21–22 Use a computer graph of the function to explain why the limit does not exist.

21. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2 + 3xy + 4y^2}{3x^2 + 5y^2}$

22. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2 + y^6}$

23–24 Find $h(x, y) = g(f(x, y))$ and the set on which h is continuous.

23. $g(t) = t^2 + \sqrt{t}$, $f(x, y) = 2x + 3y - 6$

24. $g(t) = \dfrac{\sqrt{t} - 1}{\sqrt{t} + 1}$, $f(x, y) = x^2 - y$

25–26 Graph the function and observe where it is discontinuous. Then use the formula to explain what you have observed.

25. $f(x, y) = e^{1/(x-y)}$

26. $f(x, y) = \dfrac{1}{1 - x^2 - y^2}$

27–36 Determine the set of points at which the function is continuous.

27. $F(x, y) = \dfrac{\sin(xy)}{e^x - y^2}$

28. $F(x, y) = \dfrac{x - y}{1 + x^2 + y^2}$

29. $F(x, y) = \arctan(x + \sqrt{y})$

30. $F(x, y) = e^{x^2 y} + \sqrt{x + y^2}$

31. $G(x, y) = \ln(x^2 + y^2 - 4)$

32. $G(x, y) = \sin^{-1}(x^2 + y^2)$

33. $f(x, y, z) = \dfrac{\sqrt{y}}{x^2 - y^2 + z^2}$

34. $f(x, y, z) = \sqrt{x + y + z}$

35. $f(x, y) = \begin{cases} \dfrac{x^2 y^3}{2x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 1 & \text{if } (x, y) = (0, 0) \end{cases}$

36. $f(x, y) = \begin{cases} \dfrac{xy}{x^2 + xy + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$

37–38 Use polar coordinates to find the limit. [If (r, θ) are polar coordinates of the point (x, y) with $r \geq 0$, note that $r \to 0^+$ as $(x, y) \to (0, 0)$.]

37. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 + y^3}{x^2 + y^2}$

38. $\displaystyle\lim_{(x,y)\to(0,0)} (x^2 + y^2)\ln(x^2 + y^2)$

39. Use spherical coordinates to find

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2 + y^2 + z^2}$$

40. At the beginning of this section we considered the function

$$f(x, y) = \frac{\sin(x^2 + y^2)}{x^2 + y^2}$$

and guessed that $f(x, y) \to 1$ as $(x, y) \to (0, 0)$ on the basis of numerical evidence. Use polar coordinates to confirm the value of the limit. Then graph the function.

41. Show that the function f given by $f(\mathbf{x}) = |\mathbf{x}|$ is continuous on $\mathbb{R}^n$. [*Hint:* Consider $|\mathbf{x} - \mathbf{a}|^2 = (\mathbf{x} - \mathbf{a}) \cdot (\mathbf{x} - \mathbf{a})$.]

42. If $\mathbf{c} \in V_n$, show that the function f given by $f(\mathbf{x}) = \mathbf{c} \cdot \mathbf{x}$ is continuous on $\mathbb{R}^n$.