MATH 310 HW #6

6.1

7. Integration is a linear operation. It follows that

$$\int_0^A \cosh bt \cdot e^{-st}dt = \frac{1}{2}\int_0^A e^{bt}\cdot e^{-st}dt + \frac{1}{2}\int_0^A e^{-bt}\cdot e^{-st}dt$$
$$= \frac{1}{2}\int_0^A e^{(b-s)t}dt + \frac{1}{2}\int_0^A e^{-(b+s)t}dt.$$

Hence

$$\int_0^A \cosh bt \cdot e^{-st}dt = \frac{1}{2}\left[\frac{1-e^{(b-s)A}}{s-b}\right] + \frac{1}{2}\left[\frac{1-e^{-(b+s)A}}{s+b}\right].$$

Taking a *limit*, as $A \to \infty$,

$$\int_0^\infty \cosh bt \cdot e^{-st}dt = \frac{1}{2}\left[\frac{1}{s-b}\right] + \frac{1}{2}\left[\frac{1}{s+b}\right]$$
$$= \frac{s}{s^2-b^2}.$$

Note that the above is valid for $s > |b|$.

Other solutions will be attached at end.

15. Integrating by parts;

$$\int_0^A te^{at}\cdot e^{-st}dt = -\left.\frac{t\,e^{(a-s)t}}{s-a}\right|_0^A + \int_0^A \frac{1}{s-a}e^{(a-s)t}dt$$
$$= \frac{1-e^{A(a-s)} + A(a-s)e^{A(a-s)}}{(s-a)^2}.$$

Taking a *limit*, as $A \to \infty$,

$$\int_0^\infty te^{at}\cdot e^{-st}dt = \frac{1}{(s-a)^2}.$$

Note that the limit exists as long as $s > a$.

6.2

3. Using *partial fractions*,

$$\frac{2}{s^2+3s-4} = \frac{2}{5}\left[\frac{1}{s-1} - \frac{1}{s+4}\right].$$

Hence $\mathcal{L}^{-1}[Y(s)] = \frac{2}{5}(e^t - e^{-4t})$.

5. Note that the denominator s^2+2s+5 is *irreducible* over the reals. Completing the square, $s^2+2s+5 = (s+1)^2+4$. Now convert the function to a *rational function* of the variable $\xi = s+1$. That is,

$$\frac{2s+2}{s^2+2s+5} = \frac{2(s+1)}{(s+1)^2+4}.$$

We know that

$$\mathcal{L}^{-1}\left[\frac{2\xi}{\xi^2+4}\right] = 2\cos 2t.$$

Using the fact that $\mathcal{L}[e^{at}f(t)] = \mathcal{L}[f(t)]_{s\to s-a}$,

$$\mathcal{L}^{-1}\left[\frac{2s+2}{s^2+2s+5}\right] = 2e^{-t}\cos 2t.$$

Hence $\mathcal{L}^{-1}[Y(s)] = 3 + 5\cos 2t - 2\sin 2t$.