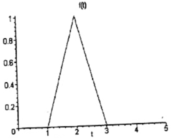
6.3 6.



9. The function can be expressed as

$$f(t) = (t-\pi)[u_\pi(t) - u_{2\pi}(t)].$$

Before invoking the *translation property* of the transform, write the function as

$$f(t) = (t-\pi)\,u_\pi(t) - (t-2\pi)\,u_{2\pi}(t) - \pi\,u_{2\pi}(t)\,.$$

It follows that

$$\mathcal{L}[f(t)] = \frac{e^{-\pi s}}{s^2} - \frac{e^{-2\pi s}}{s^2} - \frac{\pi e^{-2\pi s}}{s}.$$

15. First consider the function

$$G(s) = \frac{2(s-1)}{s^2-2s+2}.$$

Completing the square in the denominator,

$$G(s) = \frac{2(s-1)}{(s-1)^2+1}.$$

It follows that

$$\mathcal{L}^{-1}[G(s)] = 2\,e^t\cos t\,.$$

Hence

$$\mathcal{L}^{-1}\left[e^{-2s}G(s)\right] = 2\,e^{(t-2)}\cos(t-2)\,u_2(t)\,.$$

19(*a*). By definition of the Laplace transform,

$$\mathcal{L}[f(ct)] = \int_0^\infty e^{-st} f(ct)dt\,.$$

Making a change of variable, $\tau = ct$, we have

$$\begin{aligned}\mathcal{L}[f(ct)] &= \frac{1}{c}\int_0^\infty e^{-s(\tau/c)} f(\tau)d\tau \\ &= \frac{1}{c}\int_0^\infty e^{-(s/c)\tau} f(\tau)d\tau\,.\end{aligned}$$

Hence $\mathcal{L}[f(ct)] = \frac{1}{c}F\left(\frac{s}{c}\right)$, where $s/c > a$.

31. The function is *periodic*, with $T = 1$. Using the result of Prob. 28,

$$\mathcal{L}[f(t)] = \frac{1}{1-e^{-s}}\int_0^1 t\,e^{-st}dt\,.$$

It follows that

$$\mathcal{L}[f(t)] = \frac{1-e^{-s}(1+s)}{s^2(1-e^{-s})}.$$

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