9. The denominator s^2+4s+5 is *irreducible* over the reals. Completing the square, $s^2+4s+5=(s+2)^2+1$. Now convert the function to a *rational function* of the variable $\xi = s+2$. That is,

$$\frac{1-2s}{s^2+4s+5} = \frac{5-2(s+2)}{(s+2)^2+1}.$$

We find that

$$\mathcal{L}^{-1}\left[\frac{5}{\xi^2+1} - \frac{2\xi}{\xi^2+1}\right] = 5\sin t - 2\cos t.$$

Using the fact that $\mathcal{L}[e^{at}f(t)] = \mathcal{L}[f(t)]_{s\to s-a}$,

$$\mathcal{L}^{-1}\left[\frac{1-2s}{s^2+4s+5}\right] = e^{-2t}(5\sin t - 2\cos t).$$

13. Taking the Laplace transform of the ODE, we obtain

$$s^2\,Y(s) - s\,y(0) - y'(0) - 2[s\,Y(s) - y(0)] + 2\,Y(s) = 0.$$

Applying the *initial conditions*,

$$s^2\,Y(s) - 2s\,Y(s) + 2\,Y(s) - 1 = 0.$$

Solving for $Y(s)$, the transform of the solution is

$$Y(s) = \frac{1}{s^2-2s+2}.$$

Since the denominator is *irreducible*, write the transform as a function of $\xi = s-1$. That is,

$$\frac{1}{s^2-2s+2} = \frac{1}{(s-1)^2+1}.$$

First note that

$$\mathcal{L}^{-1}\left[\frac{1}{\xi^2+1}\right] = \sin t.$$

Using the fact that $\mathcal{L}[e^{at}f(t)] = \mathcal{L}[f(t)]_{s\to s-a}$,

$$\mathcal{L}^{-1}\left[\frac{1}{s^2-2s+2}\right] = e^t\sin t.$$

Hence $y(t) = e^t\sin t$.

17. Taking the Laplace transform of the ODE, we obtain

$$s^4\,Y(s) - s^3y(0) - s^2y'(0) - s\,y''(0) - y'''(0) - 4\left[s^3Y(s) - s^2\,y(0) - s\,y'(0) - y''(0)\right] +$$
$$+\,6\left[s^2\,Y(s) - s\,y(0) - y'(0)\right] - 4[s\,Y(s) - y(0)] + Y(s) = 0$$

Applying the *initial conditions*,

$$s^4Y(s) - 4s^3Y(s) + 6s^2\,Y(s) - 4s\,Y(s) + Y(s) - s^2 + 4s - 7 = 0.$$

Solving for the transform of the solution,

$$Y(s) = \frac{s^2-4s+7}{s^4-4s^3+6s^2-4s+1} = \frac{s^2-4s+7}{(s-1)^4}.$$

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Using *partial fractions*,

$$\frac{s^2-4s+7}{(s-1)^4} = \frac{4}{(s-1)^4} - \frac{2}{(s-1)^3} + \frac{1}{(s-1)^2}.$$

Note that $\mathcal{L}[t^n] = (n!)/s^{n+1}$ and $\mathcal{L}[e^{at}f(t)] = \mathcal{L}[f(t)]_{s\to s-a}$. Hence the solution of the IVP is

$$y(t) = \mathcal{L}^{-1}\left[\frac{s^2-4s+7}{(s-1)^4}\right] = \frac{2}{3}t^3e^t - t^2e^t + t\,e^t.$$