At the origin, the coefficient matrix of the linearized system is

$$\mathbf{J}(0,0) = \begin{pmatrix} -2 & 2 \\ 4 & 4 \end{pmatrix},$$

with eigenvalues $r_1 = 1 - \sqrt{17}$ and $r_2 = 1 + \sqrt{17}$. The eigenvalues are real, with opposite sign. Hence the critical point is a *saddle*, which is *unstable*. At the equilibrium point $(-2, 2)$, the coefficient matrix of the linearized system is

$$\mathbf{J}(-2,2) = \begin{pmatrix} 4 & 0 \\ 6 & 6 \end{pmatrix},$$

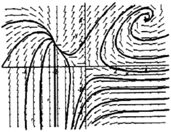
with eigenvalues $r_1 = 4$ and $r_2 = 6$. The eigenvalues are real, unequal and positive, hence the critical point is an *unstable node*. At the point $(4, 4)$, the coefficient matrix of the linearized system is

$$\mathbf{J}(4,4) = \begin{pmatrix} -6 & 6 \\ -8 & 0 \end{pmatrix},$$

with complex conjugate eigenvalues $r_{1,2} = -3 \pm i\sqrt{39}$. The critical point is a *stable spiral*, which is *asymptotically stable*.

Based on Table 9.3.1, the nonlinear terms do not affect the stability and type of each critical point.

(d).



$7(a)$. The critical points are solutions of the equations

$$\begin{aligned} 1 - y &= 0 \\ (x - y)(x + y) &= 0. \end{aligned}$$

The first equation requires that $y = 1$. Based on the second equation, $x = \pm 1$. Hence the critical points are $(-1, 1)$ and $(1, 1)$.

(b,c). $F(x,y) = 1 - y$ and $G(x,y) = x^2 - y^2$. The *Jacobian* matrix of the vector field is

$$\mathbf{J} = \begin{pmatrix} F_x(x,y) & F_y(x,y) \\ G_x(x,y) & G_y(x,y) \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 2x & -2y \end{pmatrix}.$$

At the critical point $(-1, 1)$, the coefficient matrix of the linearized system is

$$\mathbf{J}(-1,1) = \begin{pmatrix} 0 & -1 \\ -2 & -2 \end{pmatrix},$$

with eigenvalues $r_1 = -1 - \sqrt{3}$ and $r_2 = -1 + \sqrt{3}$. The eigenvalues are real, with opposite sign. Hence the critical point is a *saddle*, which is *unstable*. At the equilibrium point $(1, 1)$, the coefficient matrix of the linearized system is

$$\mathbf{J}(1,1) = \begin{pmatrix} 0 & -1 \\ 2 & -2 \end{pmatrix},$$