MATH 251 Practice Test 2

Solutions will <u>not</u> be made available.

These are only representative problems; you should review the homework problems (assigned as well as practice) too. `Check the course webpage for any corrections/updates`.

(1) Find the osculating circle at the end points (i.e., $x = 0$ and $y = 0$) of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = r^2$.

(2) Find $\mathbf{T}, \mathbf{N}, \mathbf{B}$ for $\mathbf{r}(t) = \langle te^{2t}, \cos t, t^3 \rangle$ at $t = 2$.

(3) Sketch the curve $x = -2t + 1, \quad y = \sqrt{|t|}, \quad z = t^2$.

(4) Find the tangential and normal components of the acceleration vector of a particle with position $\mathbf{r}(t) = (2t - 1)\mathbf{i} - t^2\mathbf{j} + t\mathbf{k}$.

(5) Make a rough sketch of a contour map for the function shown in Figure 1.

(6) Sketch the domain of the function $f(x, y) = \sqrt{y - x}\,\ln(x + y)$.

(7) Sketch the graph of $z = \dfrac{xy}{\sqrt{x^2 + y^2 - 1}}$.

(8) Evaluate the limits or show that they do not exist.

(a) $\displaystyle \lim_{(x,y)\to(1,1)} \frac{2xy}{x^2 + 2y^2}$ (b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{2xy}{x^2 + 2y^2}$

(9) Find all first and second partial derivatives of the following functions;

(a) $f(x, y) = \sqrt[3]{2x^2 - y^3x + 1}$ (b) $u = e^{-2t}\sin(\theta + \sqrt{t})$

(c) $w = \frac{xy}{y-z}$ (d) $T = p^2\ln(q + \ln r + e^{qr})$

(10) Find the equation of the tangent plane to the surface at the given point, and use it to approximate f at a nearby point

$$f(x, y) = z = e^x\cos y, \quad (0, 0, 1), \quad \text{nearby point} = (-0.2, 0.3, 0.96)$$

(11) Find the linearization of $f(x, y) = \frac{x^2y}{x-y}$ at $(1, 2)$.

(12) If $z = \cos xy + x\cos y$, where $x = u^3 + v^2$ and $y = v^2 - u$, use the Chain Rule to find $\partial z/\partial u$ and $\partial z/\partial v$.

(13) Find the gradient of $f(x, y, z) = z^2xe^{x\sqrt{y}}$ at the point $(1, 2, 3)$.

(14) Find the directional derivative $\mathbf{D_u}f$ of $f(x, y, z) = x^2y^2 + x\sqrt{1 + 2z}$ at the point $(-1, 2, 3)$ in the direction of $\mathbf{u} = \langle 2, 1, -2 \rangle$.

(15) You are standing on a mountain whose altitude A in metres at point x *km* north and y *km* east of the parking lot is $A(x, y) = 1445 + (0.01)e^{-(x^2+4y^2)/100}[(0.02)x^2y - (0.43)e^{-(x-y)^2}]$. You walk southwest from the parking lot at a horizontal speed of 2m/s. How quickly is your altitude changing 10 minutes later (as you are climbing over the mountain)?

(16) On the contour map given in Figure 2, sketch the path of steepest ascent from a point P to point Q, and from point P to point R.

(17) Find the minimum rate of change of $f(x, y) = x^2y - \sqrt{y+1}$ at the point $(2, 1)$. Find a unit vector $\mathbf{u}$ in that direction.

(18) Find the local maximum and minimum points and saddle points of the function $f(x, y) = x^3 - 6xy + 8y^3$.

(19) Find the absolute maximum and minimum values of $f(x, y) = e^{-x^2-y^2}(x^2 + 2y^2)$ on the set $D =$ the disc $x^2 + y^2 \leq 4$.

(20) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = x^2 + 2y^2 + 3z^2$ subject to the constraints $x + y + z = 1$ and $x - y + 2z = 2$.

(21) Find the point on the surface of $z = 2x^2 + 3y^2 - 3$ that is closest to the point $(10, 7, -3)$.

Fig 1 Fig 2