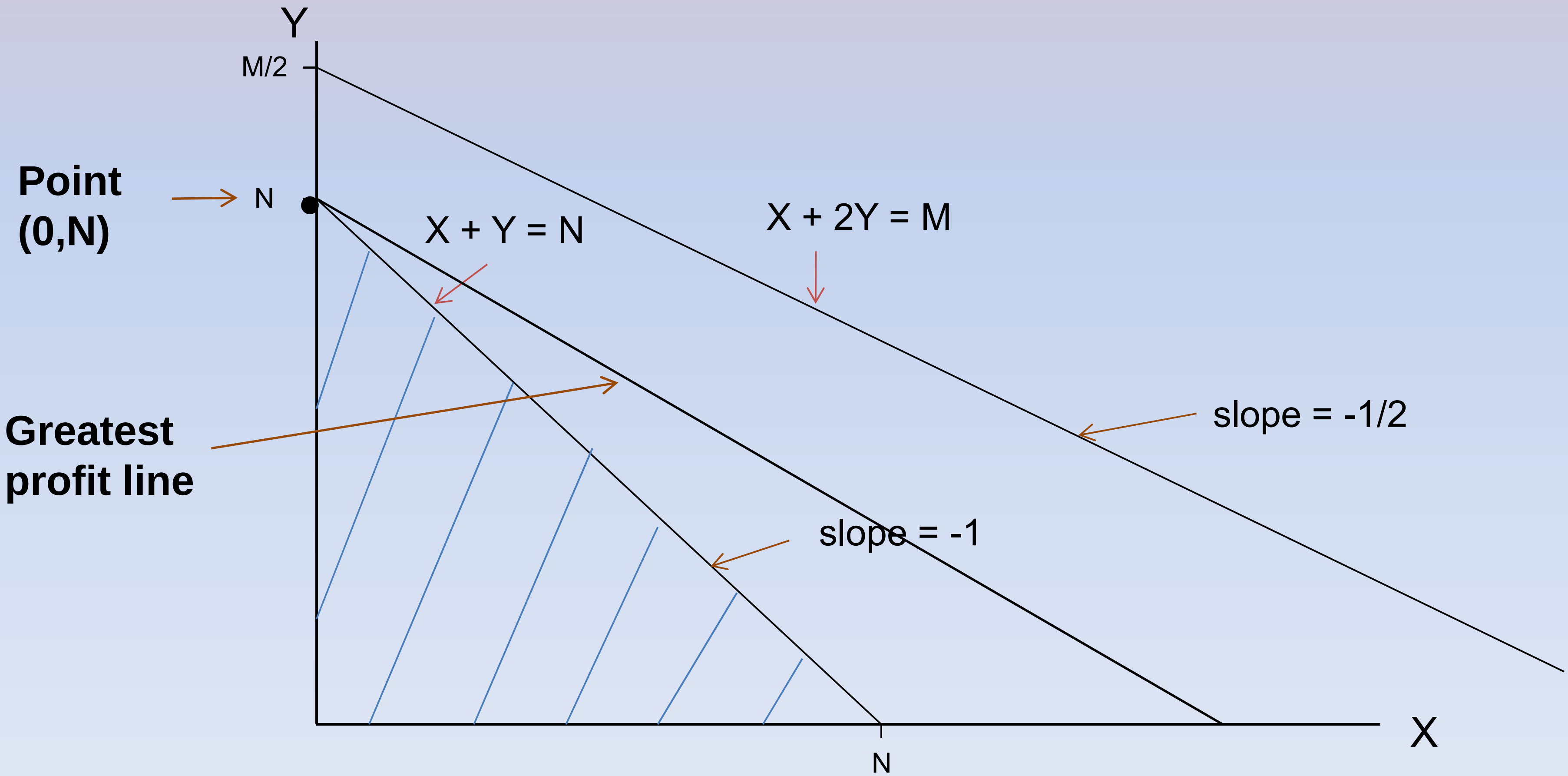


Now let's look at Case 1: M >=2N → N< M/2

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

X + 2Y <= M
X + Y <= N

So the greatest profit is achieved when X = 0, Y = N



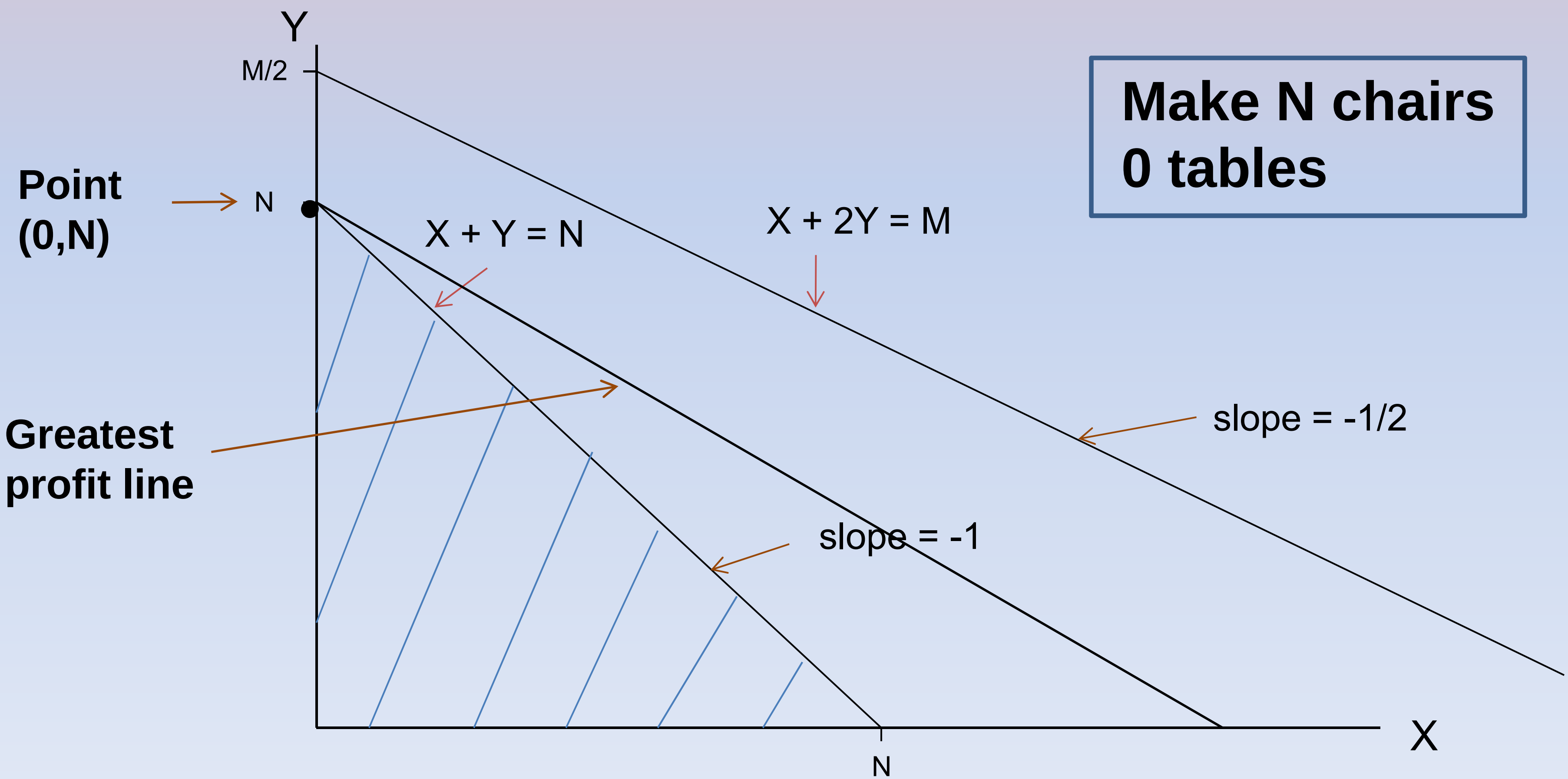
Now let's look at Case 1: M >=2N → N< M/2

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

$X + 2Y <= M$
$X + Y <= N$

So the greatest profit is achieved when X = 0, Y = N

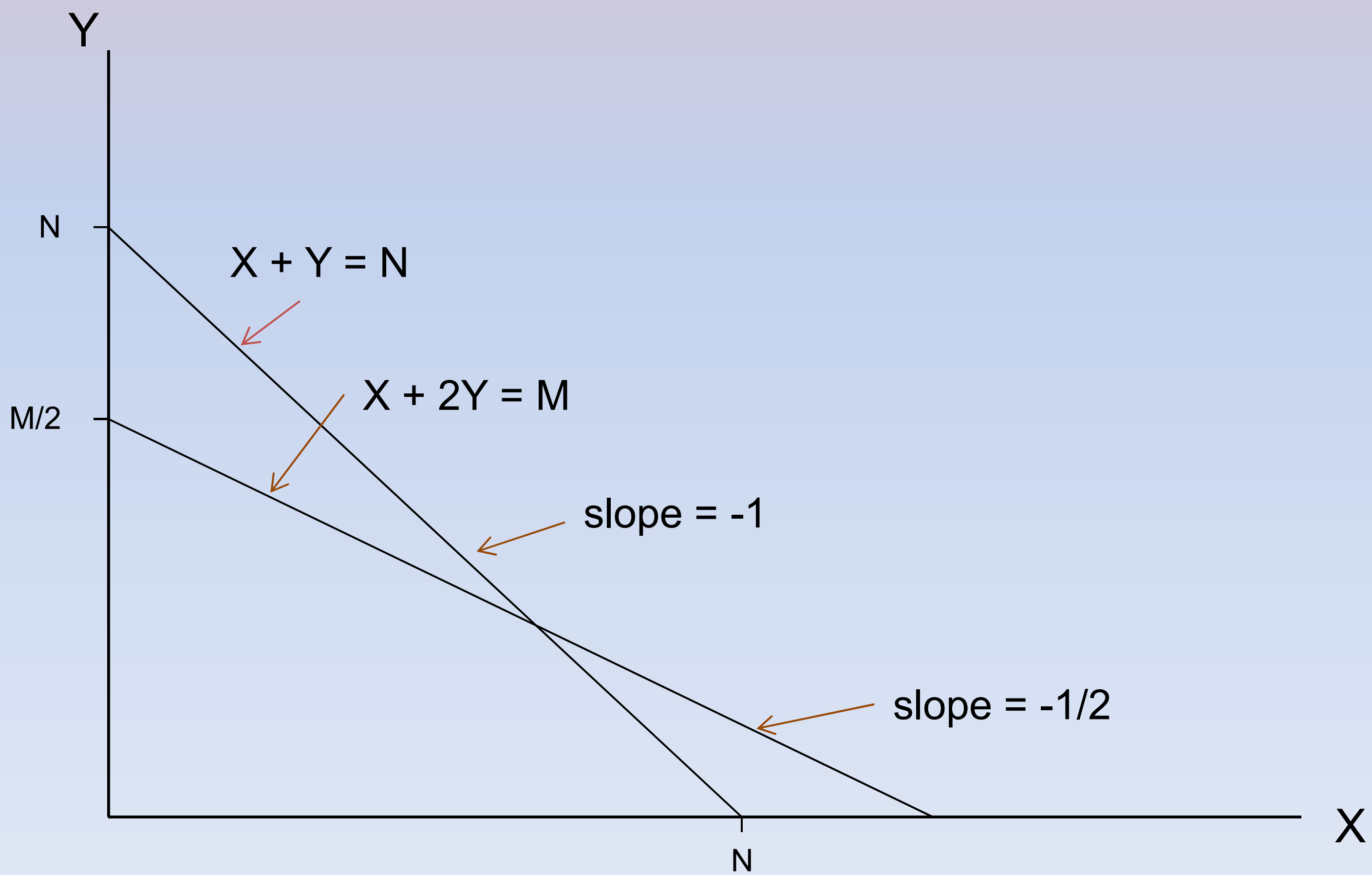
Make N chairs
0 tables



Now let's look at Case 2: M < 2N → N > M/2

X + 2Y <= M	→	Y = -(1/2) X + M/2	slope = -1/2
X + Y <= N	→	Y = -(1) X + N	slope = -1

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

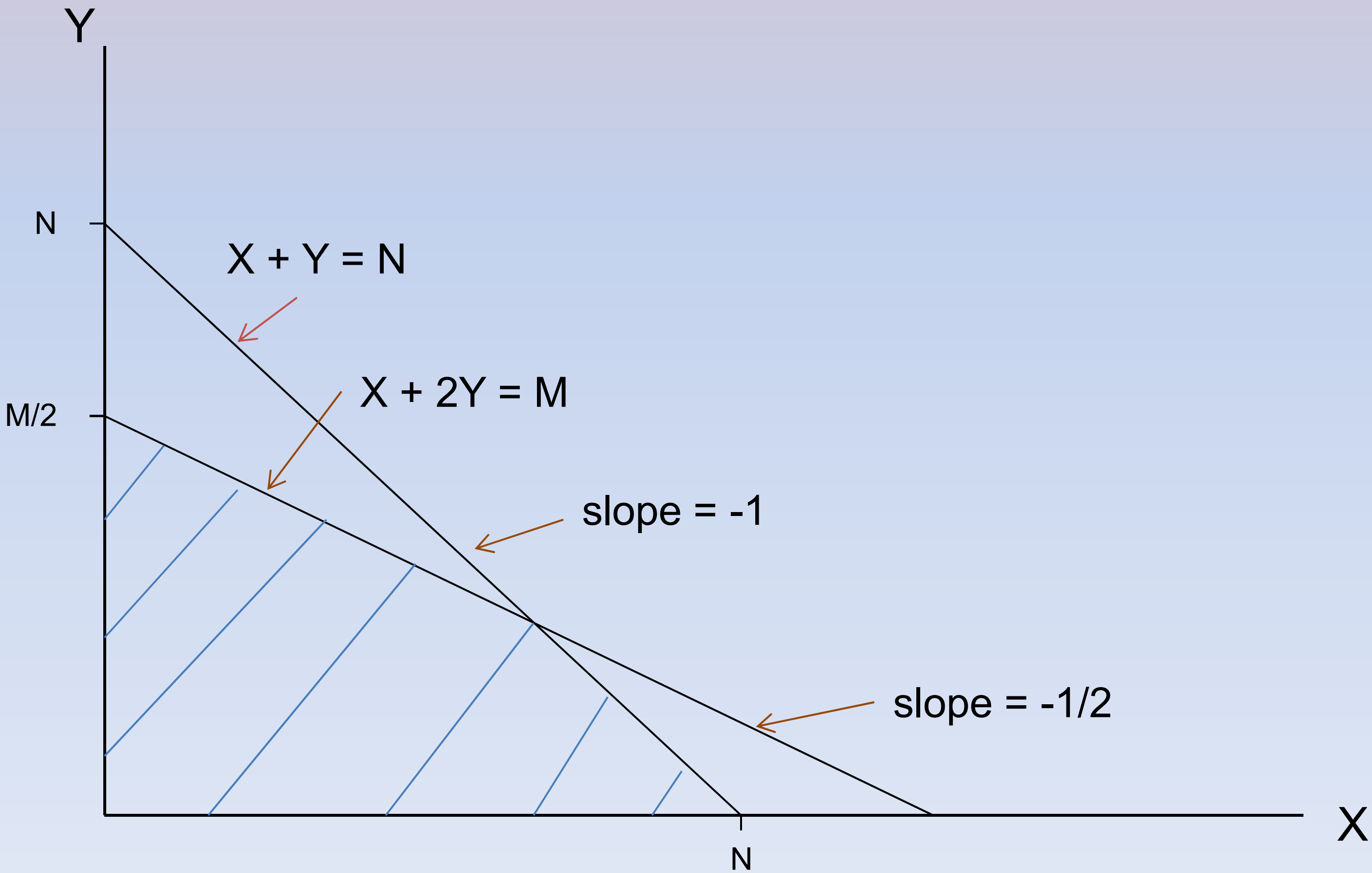


Now let's look at Case 2: M < 2N → N > M/2

X + 2Y <= M
X + Y <= N

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

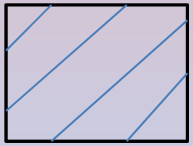
The feasible region is:



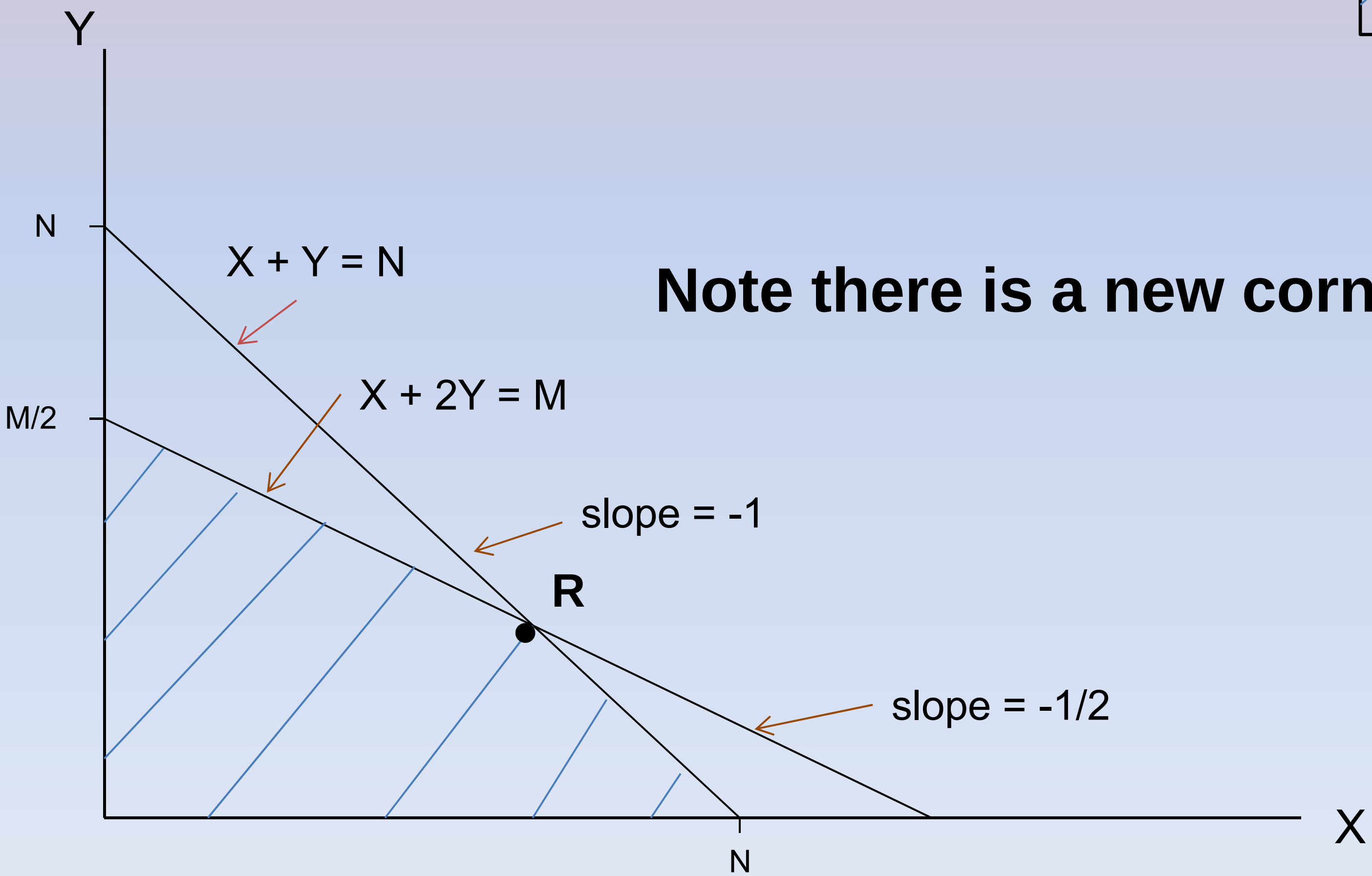
Now let's look at Case 2: M < 2N → N > M/2

X + 2Y <= M
X + Y <= N

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

The feasible region is 

Note there is a new corner; R

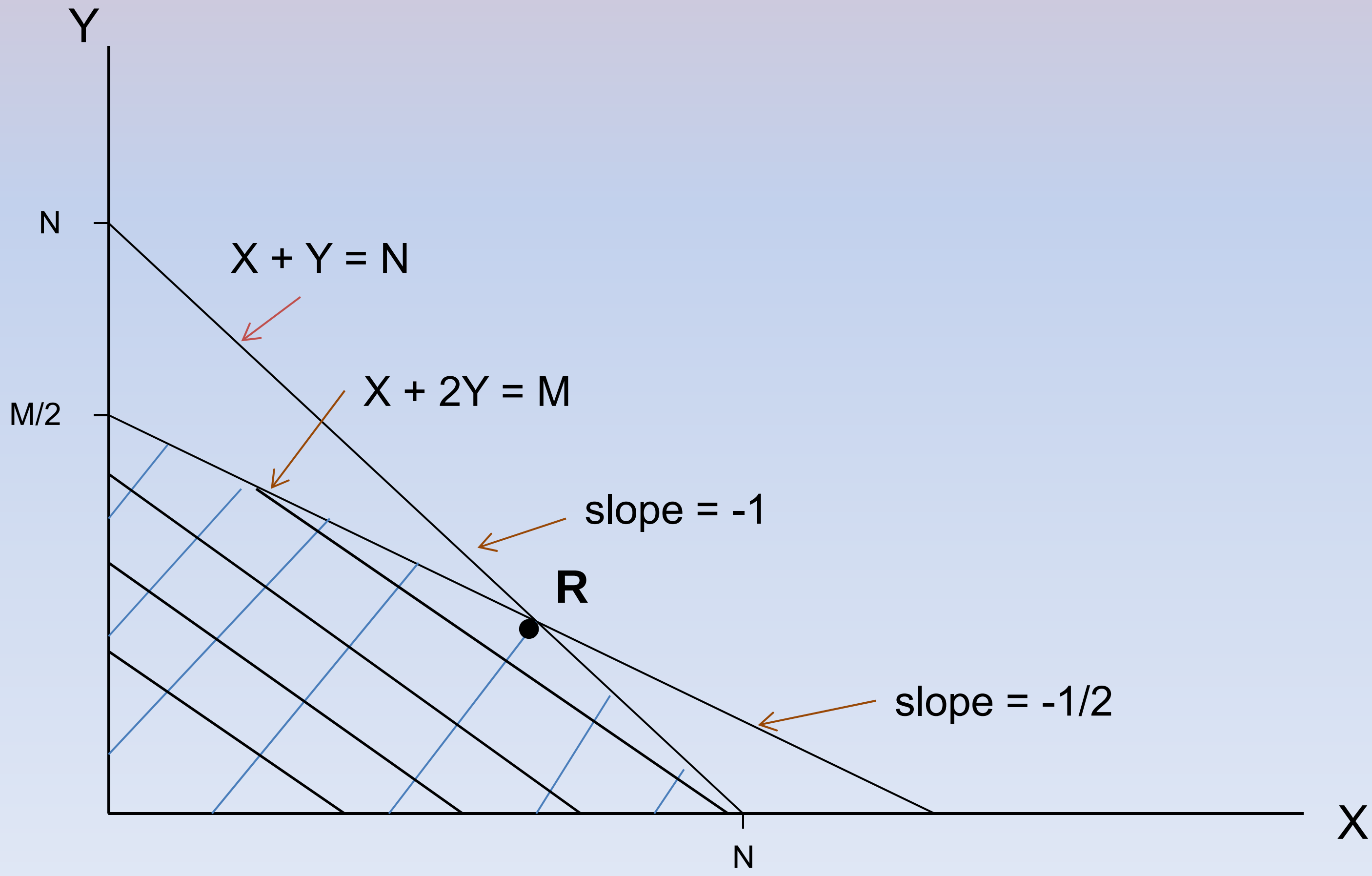


Now let's look at Case 2: M < 2N → N > M/2

X + 2Y <= M
X + Y <= N

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

Now let's add the profit lines



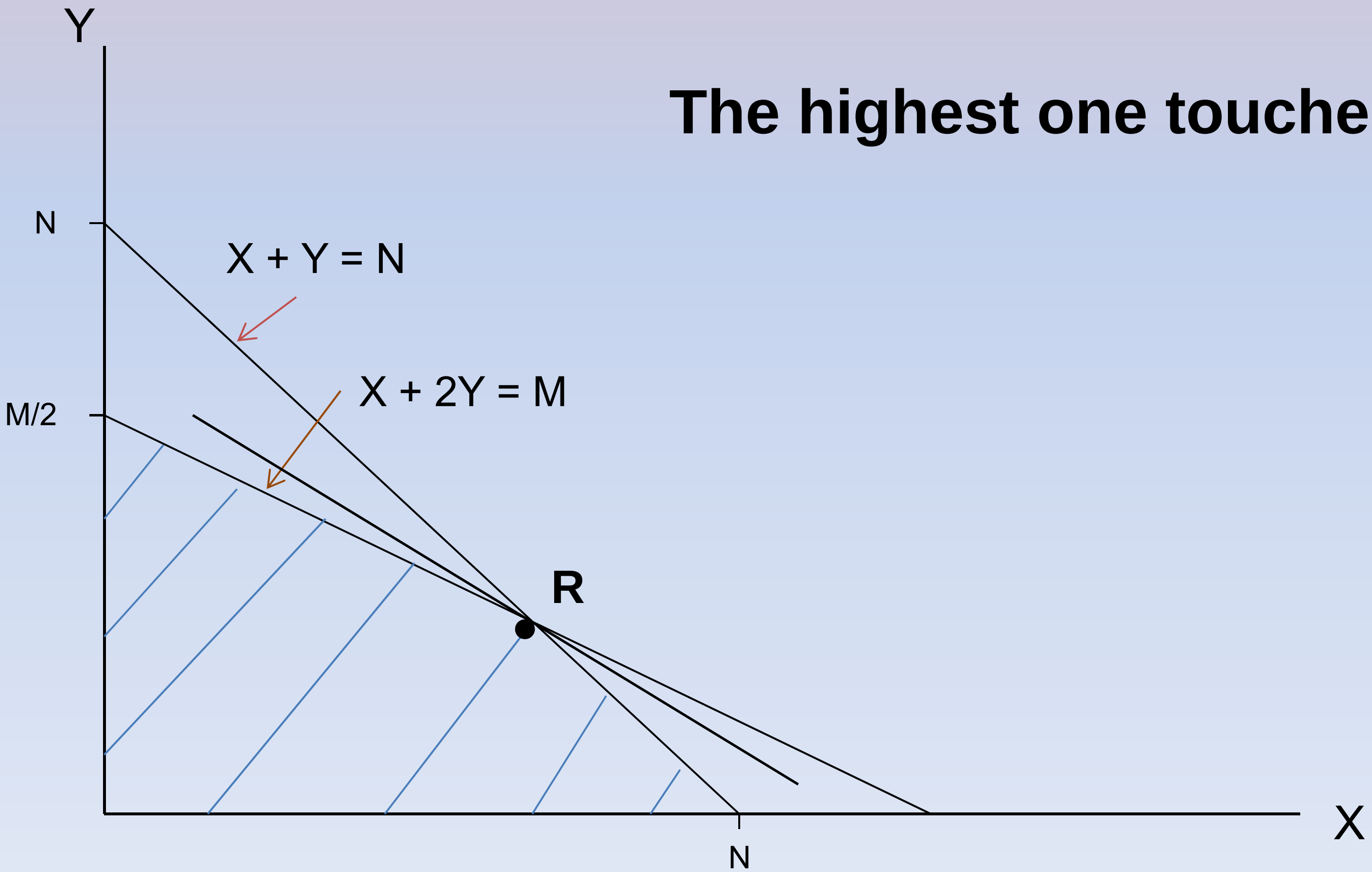
Now let's look at Case 2: M < 2N → N > M/2

X + 2Y <= M
X + Y <= N

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

Now let's add the profit lines

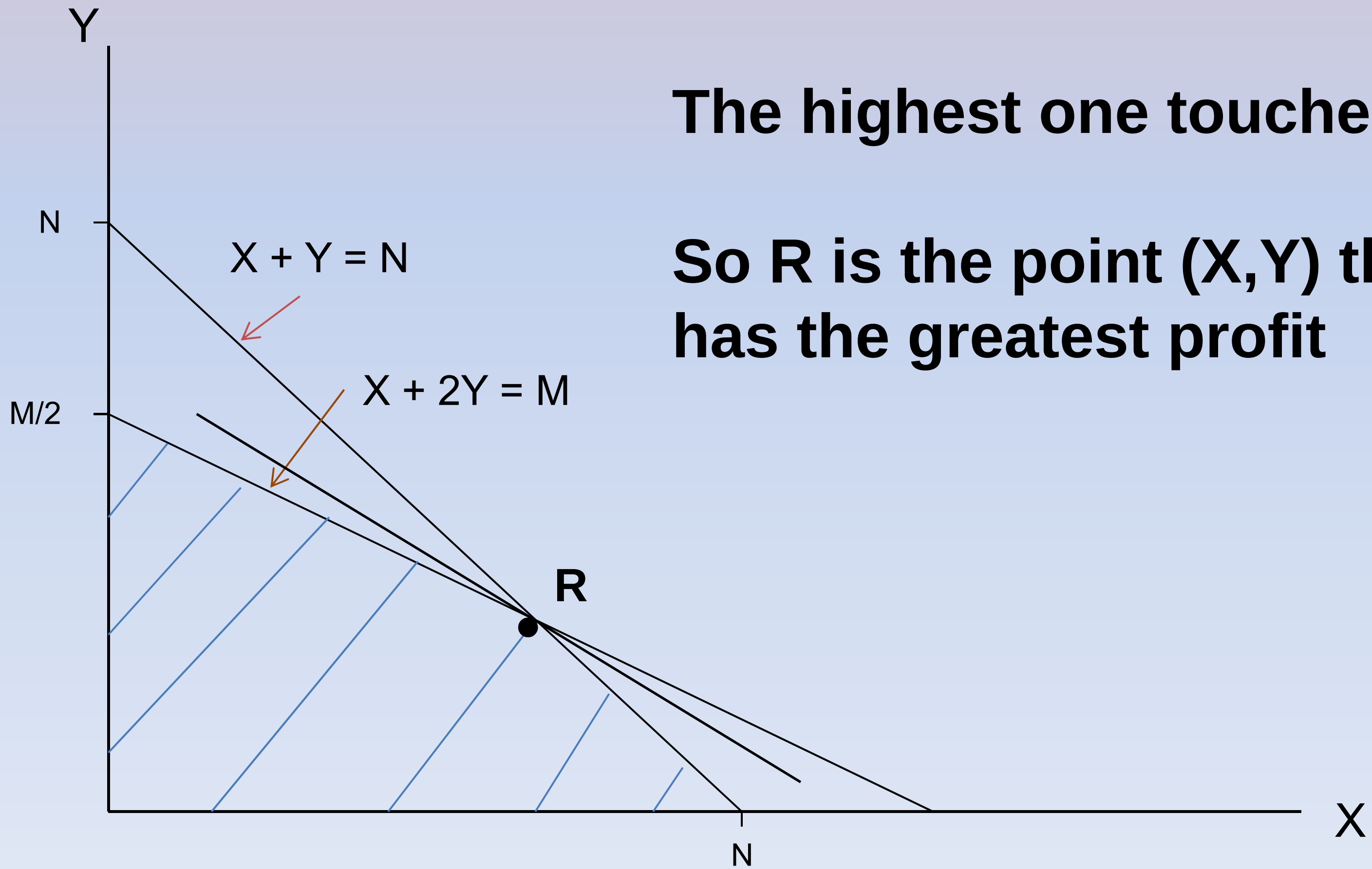
The highest one touches R!



Now let's look at Case 2: M < 2N → N > M/2

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

X + 2Y <= M
X + Y <= N



Now let's add the profit lines

The highest one touches R!

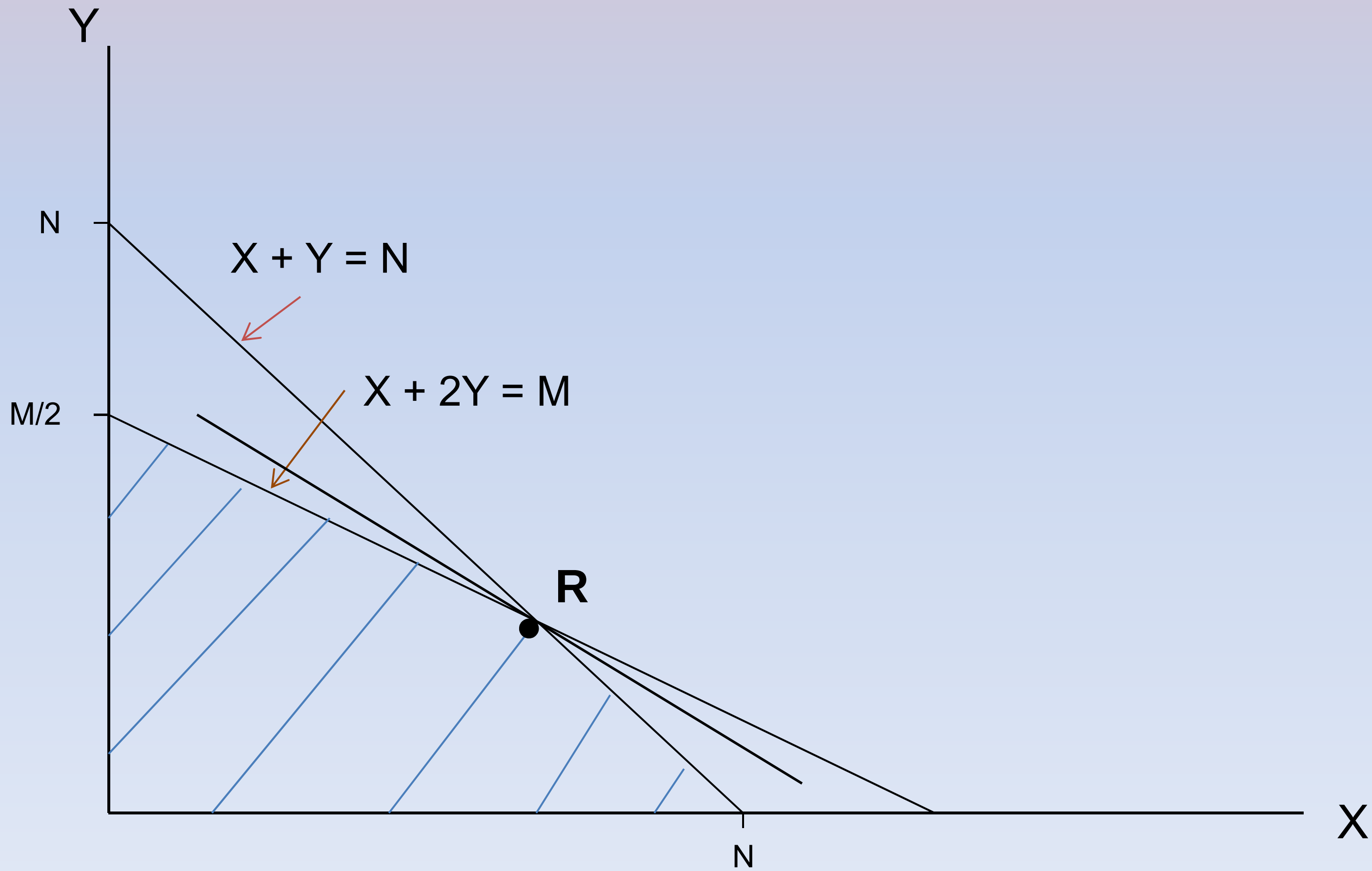
So R is the point (X,Y) that has the greatest profit

Now let's look at Case 2: M < 2N → N > M/2

X + 2Y <= M
X + Y <= N

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

What is (X,Y) at point R?



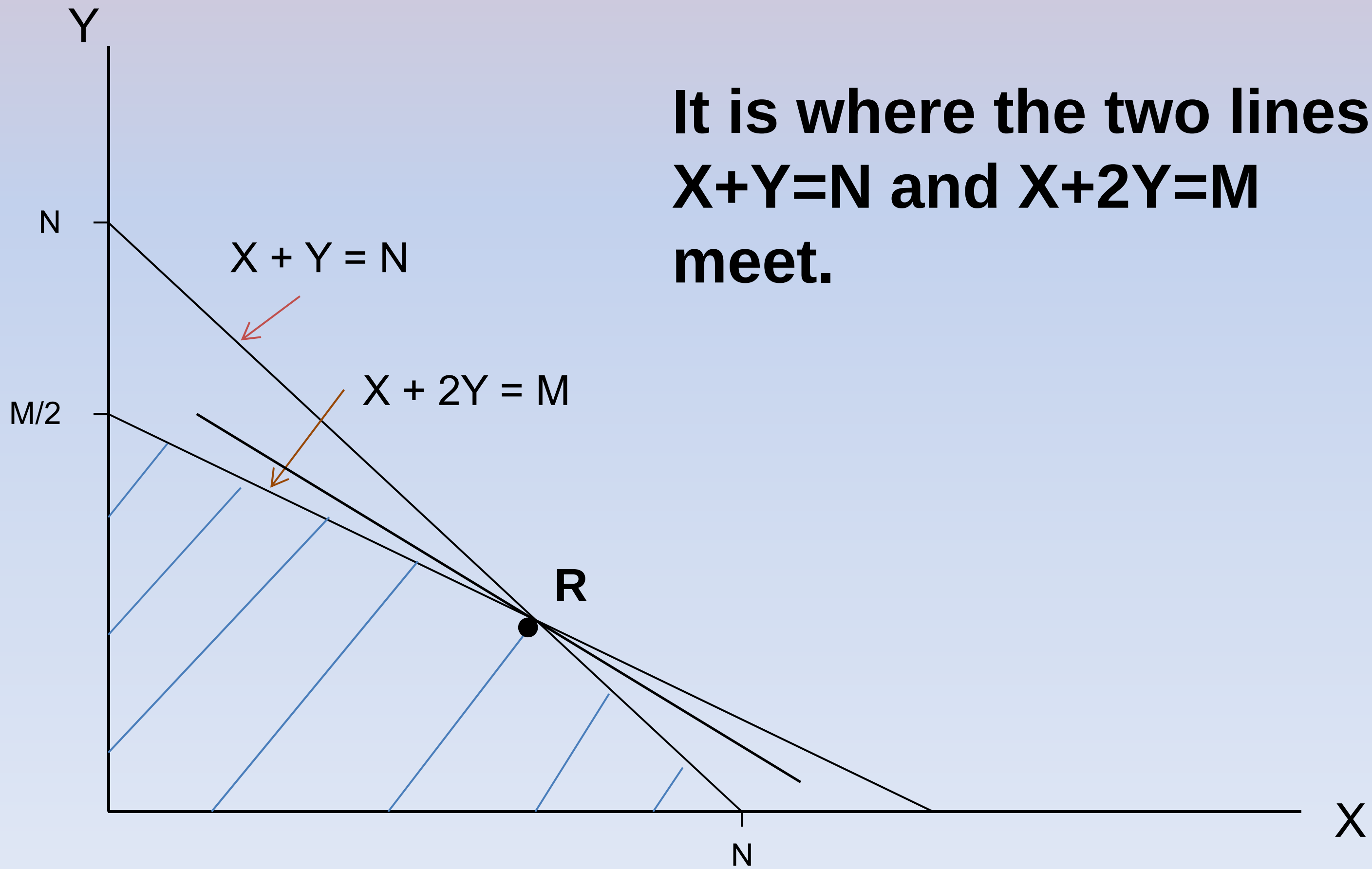
Now let's look at Case 2: M < 2N → N > M/2

X + 2Y <= M
X + Y <= N

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

What is (X,Y) at point R?

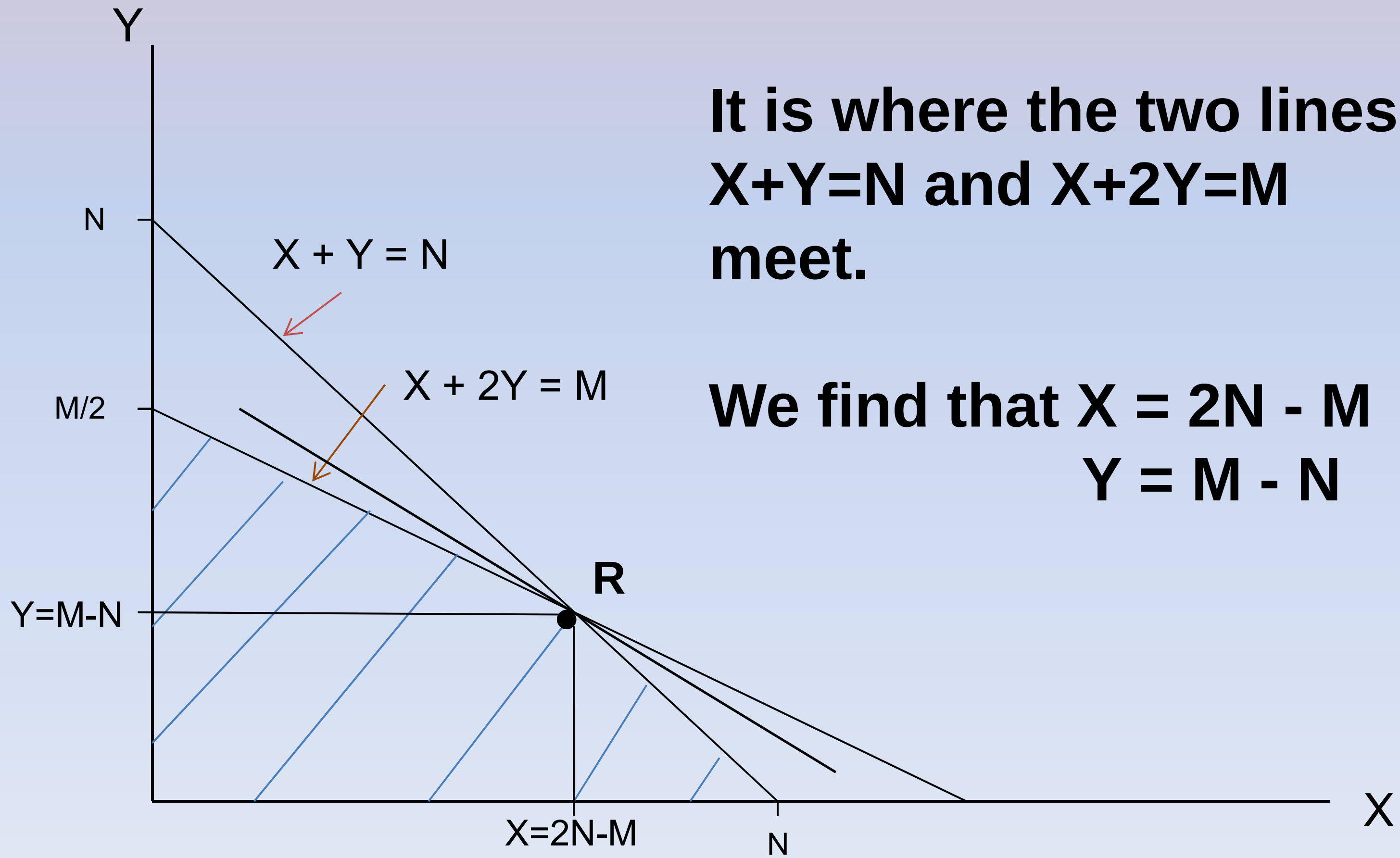
It is where the two lines X+Y=N and X+2Y=M meet.



Now let's look at Case 2: M < 2N → N > M/2

X + 2Y <= M
X + Y <= N

X = # tables
Y = # chairs
M = # small blocks
N = # large blocks



What is (X,Y) at point R?

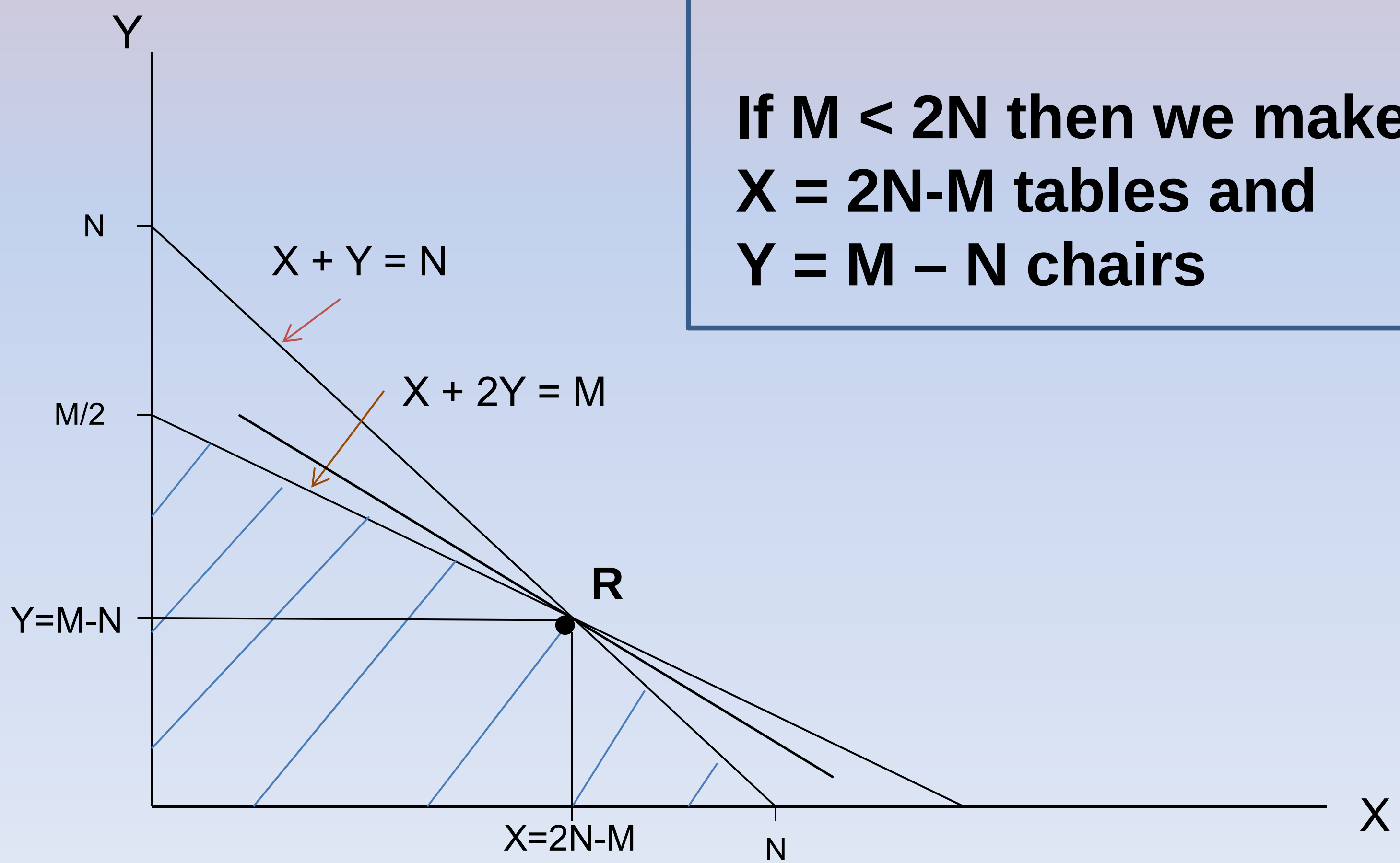
It is where the two lines X+Y=N and X+2Y=M meet.

We find that X = 2N - M
Y = M - N

Now let's look at Case 2: M < 2N → N > M/2

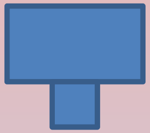
X = # tables
Y = # chairs
M = # small blocks
N = # large blocks

X + 2Y <= M
X + Y <= N

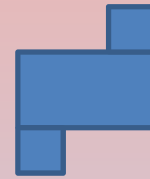


So we have our answer:

If M < 2N then we make
X = 2N-M tables and
Y = M – N chairs

table  $3

X

chair  $5

Y

M = 12
N = 12
M < 2N

tables	0	2	3	4	6	8	10	11	12
chairs	6	5	4	4	3	2	1	0	**0**
profit	30	31	29	32	33	34	35	33	**36**

X = 2N – M = 12
Y = M – N = 0

M = 20
N = 12
M < 2N

tables	2	4	5	6	7	8	10	11	12
chairs	9	**8**	7	6	5	4	2	1	0
profit	51	**52**	50	48	46	44	40	38	36

X = 2N – M = 4
Y = M – N = 8

M = 25
N = 12
M > 2N

tables	0	4	5	6	7	8	10	11	12
chairs	**12**	8	7	6	5	4	2	1	0
profit	**60**	52	50	48	46	44	40	38	36

X = 0
Y = N = 12

A problem for you . . .

A problem for you . . .

	hours per kg		hours available
	Columbian	Mexican	
roaster A	1	0	4
roaster B	0	2	12
grind/package	3	2	18
profit/kg	3	2.5	

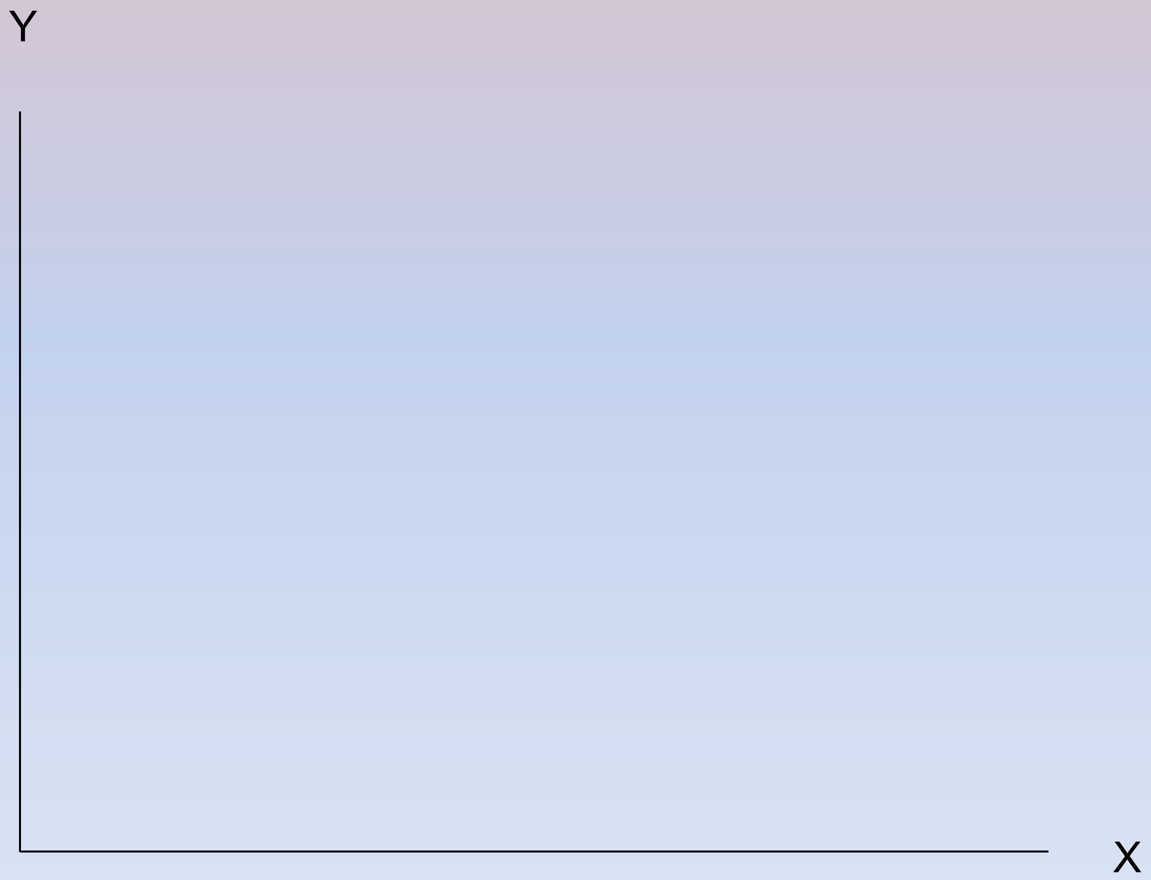
A problem for you . . .

	hours per kg		hours available
	Columbian	Mexican	
roaster A	1	0	4
roaster B	0	2	12
grind/package	3	2	18
profit/kg	3	2.5	

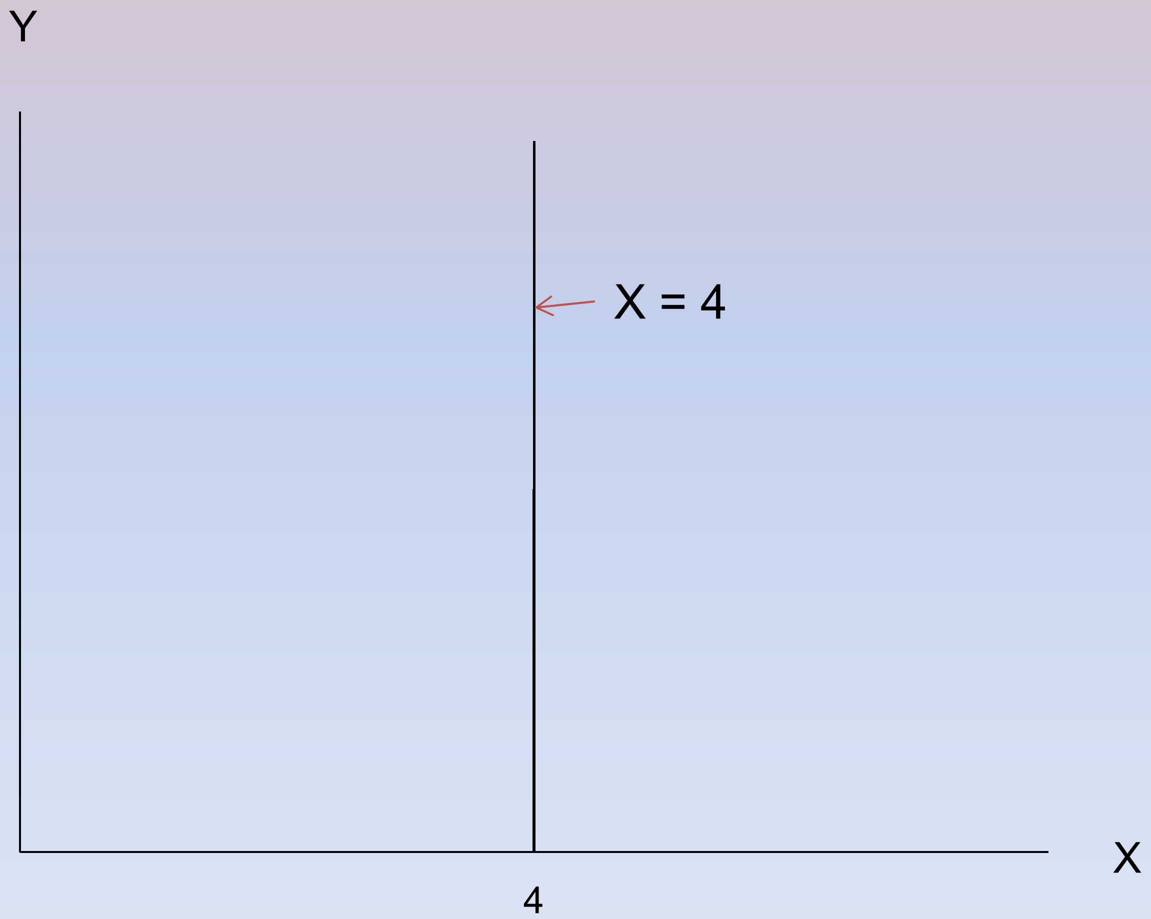
Variables: **X** **Y**

X <= 4 roaster A

X <= 4 roaster A

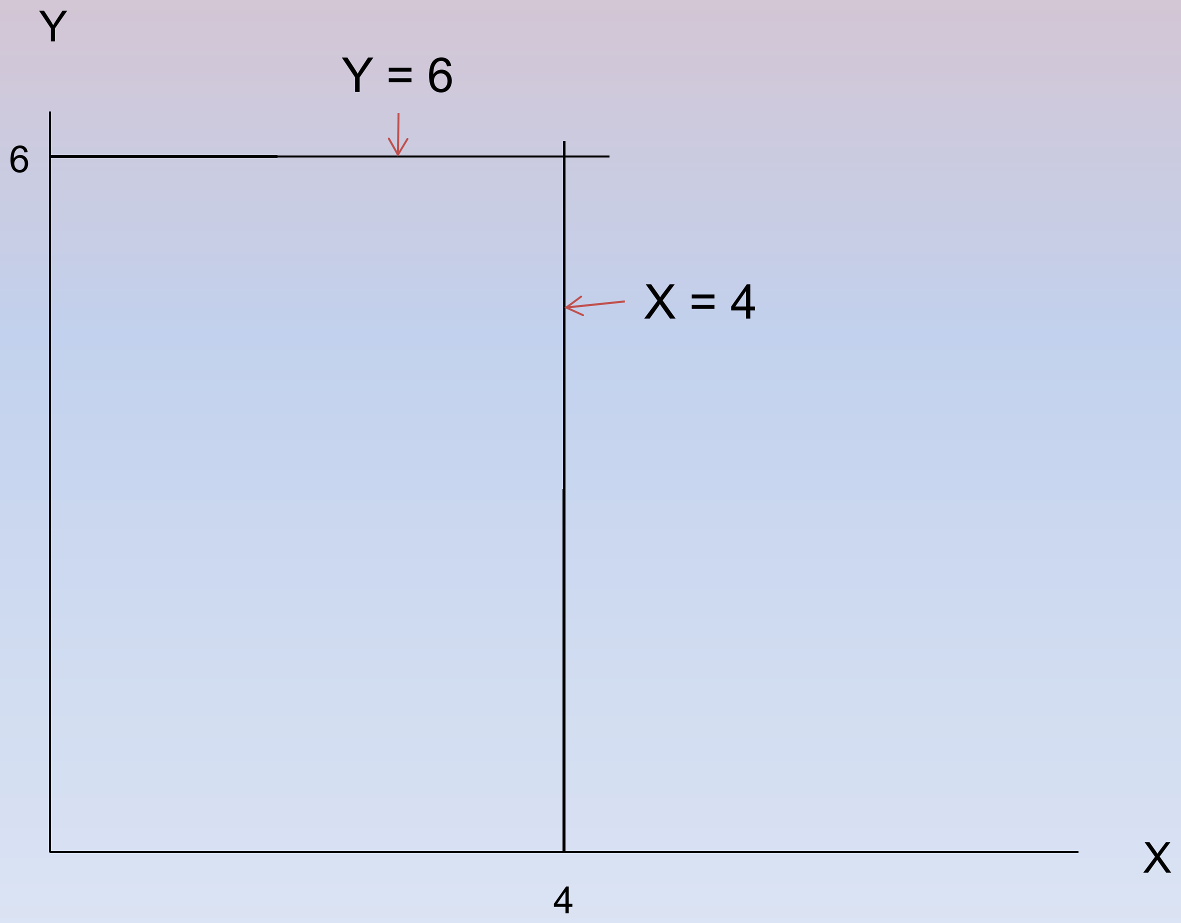


X <= 4 roaster A



X <= 4 roaster A

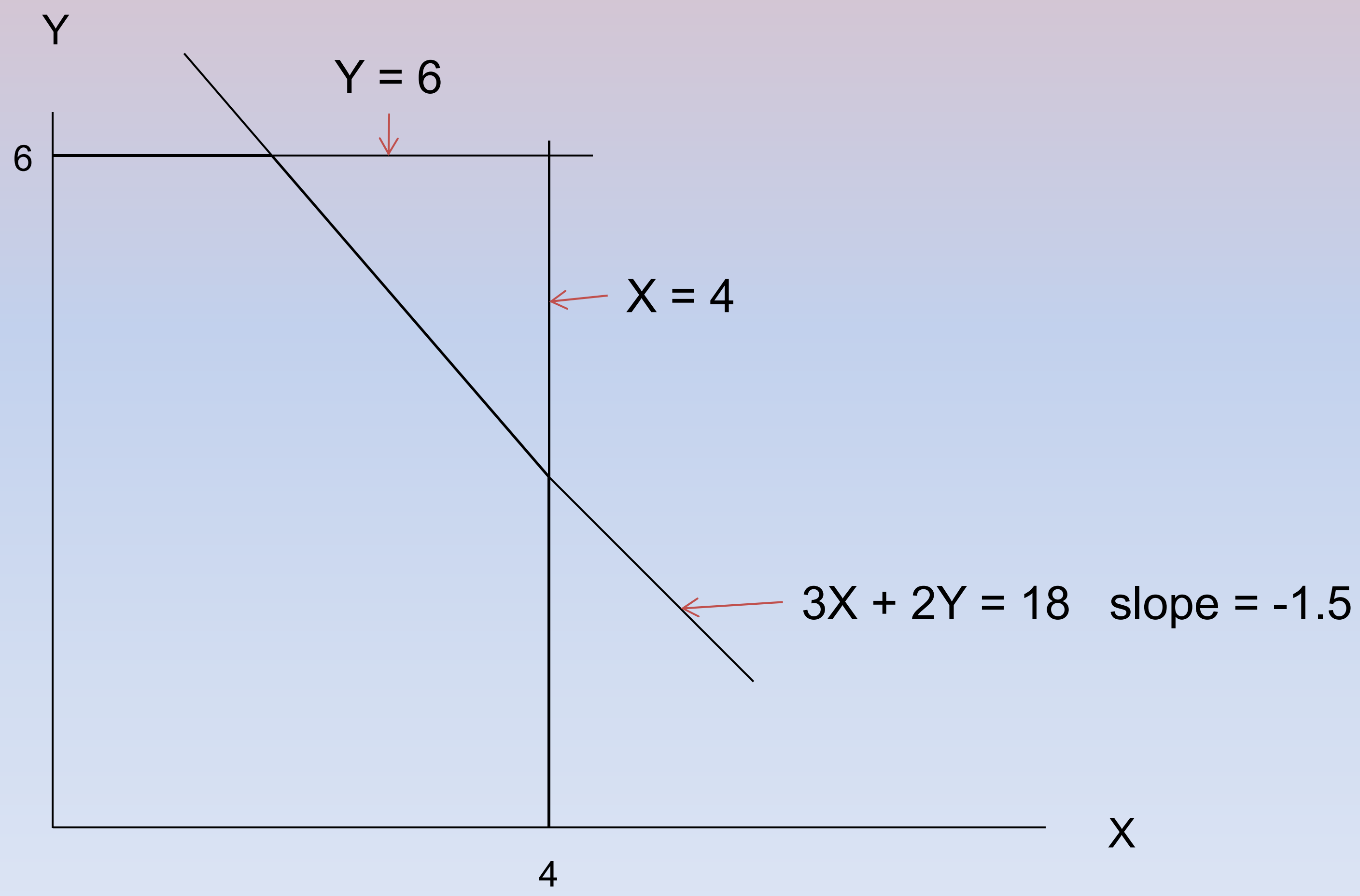
2Y <= 12 roaster B



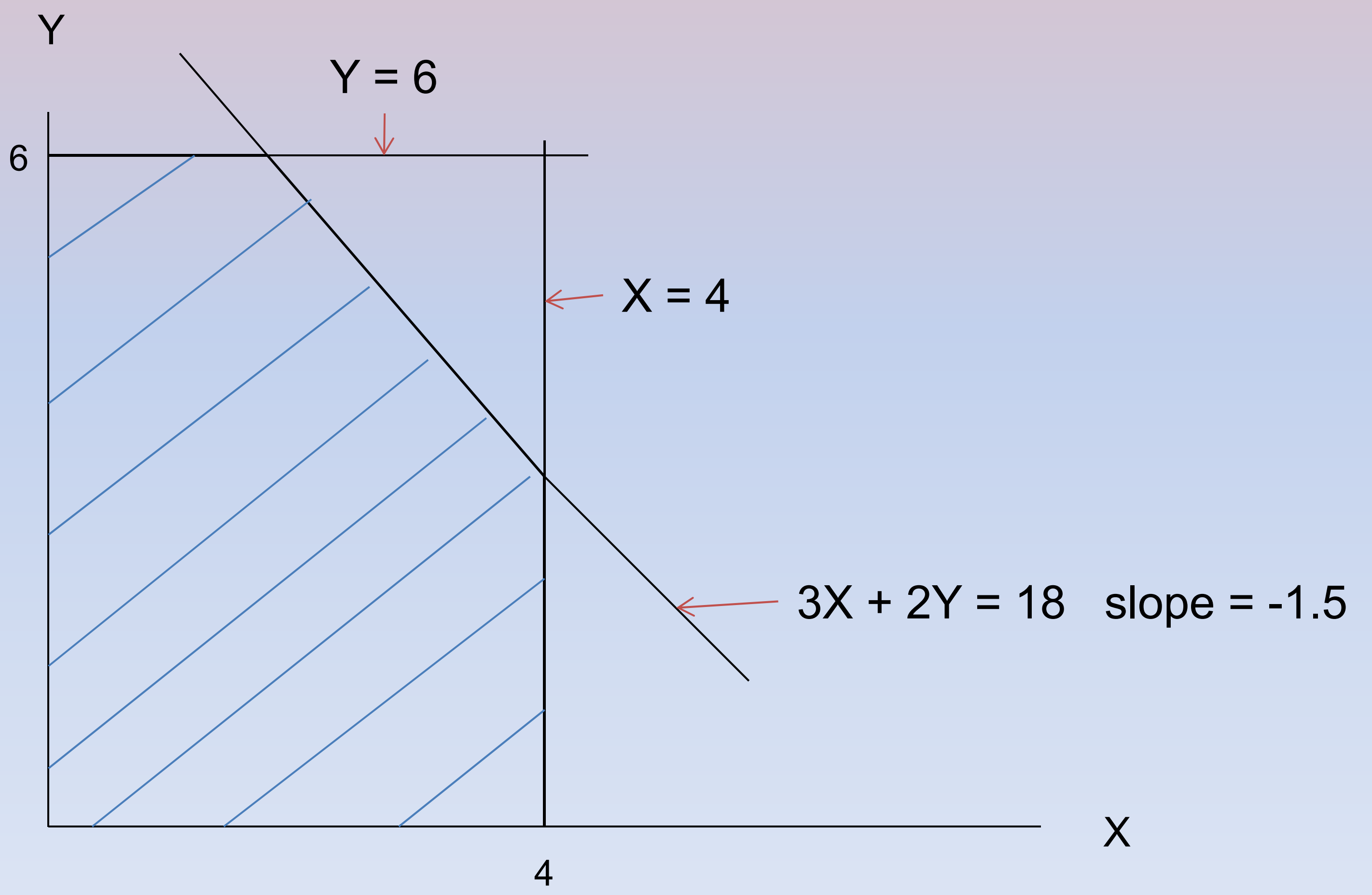
$X \le 4$ roaster A

$2Y \le 12$ roaster B

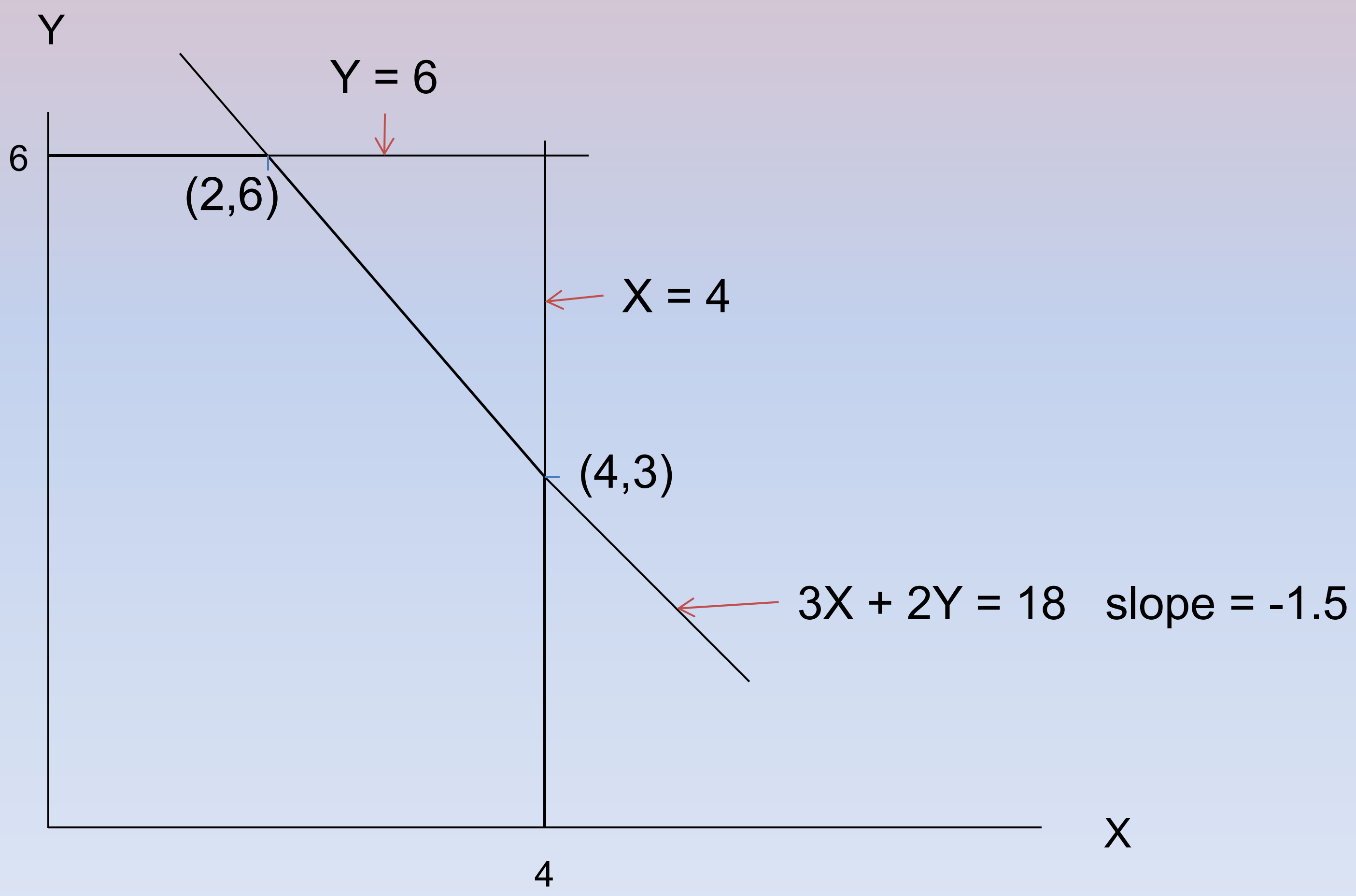
$3X + 2Y \le 18$ grinding, packaging



X <= 4 roaster A
2Y <= 12 roaster B
3X + 2Y <= 18 grinding, packaging



$X \le 4$ roaster A
$2Y \le 12$ roaster B
$3X + 2Y \le 18$ grinding, packaging

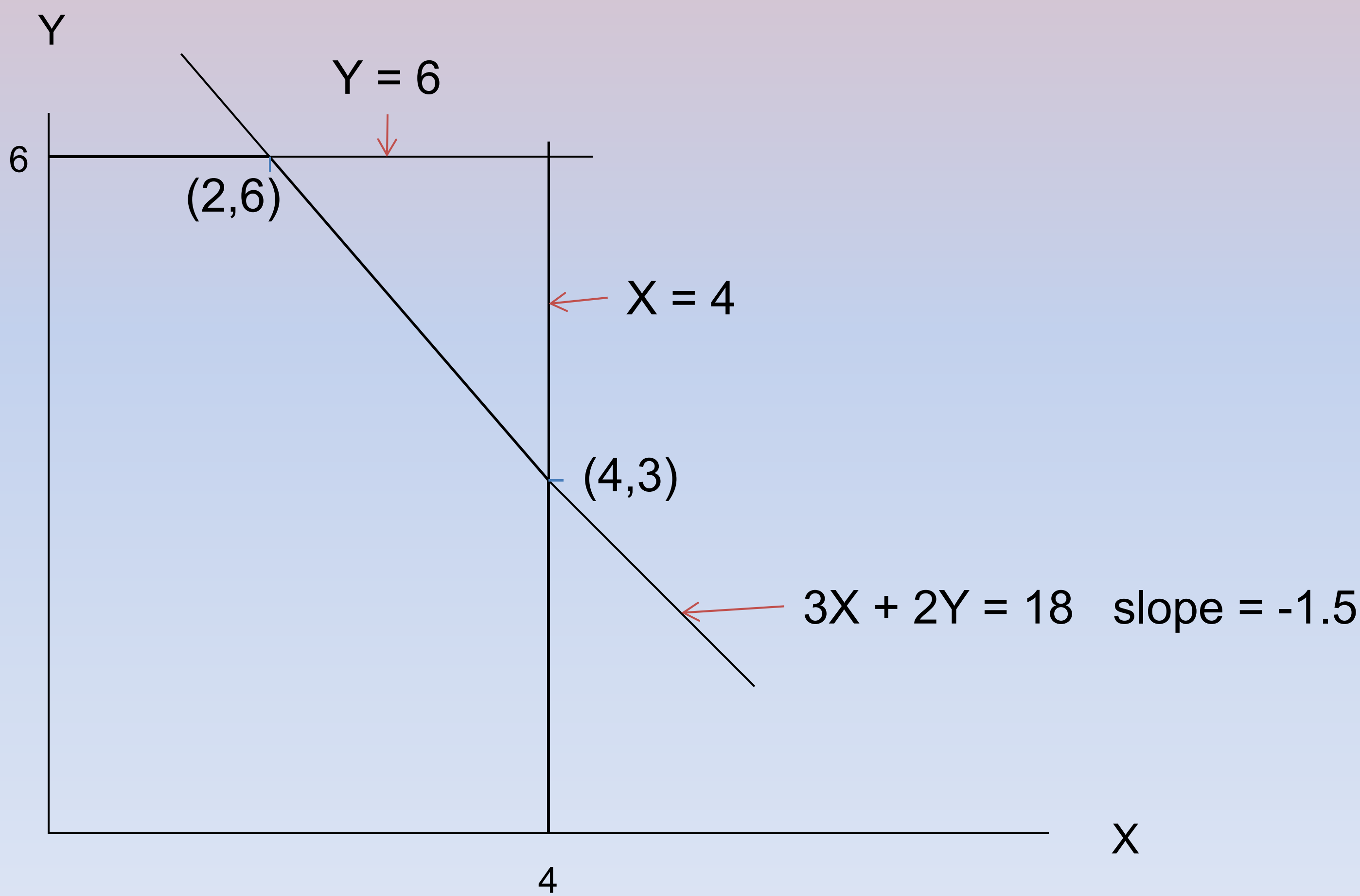


$X \le 4$ roaster A

$2Y \le 12$ roaster B

$3X + 2Y \le 18$ grinding, packaging

$3X + 2.5Y = P$ profit line, slope = -1.2



$X \le 4$ roaster A

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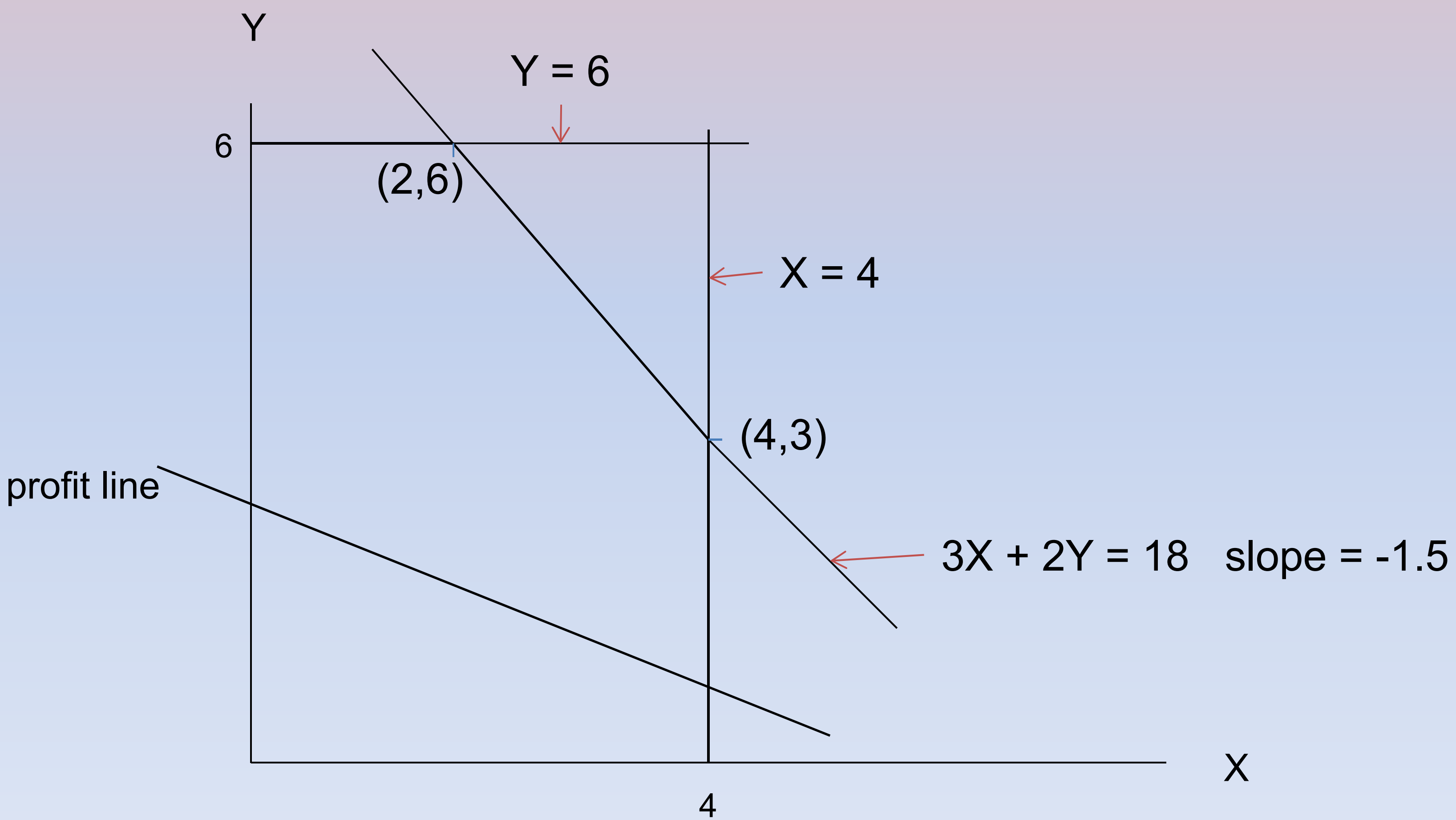
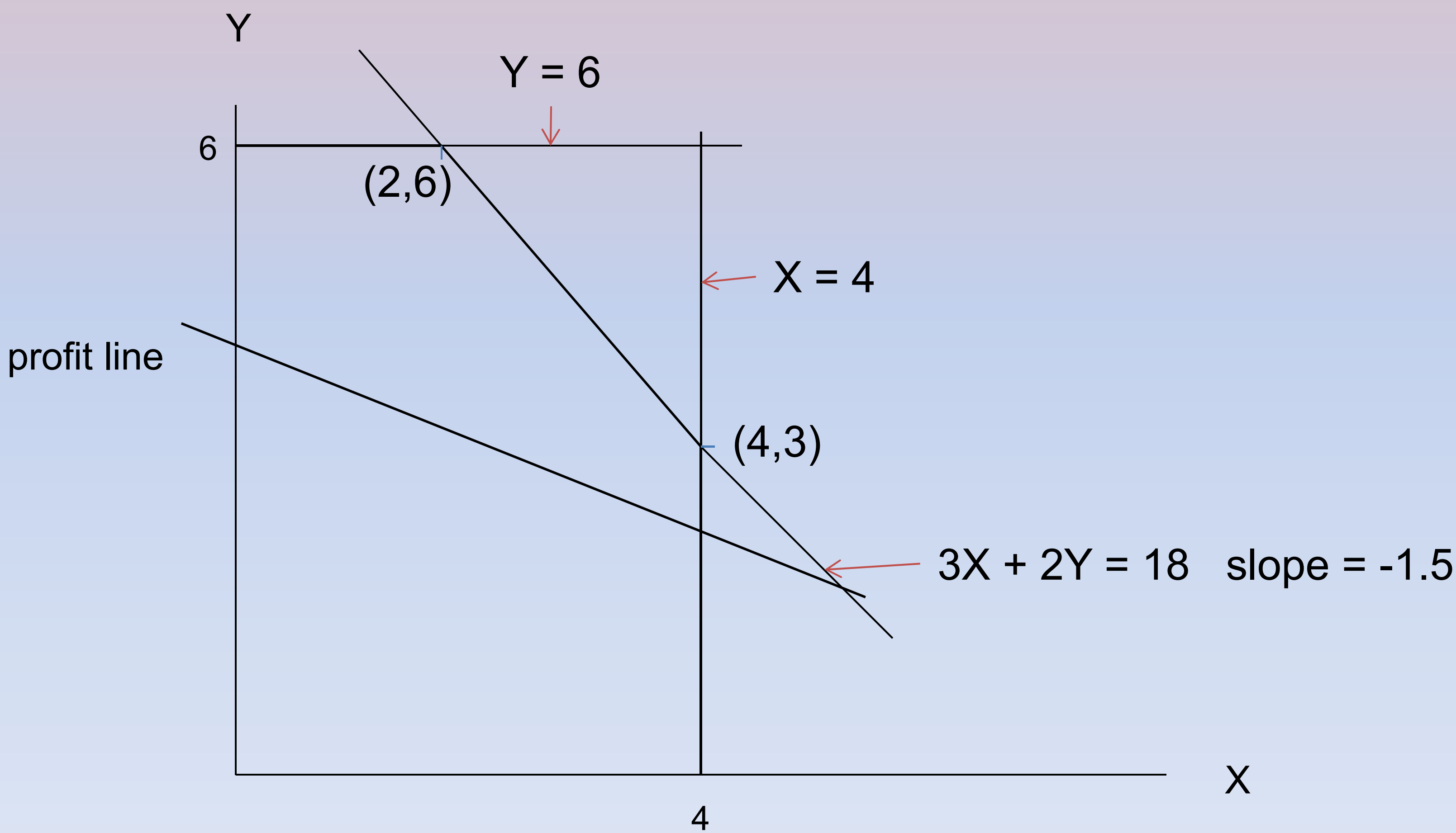


X <= 4 roaster A

2Y <= 12 roaster B

3X + 2Y <= 18 grinding, packaging

3X + 2.5Y = P profit line, slope = -1.2



$X <= 4$ roaster A

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$3X + 2Y <= 18$ grinding, packaging

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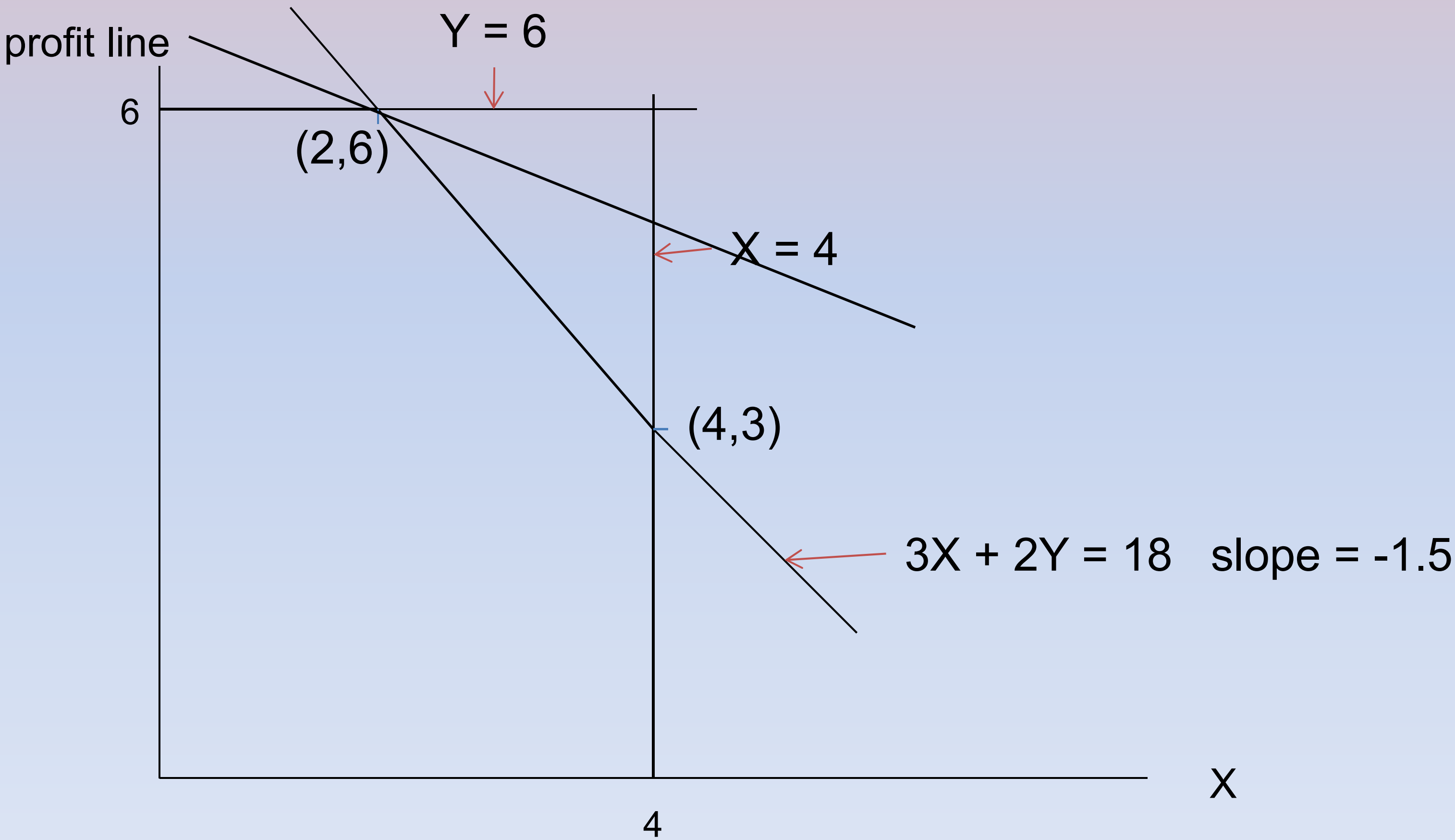
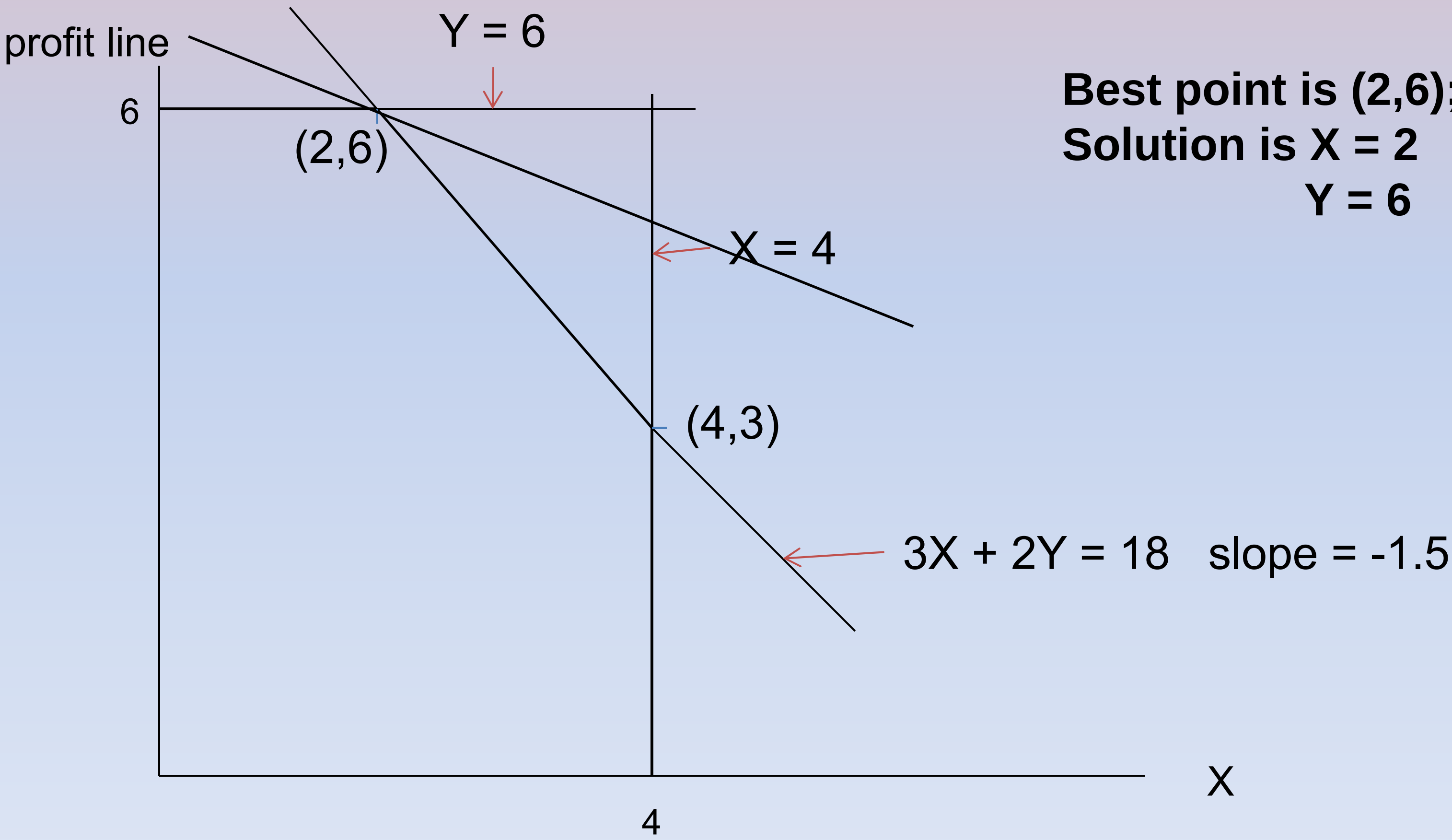


X <= 4 roaster A
2Y <= 12 roaster B
3X + 2Y <= 18 grinding, packaging
3X + 2.5Y = P profit line, slope = -1.2



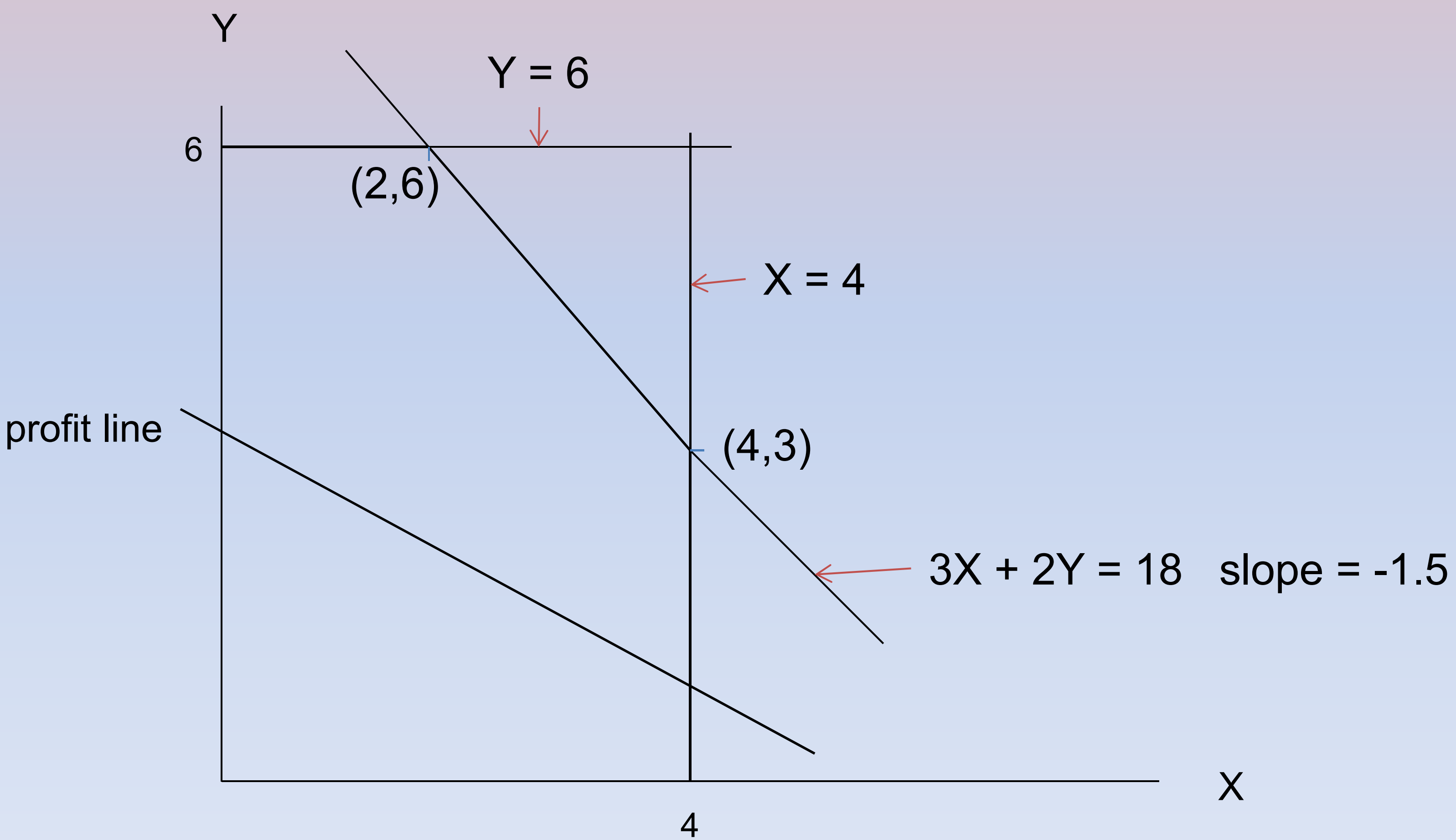
Best point is (2,6);
Solution is X = 2
Y = 6

X <= 4 roaster A

2Y <= 12 roaster B

3X + 2Y <= 18 grinding, packaging

3.5X + 2.5Y = P profit line, slope = -1.4

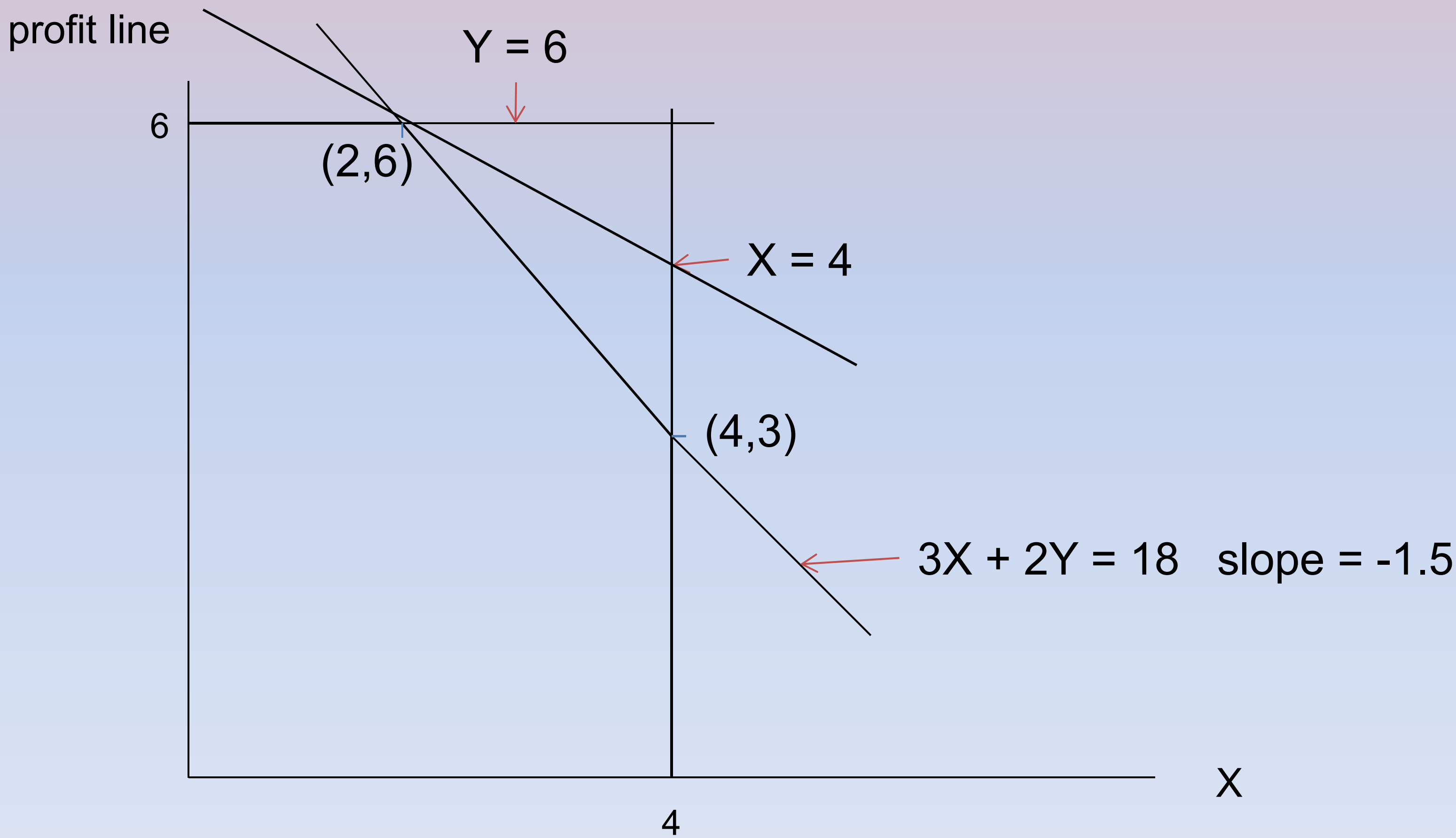


X <= 4 roaster A

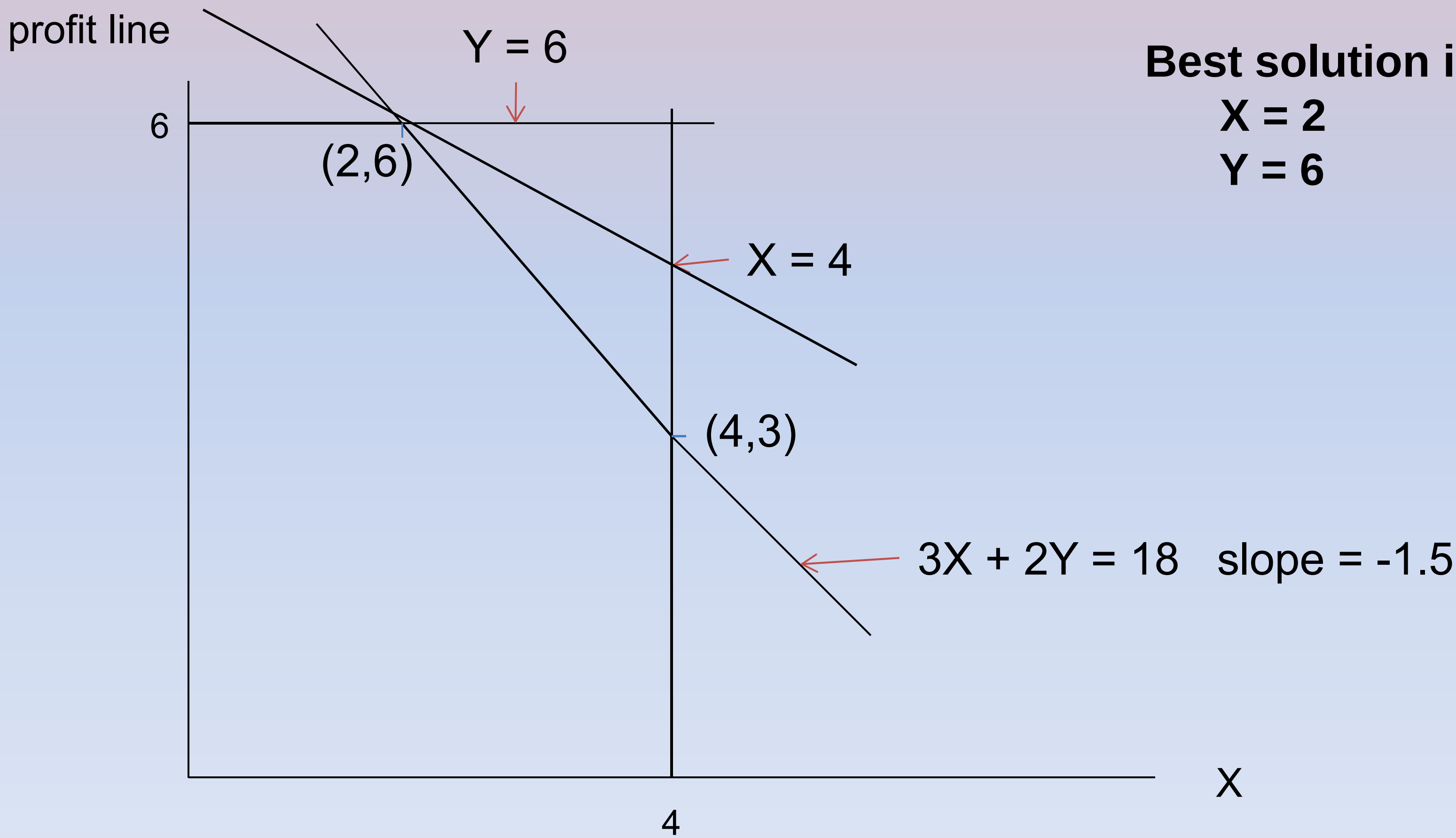
2Y <= 12 roaster B

3X + 2Y <= 18 grinding, packaging

3.5X + 2.5Y = P profit line, slope = -1.4

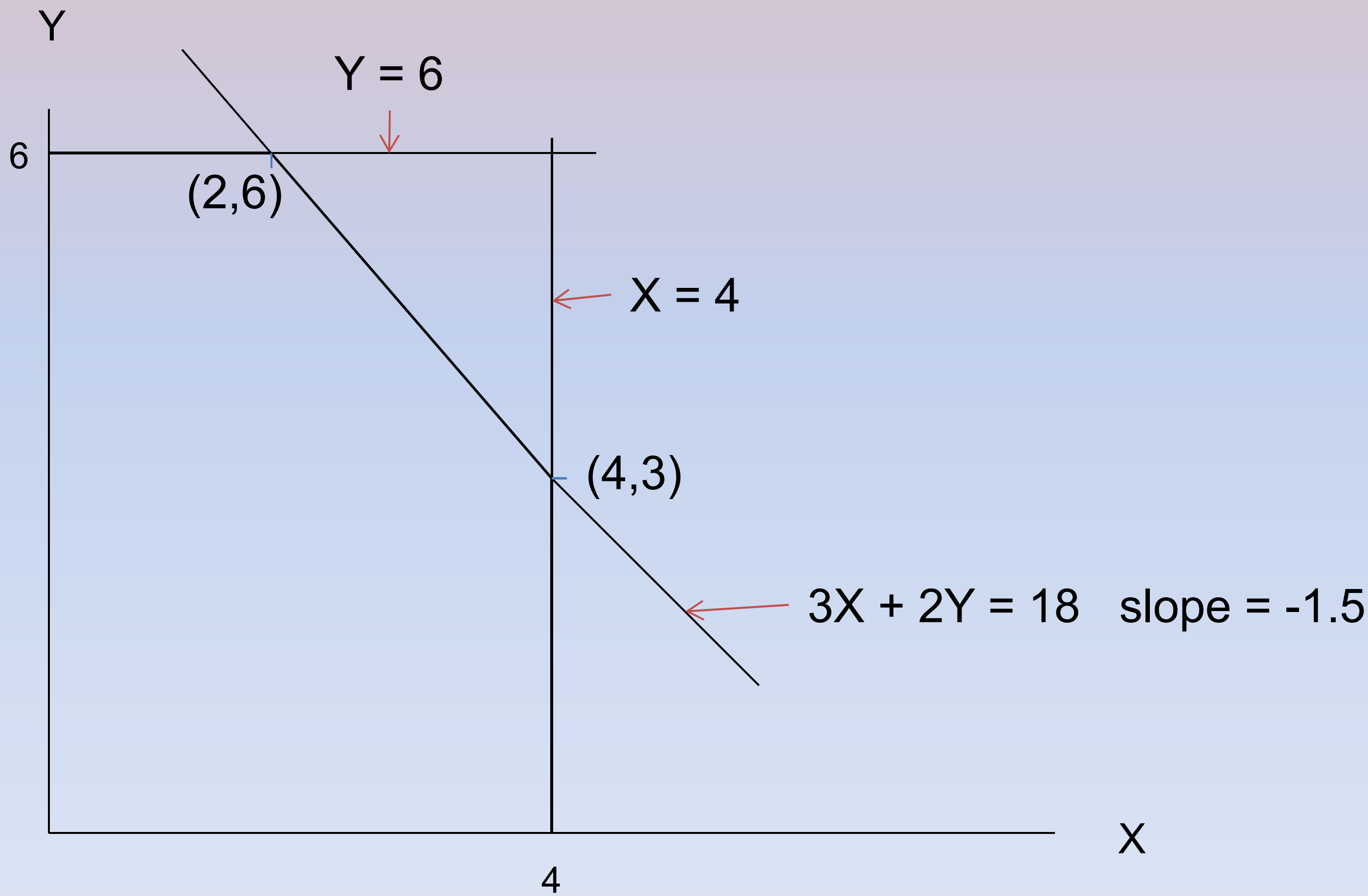


X <= 4 roaster A
2Y <= 12 roaster B
3X + 2Y <= 18 grinding, packaging
3.5X + 2.5Y = P profit line, slope = -1.4

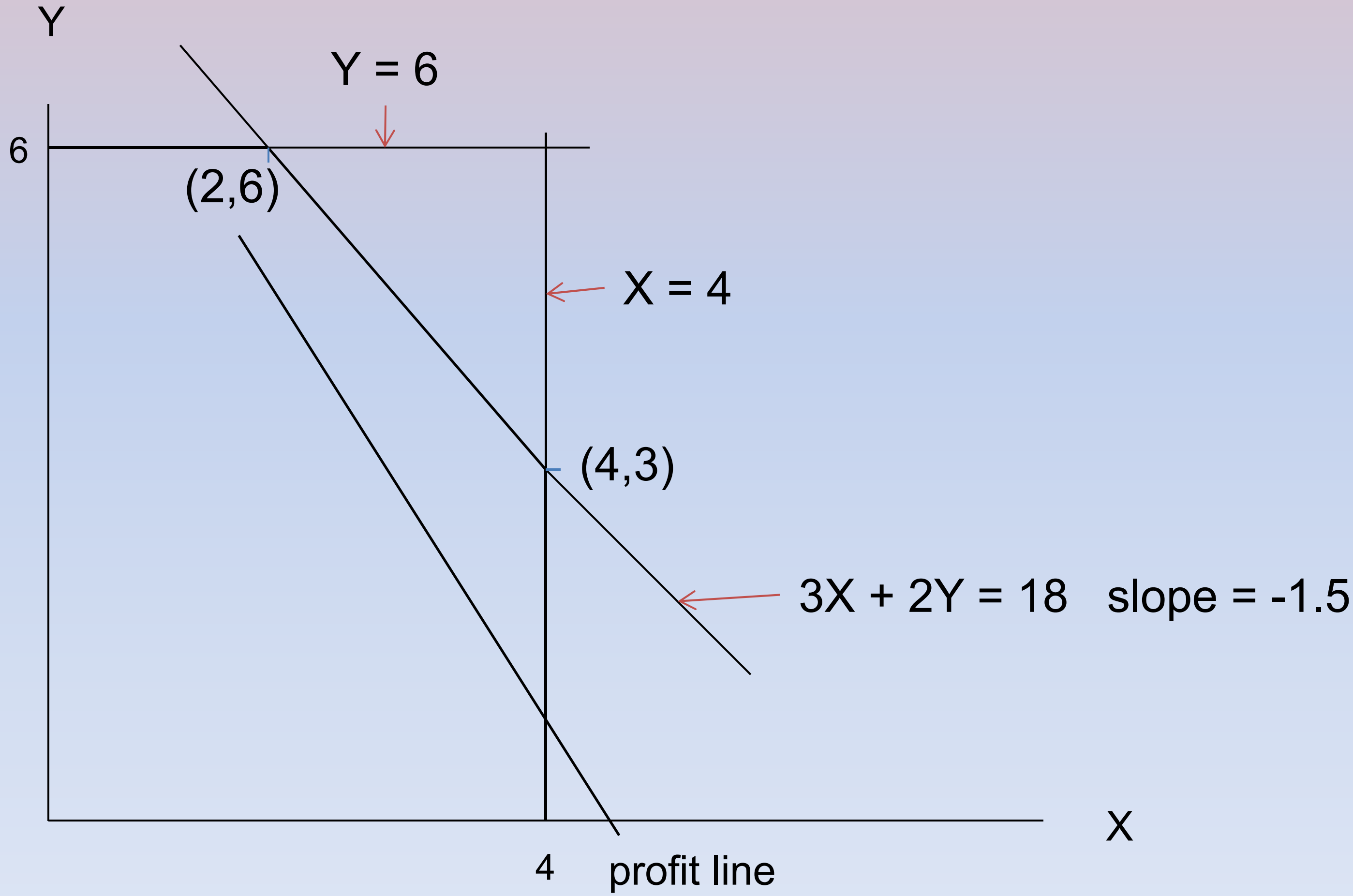


Best solution is still
X = 2
Y = 6

X <= 4 roaster A
2Y <= 12 roaster B
3X + 2Y <= 18 grinding, packaging
◊X + 2.5Y = P profit line, slope < -1.5 (steeper than -1.5)



X <= 4 roaster A
2Y <= 12 roaster B
3X + 2Y <= 18 grinding, packaging
◊X + 2.5Y = P profit line, slope < -1.5 (steeper than -1.5)

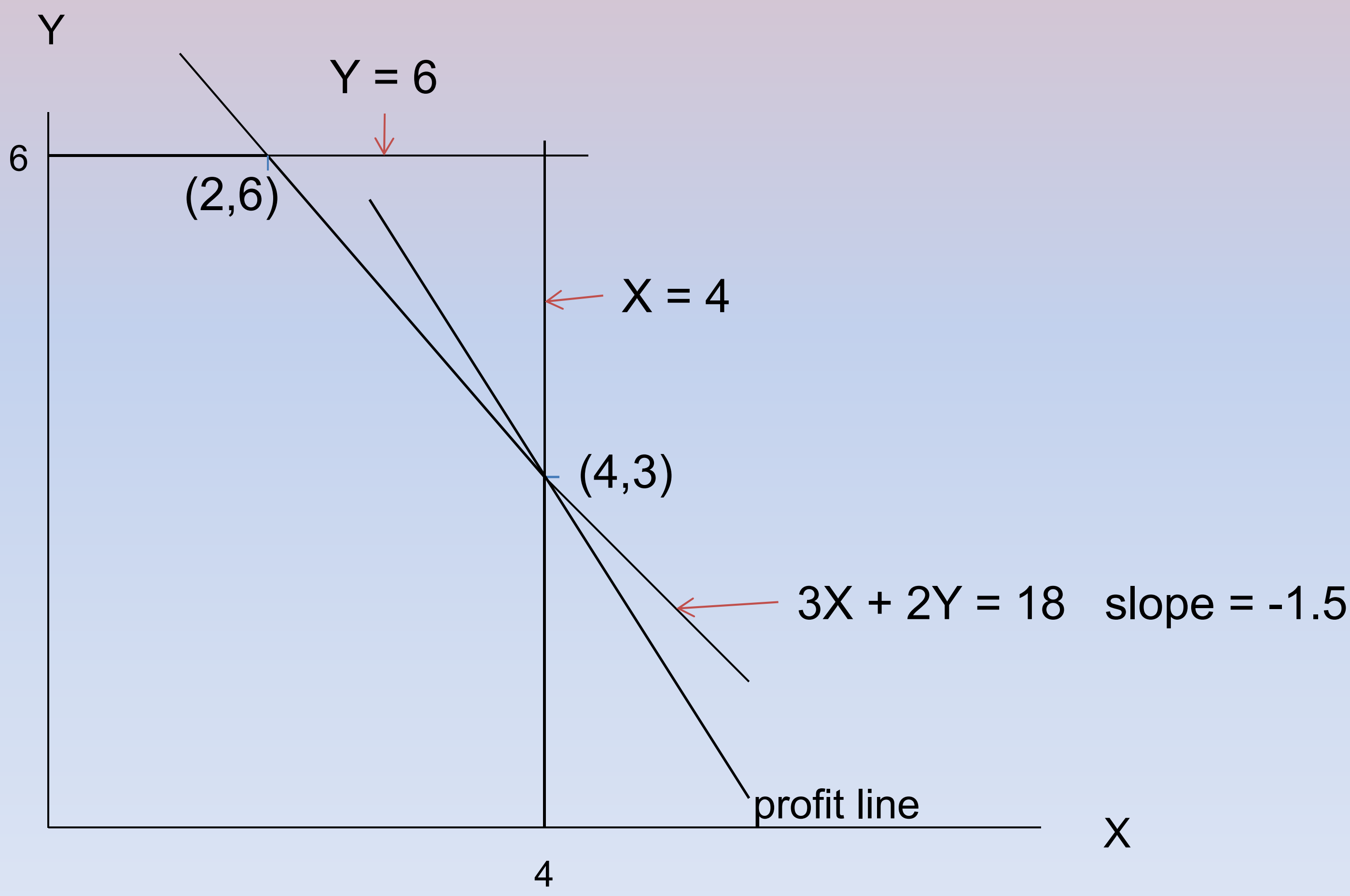


X <= 4 roaster A

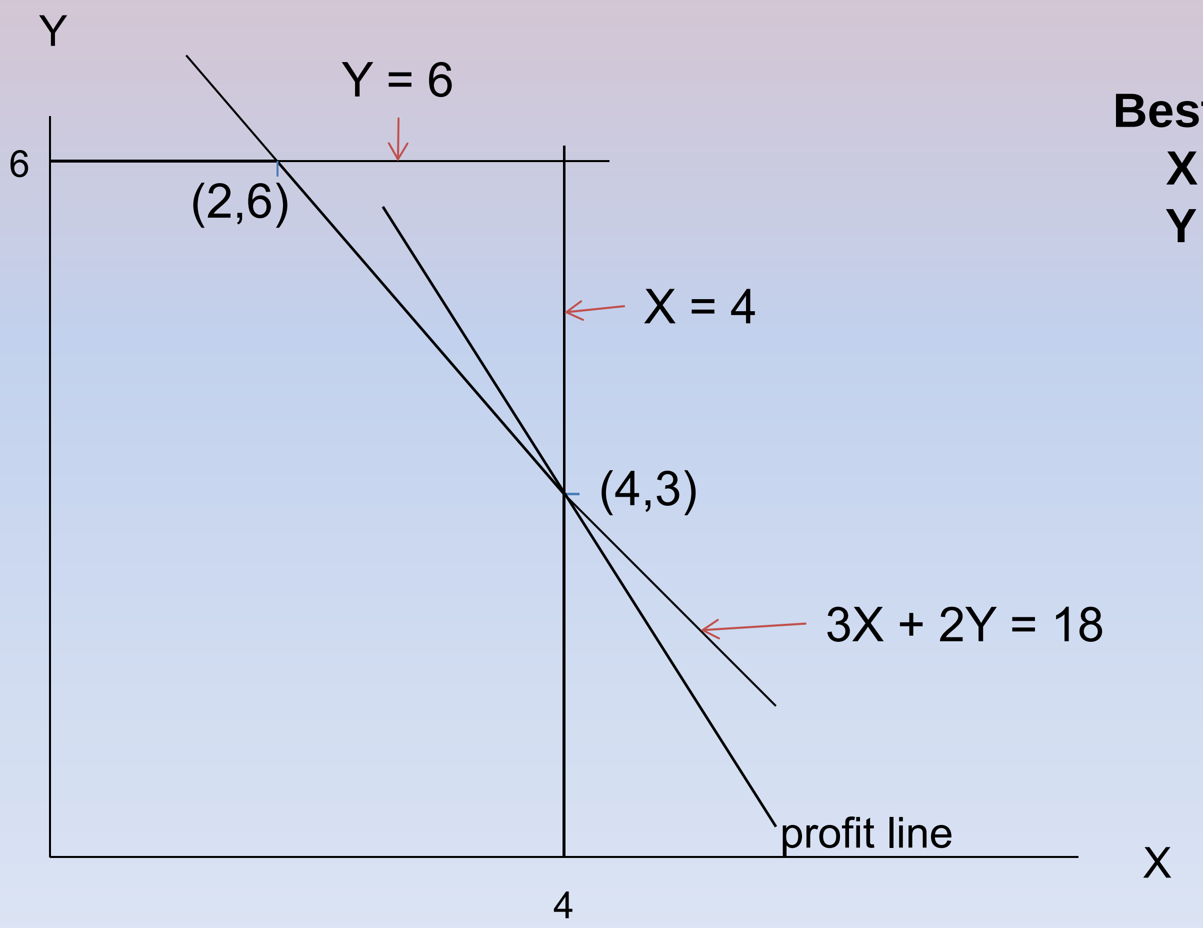
2Y <= 12 roaster B

3X + 2Y <= 18 grinding, packaging

◊X + 2.5Y = P profit line, slope < -1.5 (steeper than -1.5)

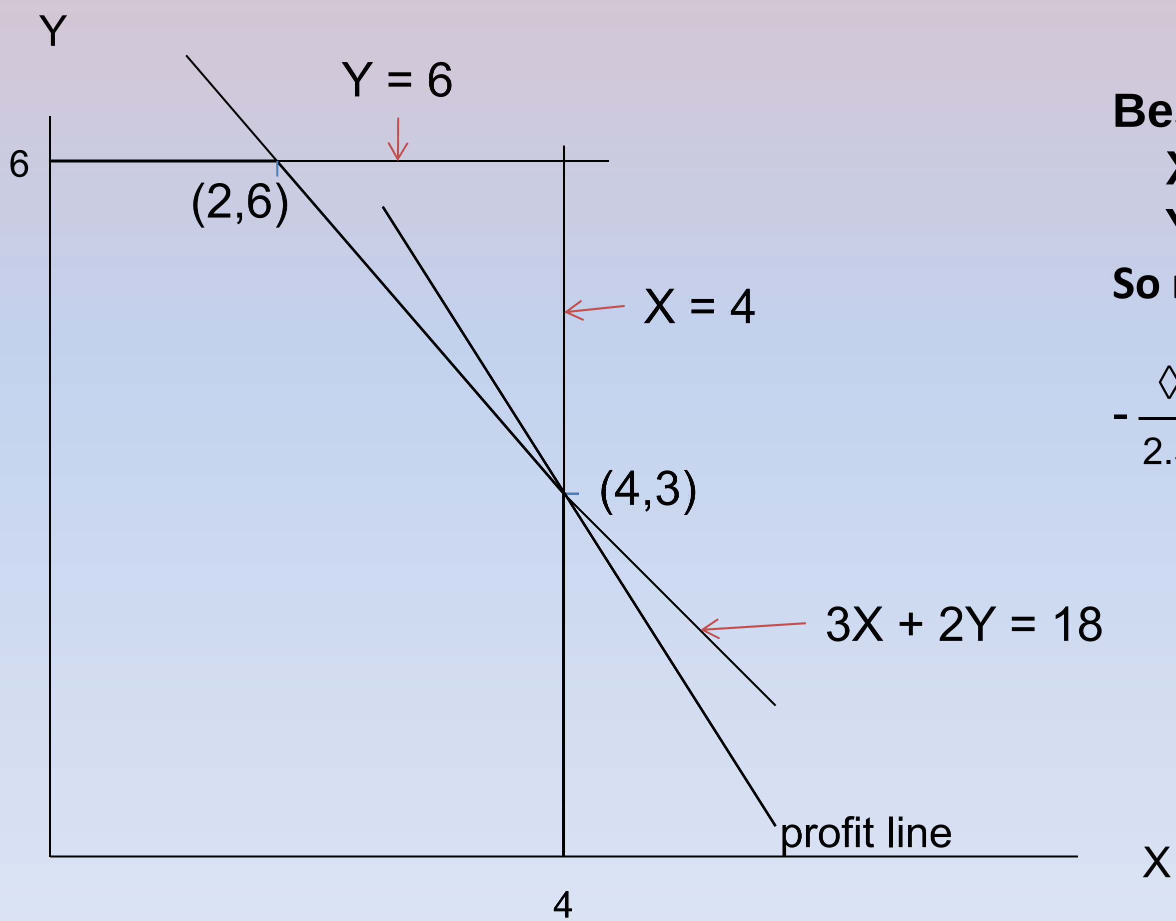


X <= 4 roaster A
2Y <= 12 roaster B
3X + 2Y <= 18 grinding, packaging
◊X + 2.5Y = P profit line, slope < -1.5 (steeper than -1.5)



Best solution now is
X = 4
Y = 3

X <= 4 roaster A
2Y <= 12 roaster B
3X + 2Y <= 18 grinding, packaging
◊X + 2.5Y = P profit line, slope < -1.5 (steeper than -1.5)



Best solution now is

X = 4

Y = 3

So need

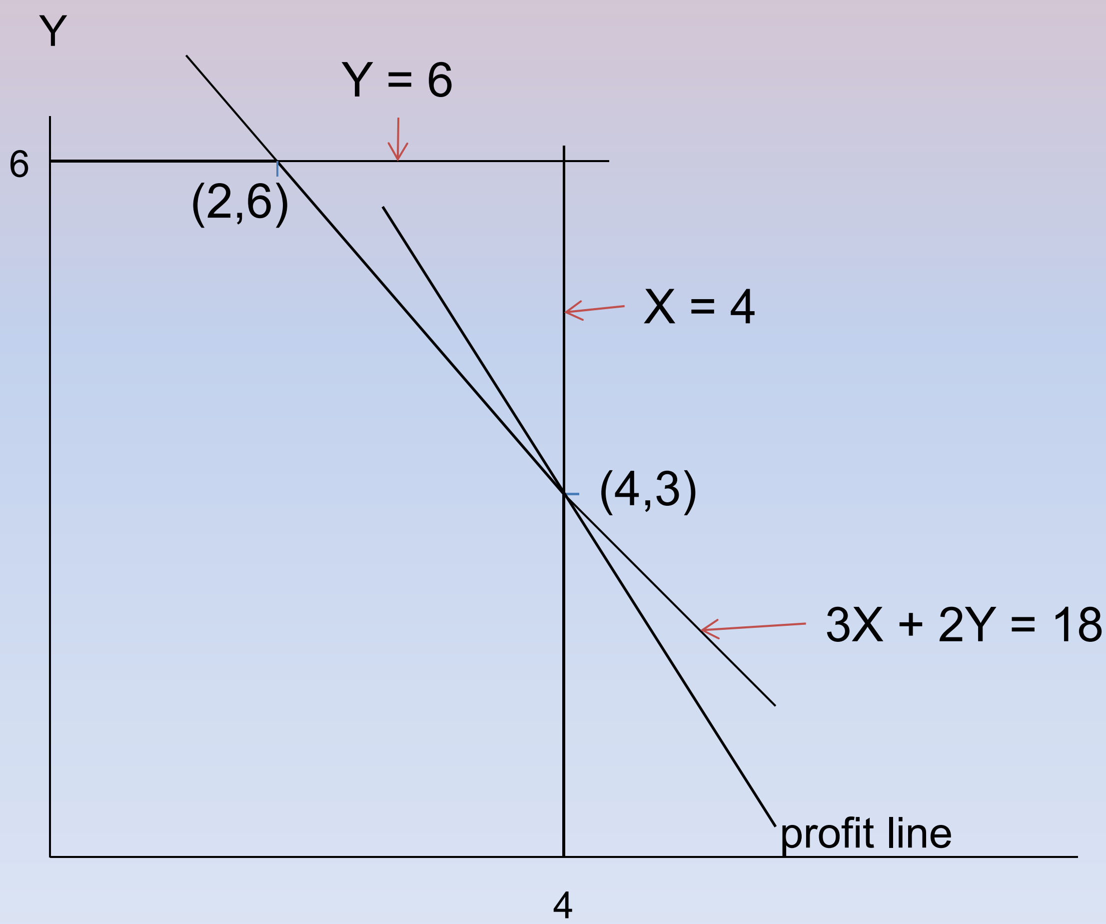
$$-\frac{◊}{2.5} < -\frac{3}{2}$$

X <= 4 roaster A

2Y <= 12 roaster B

3X + 2Y <= 18 grinding, packaging

◊X + 2.5Y = P profit line, slope < -1.5 (steeper than -1.5)



Best solution now is

X = 4

Y = 3

So need

$$-\frac{◊}{2.5} < -\frac{3}{2}$$

⟶ ◊ > 3 (2.5/2) = 3.75