6.5

4. Taking the Laplace transform of both sides of the ODE, we obtain

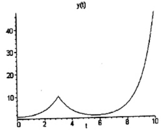
$$s^2\,Y(s) - s\,y(0) - y'(0) - Y(s) = -\,20\,e^{-3s}.$$

Applying the initial conditions,

$$s^2\,Y(s) - Y(s) - s = -\,20\,e^{-3s}.$$

Solving for the transform,

$$Y(s) = \frac{s}{s^2-1} - \frac{20\,e^{-3s}}{s^2-1}.$$



Using a *table of transforms*, and Theorem 6.3.1, the solution of the IVP is

$$y(t) = \cosh t - 20\,\sinh(t-3)u_3(t)\,.$$

19(*b*). Taking the initial conditions into consideration, the transform of the ODE is

$$s^2\,Y(s) + Y(s) = \sum_{k=1}^{20} e^{-(k\pi/2)s}.$$

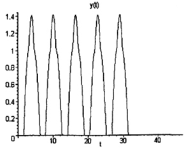
Solving for the transform of the solution,

$$Y(s) = \frac{1}{s^2+1}\sum_{k=1}^{20} e^{-(k\pi/2)s}.$$

Applying Theorem 6.3.1, term-by-term,

$$y(t) = \sum_{k=1}^{20} \sin\left(t - \frac{k\pi}{2}\right)u_{k\pi/2}(t)\,.$$

(*c*).



6.6

7. We have $f(t) = g * h\,(t)$, in which $g(t) = \sin t$ and $h(t) = \cos t$. The transform of the convolution integral is

$$\mathcal{L}\left[\int_0^t g(t-\tau)h(\tau)\,d\tau\right] = \frac{1}{s^2+1}\cdot\frac{s}{s^2+1} = \frac{s}{(s^2+1)^2}\,.$$

9. It is easy to see that

$$\mathcal{L}^{-1}\left[\frac{1}{s+1}\right] = e^{-t} \quad\text{and}\quad \mathcal{L}^{-1}\left[\frac{s}{s^2+4}\right] = \cos 2t\,.$$

Applying Theorem 6.6.1,

$$\mathcal{L}^{-1}\left[\frac{s}{(s+1)(s^2+4)}\right] = \int_0^t e^{-(t-\tau)}\cos 2\tau\,d\tau\,.$$

(7)