

27. $w = \ln \sqrt{x^2 + y^2 + z^2}$

28. $w = xye^{xz}$

29. If $z = 5x^2 + y^2$ and (x, y) changes from $(1, 2)$ to $(1.05, 2.1)$, compare the values of Δz and dz .

30. If $z = x^2 - xy + 3y^2$ and (x, y) changes from $(3, -1)$ to $(2.96, -0.95)$, compare the values of Δz and dz .

31. The length and width of a rectangle are measured as 30 cm and 24 cm, respectively, with an error in measurement of at most 0.1 cm in each. Use differentials to estimate the maximum error in the calculated area of the rectangle.

32. The dimensions of a closed rectangular box are measured as 80 cm, 60 cm, and 50 cm, respectively, with a possible error of 0.2 cm in each dimension. Use differentials to estimate the maximum error in calculating the surface area of the box.

33. Use differentials to estimate the amount of tin in a closed tin can with diameter 8 cm and height 12 cm if the tin is 0.04 cm thick.

34. Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 cm high and 4 cm in diameter if the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick.

35. A boundary stripe 3 in. wide is painted around a rectangle whose dimensions are 100 ft by 200 ft. Use differentials to approximate the number of square feet of paint in the stripe.

36. The pressure, volume, and temperature of a mole of an ideal gas are related by the equation $PV = 8.31T$, where P is measured in kilopascals, V in liters, and T in kelvins. Use differentials to find the approximate change in the pressure if the volume increases from 12 L to 12.3 L and the temperature decreases from 310 K to 305 K.

37. If R is the total resistance of three resistors, connected in parallel, with resistances R_1, R_2, R_3 , then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If the resistances are measured in ohms as $R_1 = 25 \Omega$, $R_2 = 40 \Omega$, and $R_3 = 50 \Omega$, with a possible error of 0.5% in each case, estimate the maximum error in the calculated value of R .

38. Four positive numbers, each less than 50, are rounded to the first decimal place and then multiplied together. Use differentials to estimate the maximum possible error in the computed product that might result from the rounding.

- 39–40 ■ Show that the function is differentiable by finding values of ϵ_1 and ϵ_2 that satisfy Definition 7.

39. $f(x, y) = x^2 + y^2$

40. $f(x, y) = xy - 5y^2$

41. Prove that if f is a function of two variables that is differentiable at (a, b) , then f is continuous at (a, b) .

Hint: Show that

$$\lim_{(\Delta x, \Delta y) \rightarrow (0, 0)} f(a + \Delta x, b + \Delta y) = f(a, b)$$

42. (a) The function

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

was graphed in Figure 4. Show that $f_x(0, 0)$ and $f_y(0, 0)$ both exist but f is not differentiable at $(0, 0)$. [*Hint:* Use the result of Exercise 41.]

- (b) Explain why f_x and f_y are not continuous at $(0, 0)$.