12. The characteristic equation of the system is $8r^3 + 60r^2 + 126r + 49 = 0$. The eigenvalues are $r_1 = -1/2$ and $r_{2,3} = -7/2$. The eigenvector associated with r_1 is $\boldsymbol{\xi}^{(1)} = (1, 1, 1)^T$. Setting $r = -7/2$, the components of the eigenvectors must satisfy the relation

$$\xi_1 + \xi_2 + \xi_3 = 0.$$

An eigenvector vector is given by $\boldsymbol{\xi}^{(2)} = (1, 0, -1)^T$. Since the last equation has two free variables, a third linearly independent eigenvector (associated with $r = -7/2$) is $\boldsymbol{\xi}^{(3)} = (0, 1, -1)^T$. Therefore the general solution may be written as

$$\mathbf{x} = c_1\begin{pmatrix}1\\1\\1\end{pmatrix}e^{-t/2} + c_2\begin{pmatrix}1\\0\\-1\end{pmatrix}e^{-7t/2} + c_3\begin{pmatrix}0\\1\\-1\end{pmatrix}e^{-7t/2}.$$

Invoking the initial conditions, we require that

$$\begin{aligned} c_1 + c_2 &= 2 \\ c_1 + c_3 &= 3 \\ c_1 - c_2 - c_3 &= -1. \end{aligned}$$

Hence the solution of the IVP is

$$\mathbf{x} = \frac{4}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}e^{-t/2} + \frac{2}{3}\begin{pmatrix}1\\0\\-1\end{pmatrix}e^{-7t/2} + \frac{5}{3}\begin{pmatrix}0\\1\\-1\end{pmatrix}e^{-7t/2}.$$

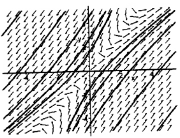
9.1 3(a). Solution of the ODE requires analysis of the algebraic equations

$$\begin{pmatrix}2-r & -1\\ 3 & -2-r\end{pmatrix}\begin{pmatrix}\xi_1\\ \xi_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}.$$

For a nonzero solution, we must have $det(\mathbf{A} - r\mathbf{I}) = r^2 - 1 = 0$. The roots of the characteristic equation are $r_1 = 1$ and $r_2 = -1$. For $r = 1$, the system of equations reduces to $\xi_1 = \xi_2$. The corresponding eigenvector is $\boldsymbol{\xi}^{(1)} = (1, 1)^T$. Substitution of $r = -1$ results in the single equation $3\xi_1 - \xi_2 = 0$. A corresponding eigenvector is $\boldsymbol{\xi}^{(2)} = (1, 3)^T$.

(b). The eigenvalues are *real*, with $r_1 r_2 < 0$. Hence the critical point is a *saddle*.

(c, d).



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(11)

5(a). The characteristic equation is given by

$$\begin{vmatrix}1-r & -5\\ 1 & -3-r\end{vmatrix} = r^2 + 2r + 2 = 0.$$

The equation has *complex* roots $r_1 = -1 + i$ and $r_2 = -1 - i$. For $r = -1 + i$, the components of the solution vector must satisfy $\xi_1 - (2+i)\xi_2 = 0$. Thus the corresponding eigenvector is $\boldsymbol{\xi}^{(1)} = (2+i, 1)^T$. Substitution of $r = -1 - i$ results in the single equation $\xi_1 - (2-i)\xi_2 = 0$. A corresponding eigenvector is $\boldsymbol{\xi}^{(2)} = (2-i, 1)^T$.

(b). The eigenvalues are *complex conjugates*, with negative real part. Hence the origin is a *stable spiral*.