

# Assignment2

Due Friday, June 29

1. 4.01. Chapter 4, page 70. Obtain the pdf of  $W$ .
2. 4.06. Chapter 4, page 71.
3. Among all the travellers who book a hotel for a particular destination through Expedia.com, 10% book 2-star hotels, 30% book 3-star hotels, 45% book 4-star hotels and 15% book 5-star hotels. Let  $X$  be the number of stars in the rating of the hotel that a randomly selected traveller books.
  - (a) Obtain pmf of  $X$ .
  - (b) Obtain cdf of  $X$ .
  - (c) Draw the plot of the cdf of  $X$ .
  - (d) What is the probability that the hotel booked by a randomly selected traveller is at least a 4-star hotel?
4. The probability that I start my day with a cup of coffee is 0.89. Let  $X = 1$  if I drink a cup of coffee in the morning and  $X = 0$  otherwise.
  - (a) Compute  $E(X)$ .
  - (b) Compute  $var(X)$ .
  - (c) Compute  $E(X^{59})$ .
5. 4.13. Chapter 4, page 72.

6. Suppose that 60% of the people who go to "The Ladle" for lunch order the famous delicious Mental Lentil soup.
  - (a) What is the probability that among the next 10 people in the line,
    - at least two people will order something other than the lentil soup?
    - fewer than half (fewer than 5 people) will order the lentil soup?
  - (b) What is the probability that the number of people who will order the lentil soup among the next ten people is more than two standard deviations away from the mean number of lentil soup fans among every ten people?
7. 4.17. Chapter 4, page 73.
8. Suppose that only .5% of the statistics students quit the program. Consider a sample of 20,000 statistics students.
  - (a) What is the expected value and standard deviation of the number of statistics students who quit the program?
  - (b) What is the probability that fewer than 3 students quit the program?
9. The number of buses stopping at a certain bus stop is a Poisson process with rate  $\alpha = 6$  per hour.
  - (a) What is the probability that exactly 10 buses stop at the bus stop in a 2-hours time period?
  - (b) What is the probability that no buses stop in a 30-minutes time period?
  - (c) What is the expected number of the buses stopping in a 2 hours and 30 minutes time period?
10. 5.01. Chapter 5, page 105.
11. 5.07. Chapter 5, page 107.
12. Suppose that the duration of my phone conversations with my sister has a uniform distribution with parameters  $a = 10$  minutes and  $b = 40$  minutes.

- (a) What is the probability that my conversation lasts less than 15 minutes?
  - (b) What is the probability that my conversation lasts 15 to 30 minutes?
  - (c) What is the expected length of a conversation?
  - (d) What is the variance of the conversation length?
13. Let  $X$  have a Pareto distribution with parameters  $k$  and  $\theta$ , i.e.,  $f(x) = \frac{k\theta^k}{x^{k+1}}$  if  $x \geq \theta$  and  $f(x) = 0$  if  $x < \theta$ .
- (a) For what values of  $k$  does  $E(X)$  exist? Obtain  $E(X)$  for those values. (Hint: What is  $E(X)$  when  $k = 1$ ?)
  - (b) For what values of  $k$  does  $\text{var}(X)$  exist? Obtain  $\text{var}(X)$  for those values. (Hint: What is  $\text{var}(X)$  when  $k = 2$ ?)
14. 5.16. Chapter 5, page 108.
15. 5.22. Chapter 5, page 108.
16. There are two machines available for cutting corks intended for use in wine bottles. The first produces corks with diameters that are normally distributed with mean 3 cm and standard deviation 0.1 cm. The second machine produces corks with diameters that have a normal distribution with mean 3.04 cm and standard deviation .02 cm. Acceptable corks have diameters between 2.9 cm and 3.1 cm. Which machine is more likely to produce an acceptable cork?
17. Let  $X$  and  $Y$  have the joint distribution  $f(x, y)$  where  $f(x, y) = x + y$  if  $0 < x < 1$  and  $0 < y < 1$ , and  $f(x, y) = 0$  otherwise.
- (a) Obtain the marginal pdf's of  $X$  and  $Y$ .
  - (b) Calculate  $P(X \geq 0.3|Y \geq 0.3)$ .
18. 5.37. Chapter 5, page 110.
19. 5.43. Chapter 5, page 111.
20. Suppose 25% of customers of a bakery go there for the special olive loaf. A random sample of 600 customers is selected. What is the probability that

- (a) between 100 and 200 of the selected customers are olive loaf lovers?
- (b) fewer than 200 of the selected customers buy the olive loaf?