STAT 270- Reviw Examples

July 23, 2012

Example 1

It is known that 65% of SFU Business School professors read the Wall Street Journal, 55% read the Vancouver Sun and 45% read both.
(a) What is the probability that a randomly selected professor reads exactly one of the two papers?
(b) Consider the hypotheses, H_0: the given proportions are correct, versus $H_1 : P(WS) = .55, P(VS) = .55, P(WS \cap VS) = .25$. If a randomly selected professor reads none of the two papers which one of H_0 or H_1 does the data support?

Example 2

Let X, Y, and Z be rv's with the following joint pdf

$$f(x,y,z) = \begin{cases} 8xyz & 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute the covariance between X and Y.

(b) Are X and Y independent?

Example 3

Let X and Y have the joint pdf

$$f(x,y) = \begin{cases} 8xy & 0 \le x \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Calculate $E(XY^2)$.
(b) Define $Z = \frac{X}{Y}$. Obtain cdf and pdf of Z.

Example 4 (truncated Poisson)

Consider the rv X with pmf

$$p(x) = \begin{cases} c\frac{e^{-\lambda}\lambda^x}{x!} & x = 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}$$

(a) Obtain c.

(b) Define $Y = X - 1$. Obtain pmf of Y. (c) Calculate $E(X)$.

Example 5

Let $\bar{X}$ be the mean of a random sample of size *n* from a *Normal*$(\mu, 9)$ distribution. Find *n* such that $(\bar{X} - 1, \bar{X} + 1)$ is a 90% CI for μ.

Example 6

Let X have a pdf of the form $f(x) = \theta x^{\theta-1}$, $0 < x < 1$ where $\theta \in \{\theta : \theta = 1, 2\}$. To test the simple hypothesis $H_0 : \theta = 1$ against the alternative simple hypotehsis $H_1 : \theta = 2$. Use a random sample of size $n = 2$: X_1, X_2 and a critical region defined as $C = \{(x_1, x_2) : x_1 + x_2 \geq \frac{3}{4}\}$. Find the power of the test.

Example 7

The lifetime of a tire in miles, X, is normally distributed with mean θ and standard deviation 5000. Past experience indicates that $\theta = 30,000$. The manufacturer claimes that tires made by a new process have mean $\theta > 30,000$. To check this claim we test the hypothesis $H_0 : \theta = 30,000$ versus $H_1 : \theta > 30,000$. A random sample of size n, $x_1, \ldots, x_n$ is observed and H_0 is rejected if $\bar{x} > c$. Determine n and c such that $1 - \beta(30,000) = 0.01$, $1 - \beta(35,000) = 0.98$.