6.2 #5 $\frac{2s+2}{s^2+2s+5} = \frac{2(s+1)}{(s+1)^2+4}$ So according to line #14 Table 6.2.1,

$$\mathcal{L}^{-1}\left\{\frac{2(s+1)}{(s+1)^2+4}\right\} = \mathcal{L}^{-1}\left\{2F(s+1)\right\} = 2e^{-t}f(t) \text{ where}$$

$$\mathcal{L}\left\{f(t)\right\} = \frac{s}{s^2+4} \quad \text{so} \quad f(t) = \cos 2t$$

Thus, $\mathcal{L}^{-1}\left\{\frac{2s+2}{s^2+2s+5}\right\} = 2e^{-t}\cos 2t$

6.3 #28 $f(t+T) = f(t)$. $\mathcal{L}\left\{f(t)\right\} = \int_0^\infty e^{-st} f(t)\,dt$

$$= \underbrace{\int_0^T e^{-st} f(t)\,dt}_{=L} + \int_T^{2T} e^{-st} f(t)\,dt + \int_{2T}^{3T} e^{-st} f(t)\,dt + \cdots$$

$$= L + e^{-sT}L + e^{-2sT}L + \cdots$$

via substitution $\tau = t - kT$, so

$$\int_{kT}^{(k+1)T} e^{-st} f(t)\,dt = e^{-kT}\int_0^T e^{-s\tau}\underbrace{f(\tau+kT)}_{=f(\tau)}\,d\tau = e^{-kT}L$$

$$= L\left(1 + e^{-sT} + (e^{-sT})^2 + \cdots\right)$$

$$= \frac{L}{1-e^{-sT}} \checkmark$$

geometric series

6.6 #5 From line #16 Table 6.2.1, we see that if $F(s) = \mathcal{L}\left\{e^{-t}\right\}$ and $G(s) = \mathcal{L}\left\{\sin t\right\}$, then $\mathcal{L}\left\{f(t)\right\} = F(s)G(s) = \frac{1}{(s+1)(s^2+1)}$

line #2 line #5

→