9. Let $g(t)$ be the *forcing function* on the right-hand-side. Taking the Laplace transform of both sides of the ODE, we obtain

$$s^2 Y(s) - s\,y(0) - y'(0) + Y(s) = \mathcal{L}[g(t)].$$

Applying the initial conditions,

$$s^2 Y(s) + Y(s) - 1 = \mathcal{L}[g(t)].$$

The forcing function can be written as

$$\begin{aligned} g(t) &= \frac{t}{2}[1 - u_6(t)] + 3\,u_6(t) \\ &= \frac{t}{2} - \frac{1}{2}(t-6)u_6(t) \end{aligned}$$

with Laplace transform

$$\mathcal{L}[g(t)] = \frac{1}{2s^2} - \frac{e^{-6s}}{2s^2}.$$

Solving for the transform,

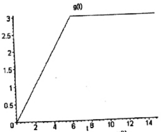
$$Y(s) = \frac{1}{s^2+1} + \frac{1}{2s^2(s^2+1)} - \frac{e^{-6s}}{2s^2(s^2+1)}.$$

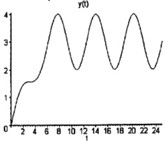
Using partial fractions,

$$\frac{1}{2s^2(s^2+1)} = \frac{1}{2}\left[\frac{1}{s^2} - \frac{1}{s^2+1}\right].$$

Taking the inverse transform, and using Theorem 6.3.1, the solution of the IVP is

$$\begin{aligned} y(t) &= \sin t + \frac{1}{2}[t - \sin t] - \frac{1}{2}[(t-6) - \sin(t-6)]u_6(t) \\ &= \frac{1}{2}[t + \sin t] - \frac{1}{2}[(t-6) - \sin(t-6)]u_6(t). \end{aligned}$$





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The solution increases, in response to the *ramp input*, and thereafter oscillates about a mean value of $y_m = 3$.