

Magic Mathematics

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SFU-CMS Math Camp Surrey 2015

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- Multiply it by 2 and then add 5

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the first digit is the number you picked at the start
the last two digits are your age!

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HOW??

Let x be the number you picked at the start

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Let's go to the end.....

“three digits; the number then your age”; this is $100x + \text{age}$

So this is where we need to end.....

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“Pick a number, multiply it by 2 and add 5”; $2x+5$

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“Pick a number, multiply it by 2 and add 5”; $2x+5$

“multiply the result by 50”; $(2x+5) \times 50$

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Back to the beginning;

“Pick a number, multiply it by 2 and add 5”; $2x+5$

“multiply the result by 50”; $(2x+5) \times 50$

“add 1765”; $(2x+5) \times 50 + 1764$

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So what do we have;

$$(2x+5) \times 50 + 1765 - \text{birth year} = 100x + 250 + 1765 - \text{birth year}$$

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So what do we have;

$$\begin{aligned}(2x+5) \times 50 + 1765 - \text{birth year} &= 100x + 250 + 1765 - \text{birth year} \\ &= 100x + 2015 - \text{birth year}\end{aligned}$$

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It's just math.....

Now let's make another one.....

- “Pick two numbers from 1,...,9; a first number and a second number”
-
- “... your age...”
-
-
- “The answer is 6 digits; __ __ __ __ __ __
where the first 4 digits is your birth year and the last two digits are the first and second numbers your picked, in that order”

Fill in the missing steps.....

Here's one way:

- Pick two numbers from 1,...,9; a first number and a second number.
- Add them together then multiply the sum by 2.
- Add 8 times the first number to this sum and then subtract the second number.
Call the result A.
- Now multiply your age by 2 and subtract it from 4030 (the year of Star Trek!).
- Multiply this number by 50. Call the result B.
- Now add the numbers A and B together.
- The answer is 6 digits;
where the first 4 digits is your birth year and the last two
digits are the first and second numbers your picked, in that order.

Can you explain why this works??

- Pick two numbers from 1,...,9; a, b
- Add them together then multiply the sum by 2; $(a+b) \times 2$
- Add 8 times the first number to this sum and then subtract the second number.
Call the result A; $A = 2a + 2b + 8a - b = \underline{10a + b}$
- Now multiply your age by 2 and subtract it from 4030; $4030 - 2 \times (\text{age})$
- Multiply this number by 50. Call the result B;

$$B = (4030 - 2 \times (\text{age})) \times 50 = 201500 - 100 \times (\text{age})$$

$$= (2015 - \text{age}) \times 100$$

$$= \text{birth year} \times 100$$
- Now add the numbers A and B together; $A + B =$

$$A + B = (\text{birth year}) \times 100 + 10a + b$$
- The answer is 6 digits;
 where the first 4 digits is your birth year and the last two digits are the first and second numbers your picked, in that order.

Magic Squares...

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	13	25	2	9

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17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	13	25	2	9

Each row, column and diagonal sums to 65, using the digits 1,2,...,25 exactly once.

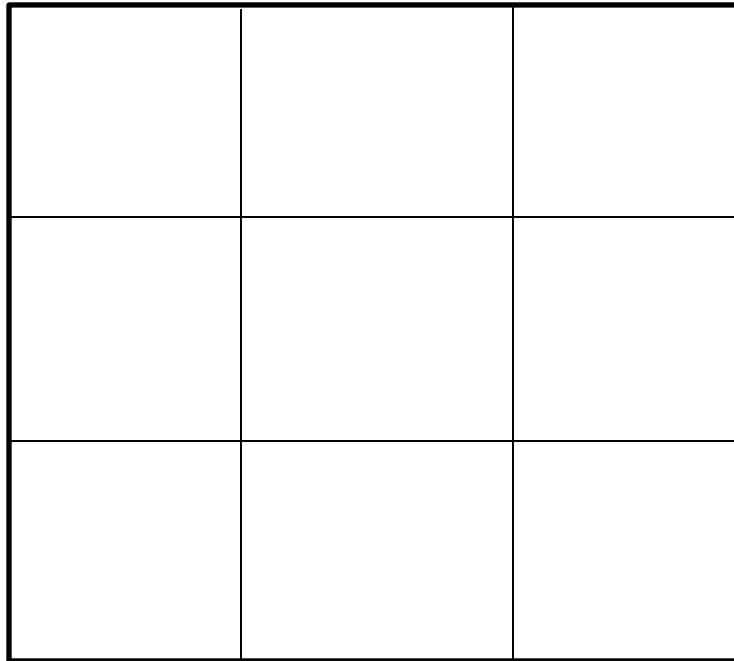
Have been around since 1125 AD, at least..... (China)
(See "Magic Squares and Cubes", by W.S. Andrews)

Magic Squares...

Put the digits 1,...,9 into the square using each digit only once, so that each row and column and diagonal add up to 15

Magic Squares...

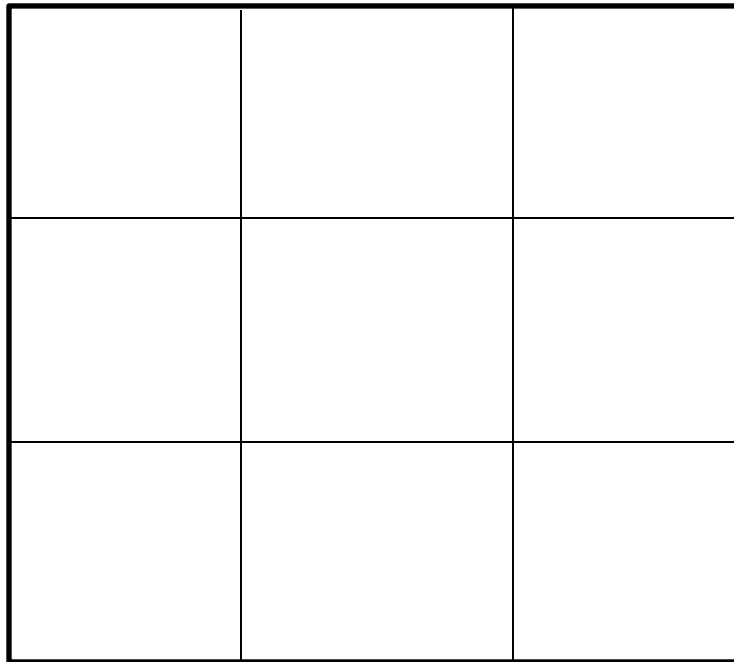
Put the digits 1,...,9 into the square using each digit only once, so that each row and column and diagonal add up to 15



First: How many ways are there to put the 9 digits into the square?

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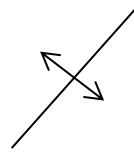
A LOT!! $9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362,880$

The 8 symmetries of the square!

2 9 4
7 5 3
6 1 8



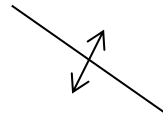
8 3 4
1 5 9
6 7 2



reflection



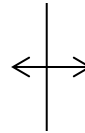
2 7 6
9 5 1
4 3 8



reflection



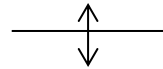
4 9 2
3 5 7
8 1 6



reflection



6 1 8
7 5 3
2 9 4



reflection



6 7 2
1 5 9
8 3 4



90 rotation



8 1 6
3 5 7
4 9 2



180 rotation



4 3 8
9 5 1
2 7 6



270 rotation

Let's analyze.....

Write out all the sums of three digits that add to 15:

All sums with 1:	(1 9 5), (1 8 6)	TWO
2:	(2 9 4), (2 6 7), (2 5 8)	THREE
3:	(3 8 4), (3 7 5)	TWO
4:	(4 9 2), (4 5 6), (4 8 3)	THREE
5:	(5 9 1), (5 6 4), (5 8 2), (5 7 3)	FOUR!
6:	(6 5 4), (6 7 2), (6 8 1)	THREE
7:	(7 6 2), (7 5 3)	TWO
8:	(8 6 1), (8 8 5 2), (8 4 3)	THREE
9:	(9 5 1), (9 4 2)	TWO

Let's analyze.....

Write out all the sums of three digits that add to 15:

All sums with 1:	(1 9 5), (1 8 6)	TWO	middle side
2:	(2 9 4), (2 6 7), (2 5 8)	THREE	corner
3:	(3 8 4), (3 7 5)	TWO	middle side
4:	(4 9 2), (4 5 6), (4 8 3)	THREE	corner
5:	(5 9 1), (5 6 4), (5 8 2), (5 7 3)	FOUR!	CENTRE
6:	(6 5 4), (6 7 2), (6 8 1)	THREE	corner
7:	(7 6 2), (7 5 3)	TWO	middle side
8:	(8 6 1), (8 8 5 2), (8 4 3)	THREE	corner
9:	(9 5 1), (9 4 2)	TWO	middle side

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7:	(7 6 2), (7 5 3)	TWO	middle side
8:	(8 6 1), (8 8 5 2), (8 4 3)	THREE	corner
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Can you make a 3 x 3 magic square with sum = 16 using 1,2 ...9? No!

Let's analyze.....

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How about sum to 16 using 2,3,..., 10? No!

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3:	(3 8 4), (3 7 5)	TWO	middle side
4:	(4 9 2), (4 5 6), (4 8 3)	THREE	corner
5:	(5 9 1), (5 6 4), (5 8 2), (5 7 3)	FOUR!	CENTRE
6:	(6 5 4), (6 7 2), (6 8 1)	THREE	corner
7:	(7 6 2), (7 5 3)	TWO	middle side
8:	(8 6 1), (8 8 5 2), (8 4 3)	THREE	corner
9:	(9 5 1), (9 4 2)	TWO	middle side

Can you make a 3 x 3 magic square with sum = 16 using 1,2 ...9? No!

How about sum to 16 using 2,3,..., 10? No!

They are not easy to make..... But, you can make one that adds to 18 using the digits 2,3,...,10 - why is this??

Here is a 4 x 4 magic square using the digits 1,2,..., 16;

1		14	
	6		9
		11	5
13		2	

Sums add up to 34.....

Here is a 4 x 4 magic square using the digits 1,2,..., 16;

1		14	
	6		9
		11	5
13		2	

Sums add up to 34.....

This presentation posted here;

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