37. $f(x, y, z) = x/(y + z)$; $f_z(3, 2, 1)$

38. $f(u, v, w) = w \tan(uv)$; $f_v(2, 0, 3)$

39–40 Use the definition of partial derivatives as limits (4) to find $f_x(x, y)$ and $f_y(x, y)$.

39. $f(x, y) = x^2 - xy + 2y^2$

40. $f(x, y) = \sqrt{3x - y}$

41–44 Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$. (You can see what these surfaces look like in TEC Visual 14.3.)

41. $x^2 + y^2 + z^2 = 3xyz$

42. $yz = \ln(x + z)$

43. $x - z = \arctan(yz)$

44. $\sin(xyz) = x + 2y + 3z$

45–46 Find $\partial z/\partial x$ and $\partial z/\partial y$.

45. (a) $z = f(x) + g(y)$ (b) $z = f(x + y)$

46. (a) $z = f(x)g(y)$ (b) $z = f(xy)$
(c) $z = f(x/y)$

47–52 Find all the second partial derivatives.

47. $f(x, y) = x^4 - 3x^2y^3$

48. $f(x, y) = \ln(3x + 5y)$

49. $z = x/(x + y)$

50. $z = y \tan 2x$

51. $u = e^{-s} \sin t$

52. $v = \sqrt{x + y^2}$

53–56 Verify that the conclusion of Clairaut's Theorem holds, that is, $u_{xy} = u_{yx}$.

53. $u = x \sin(x + 2y)$

54. $u = x^4y^2 - 2xy^5$

55. $u = \ln \sqrt{x^2 + y^2}$

56. $u = xye^y$

57–64 Find the indicated partial derivative.

57. $f(x, y) = 3xy^4 + x^3y^2$; f_{xxy}, f_{yyy}

58. $f(x, t) = x^2e^{-ct}$; f_{ttt}, f_{txx}

59. $f(x, y, z) = \cos(4x + 3y + 2z)$; f_{xyz}, f_{yzz}

60. $f(r, s, t) = r\ln(rs^2t^3)$; f_{rss}, f_{rst}

61. $u = e^{r\theta}\sin\theta$; $\dfrac{\partial^3 u}{\partial r^2\,\partial\theta}$

62. $z = u\sqrt{v - w}$; $\dfrac{\partial^3 z}{\partial u\,\partial v\,\partial w}$

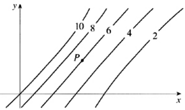
63. $w = \dfrac{x}{y + 2z}$; $\dfrac{\partial^3 w}{\partial z\,\partial y\,\partial x}$, $\dfrac{\partial^3 w}{\partial x^2\,\partial y}$

64. $u = x^a y^b z^c$; $\dfrac{\partial^6 u}{\partial x\,\partial y^2\,\partial z^3}$

65. Use the table of values of $f(x, y)$ to estimate the values of $f_x(3, 2)$, $f_x(3, 2.2)$, and $f_{xy}(3, 2)$.

x \ y	1.8	2.0	2.2
2.5	12.5	10.2	9.3
3.0	18.1	17.5	15.9
3.5	20.0	22.4	26.1

66. Level curves are shown for a function f. Determine whether the following partial derivatives are positive or negative at the point P.
(a) f_x (b) f_y (c) f_{xx}
(d) f_{xy} (e) f_{yy}



67. Verify that the function $u = e^{-\alpha^2k^2t}\sin kx$ is a solution of the *heat conduction equation* $u_t = \alpha^2 u_{xx}$.

68. Determine whether each of the following functions is a solution of Laplace's equation $u_{xx} + u_{yy} = 0$.
(a) $u = x^2 + y^2$
(b) $u = x^2 - y^2$
(c) $u = x^3 + 3xy^2$
(d) $u = \ln\sqrt{x^2 + y^2}$
(e) $u = \sin x \cosh y + \cos x \sinh y$
(f) $u = e^{-x}\cos y - e^{-y}\cos x$

69. Verify that the function $u = 1/\sqrt{x^2 + y^2 + z^2}$ is a solution of the three-dimensional Laplace equation $u_{xx} + u_{yy} + u_{zz} = 0$.

70. Show that each of the following functions is a solution of the wave equation $u_{tt} = a^2u_{xx}$.
(a) $u = \sin(kx)\sin(akt)$
(b) $u = t/(a^2t^2 - x^2)$
(c) $u = (x - at)^6 + (x + at)^6$
(d) $u = \sin(x - at) + \ln(x + at)$

71. If f and g are twice differentiable functions of a single variable, show that the function

$$u(x, t) = f(x + at) + g(x - at)$$

is a solution of the wave equation given in Exercise 70.

72. If $u = e^{a_1x_1 + a_2x_2 + \cdots + a_nx_n}$, where $a_1^2 + a_2^2 + \cdots + a_n^2 = 1$, show that

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \cdots + \frac{\partial^2 u}{\partial x_n^2} = u$$