12(a). The critical points are given by the solution set of the equations

$$(1+x)\sin y = 0$$
$$1 - x - \cos y = 0\,.$$

If $x = -1$, then we must have $\cos y = 2$, which is impossible. Therefore $\sin y = 0$, which implies that $y = n\pi,\ n = 0, \pm 1, 2, \ldots$. Based on the second equation,

$$x = 1 - \cos n\pi\,.$$

It follows that the critical points are located at $(0, 2k\pi)$ and $(2, (2k+1)\pi)$, where $k = 0, \pm 1, 2, \ldots$.

(b,c). Given that $F(x,y) = (1+x)\sin y$ and $G(x,y) = 1 - x - \cos y$, the *Jacobian* matrix of the vector field is

$$\mathbf{J} = \begin{pmatrix} \sin y & (1+x)\cos y \\ -1 & \sin y \end{pmatrix}.$$

At the critical points $(0, 2k\pi)$, the coefficient matrix of the linearized system is

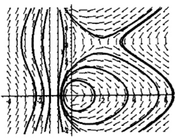
$$\mathbf{J}(0, 2k\pi) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

with purely complex eigenvalues $r_{1,2} = \pm\, i$. The critical points of the associated linear systems are *centers*, which are *stable*. Note that Theorem 9.3.2 does *not* provide a definite conclusion regarding the relation between the nature of the critical points of the nonlinear systems and their corresponding linearizations. At the points $(2, (2k+1)\pi)$, the coefficient matrix of the linearized system is

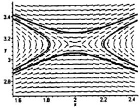
$$\mathbf{J}[2, (2k+1)\pi] = \begin{pmatrix} 0 & -3 \\ -1 & 0 \end{pmatrix},$$

with eigenvalues $r_1 = \sqrt{3}$ and $r_2 = -\sqrt{3}$. The eigenvalues are real, with opposite sign. Hence the critical points of the associated linear systems are *saddles*, which are *unstable*.

(d).



As asserted in Theorem 9.3.2, the trajectories near the critical points $(2, (2k+1)\pi)$ resemble those near a saddle.



Upon closer examination, the critical points $(0, 2k\pi)$ are indeed centers.