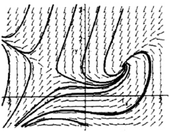
with complex conjugate eigenvalues $r_{1,2} = -1 \pm i$. The critical point is a *stable spiral*, which is *asymptotically stable.*

(d).



Based on Table 9.3.1, the nonlinear terms do not affect the stability and type of each critical point.

9(a). Based on Prob. 8, in Section 9.2, the critical points are at $(0,0)$, $(-2,-2)$, $(0,1)$ and $(3,-2)$.

(b,c). First note that $F(x,y) = -(x-y)(1-x-y)$ and $G(x,y) = x(2+y)$. The *Jacobian* matrix of the vector field is

$$\mathbf{J} = \begin{pmatrix} 2x-1 & 1-2y \\ 2+y & x \end{pmatrix}.$$

At the origin, the coefficient matrix of the linearized system is

$$\mathbf{J}(0,0) = \begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix},$$

with eigenvalues $r_1 = 1$ and $r_2 = -2$. The eigenvalues are real, with opposite sign. Hence the critical point is a *saddle*, which is *unstable*. At the critical point $(0,1)$, the coefficient matrix of the linearized system is

$$\mathbf{J}(0,1) = \begin{pmatrix} -1 & -1 \\ 3 & 0 \end{pmatrix},$$

with complex conjugate eigenvalues $r_{1,2} = -1/2 \pm i\sqrt{11}/2$. The critical point is a *stable spiral*, which is *asymptotically stable.* At the point $(-2,-2)$, the coefficient matrix of the linearized system is

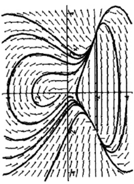
$$\mathbf{J}(-2,-2) = \begin{pmatrix} -5 & 5 \\ 0 & -2 \end{pmatrix},$$

with eigenvalues $r_1 = -2$ and $r_2 = -5$. The eigenvalues are unequal and negative, hence the critical point is a *stable node*. At the point $(3,-2)$, the coefficient matrix of the linearized system is

$$\mathbf{J}(3,-2) = \begin{pmatrix} 5 & 5 \\ 0 & 3 \end{pmatrix},$$

with eigenvalues $r_1 = 3$ and $r_2 = 5$. The eigenvalues are unequal and positive, hence the critical point is an *unstable node*.

(d).



Based on Table 9.3.1, the nonlinear terms do not affect the stability and type of each critical point.

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