14.5 Exercises

1–6 Use the Chain Rule to find dz/dt or dw/dt.

1. $z = x^2y + xy^2, \quad x = 2 + t^4, \quad y = 1 - t^3$

2. $z = \sqrt{x^2 + y^2}, \quad x = e^{2t}, \quad y = e^{-2t}$

3. $z = \sin x \cos y, \quad x = \pi t, \quad y = \sqrt{t}$

4. $z = x \ln(x + 2y), \quad x = \sin t, \quad y = \cos t$

5. $w = xe^{y/z}, \quad x = t^2, \quad y = 1 - t, \quad z = 1 + 2t$

6. $w = xy + yz^2, \quad x = e^t, \quad y = e^t \sin t, \quad z = e^t \cos t$

7–12 Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$.

7. $z = x^2 + xy + y^2, \quad x = s + t, \quad y = st$

8. $z = x/y, \quad x = se^t, \quad y = 1 + se^{-t}$

9. $z = \arctan(2x + y), \quad x = s^2t, \quad y = s \ln t$

10. $z = e^{xy} \tan y, \quad x = s + 2t, \quad y = s/t$

11. $z = e^r \cos \theta, \quad r = st, \quad \theta = \sqrt{s^2 + t^2}$

12. $z = \sin \alpha \tan \beta, \quad \alpha = 3s + t, \quad \beta = s - t$

13. If $z = f(x, y)$, where f is differentiable, $x = g(t)$, $y = h(t)$, $g(3) = 2$, $g'(3) = 5$, $h(3) = 7$, $h'(3) = -4$, $f_x(2, 7) = 6$, and $f_y(2, 7) = -8$, find dz/dt when $t = 3$.

14. Let $W(s, t) = F(u(s, t), v(s, t))$, where F, u, and v are differentiable, $u(1, 0) = 2$, $u_s(1, 0) = -2$, $u_t(1, 0) = 6$, $v(1, 0) = 3$, $v_s(1, 0) = 5$, $v_t(1, 0) = 4$, $F_u(2, 3) = -1$, and $F_v(2, 3) = 10$. Find $W_s(1, 0)$ and $W_t(1, 0)$.

15. Suppose f is a differentiable function of x and y, and $g(u, v) = f(e^u + \sin v, e^u + \cos v)$. Use the table of values to calculate $g_u(0, 0)$ and $g_v(0, 0)$.

	f	g	f_x	f_y
(0, 0)	3	6	4	8
(1, 2)	6	3	2	5

16. Suppose f is a differentiable function of x and y, and $g(r, s) = f(2r - s, s^2 - 4r)$. Use the table of values in Exercise 15 to calculate $g_r(1, 2)$ and $g_s(1, 2)$.

17–20 Use a tree diagram to write out the Chain Rule for the given case. Assume all functions are differentiable.

17. $u = f(x, y)$, where $x = x(r, s, t)$, $y = y(r, s, t)$

18. $w = f(x, y, z)$, where $x = x(t, u)$, $y = y(t, u)$, $z = z(t, u)$

19. $v = f(p, q, r)$,
where $p = p(x, y, z)$, $q = q(x, y, z)$, $r = r(x, y, z)$

20. $u = f(s, t)$, where $s = s(w, x, y, z)$, $t = t(w, x, y, z)$

21–26 Use the Chain Rule to find the indicated partial derivatives.

21. $z = x^2 + xy^3, \quad x = uv^2 + w^3, \quad y = u + ve^w;$
$\dfrac{\partial z}{\partial u}, \dfrac{\partial z}{\partial v}, \dfrac{\partial z}{\partial w}$ when $u = 2, v = 1, w = 0$

22. $u = \sqrt{r^2 + s^2}, \quad r = y + x \cos t, \quad s = x + y \sin t;$
$\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}, \dfrac{\partial u}{\partial t}$ when $x = 1, y = 2, t = 0$

23. $R = \ln(u^2 + v^2 + w^2), \quad u = x + 2y, \quad v = 2x - y,$ $w = 2xy;$
$\dfrac{\partial R}{\partial x}, \dfrac{\partial R}{\partial y}$ when $x = y = 1$

24. $M = xe^{y - z^2}, \quad x = 2uv, \quad y = u - v, \quad z = u + v;$
$\dfrac{\partial M}{\partial u}, \dfrac{\partial M}{\partial v}$ when $u = 3, v = -1$

25. $u = x^2 + yz, \quad x = pr \cos \theta, \quad y = pr \sin \theta, \quad z = p + r;$
$\dfrac{\partial u}{\partial p}, \dfrac{\partial u}{\partial r}, \dfrac{\partial u}{\partial \theta}$ when $p = 2, r = 3, \theta = 0$

26. $Y = w \tan^{-1}(uv), \quad u = r + s, \quad v = s + t, \quad w = t + r;$
$\dfrac{\partial Y}{\partial r}, \dfrac{\partial Y}{\partial s}, \dfrac{\partial Y}{\partial t}$ when $r = 1, s = 0, t = 1$

27–30 Use Equation 6 to find dy/dx.

27. $\sqrt{xy} = 1 + x^2y$

28. $y^5 + x^2y^3 = 1 + ye^{x^2}$

29. $\cos(x - y) = xe^y$

30. $\sin x + \cos y = \sin x \cos y$

31–34 Use Equations 7 to find $\partial z/\partial x$ and $\partial z/\partial y$.

31. $x^2 + y^2 + z^2 = 3xyz$

32. $xyz = \cos(x + y + z)$

33. $x - z = \arctan(yz)$

34. $yz = \ln(x + z)$

35. The temperature at a point (x, y) is $T(x, y)$, measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x = \sqrt{1 + t}$, $y = 2 + \frac{1}{3}t$, where x and y are measured in centimeters. The temperature function satisfies $T_x(2, 3) = 4$ and $T_y(2, 3) = 3$. How fast is the temperature rising on the bug's path after 3 seconds?

36. Wheat production in a given year, W, depends on the average temperature T and the annual rainfall R. Scientists estimate that the average temperature is rising at a rate of 0.15°C/year and rainfall is decreasing at a rate of 0.1 cm/year. They also estimate that, at current production levels, $\partial W/\partial T = -2$ and $\partial W/\partial R = 8$.

(a) What is the significance of the signs of these partial derivatives?

(b) Estimate the current rate of change of wheat production, dW/dt.