STAT 270
Sample Midterm 2

Name:

Student ID:

Instructions:

- There are 4 pages in this exam booklet, including cover page.
- There are 3 questions. Please read all questions carefully and write the answers in the space provided for each question.
- Define all variables/events used in your solutions
- The exam time is 50 minutes.
- You are allowed to have a one-sided cheat sheet and a non-programmable calculator.

Good luck!

1. Let X and Y have the joint pmf given by the following table

p(x,y)	x=1	x=2
y=0	0.2	0.15
y=1	0.03	c
y=2	0.34	0.05

(a) Calculate $E(Y)$. (1 mark)

(b) Calculate $P(X = 1|Y \leq 1)$. (1 mark)

(c) Calculate $corr(X,Y)$. (1 mark)

(d) Obtain distribution of $Z = X - Y$. (1 mark)

$$\sum_x \sum_y p(x,y) = 1 \implies .2 + .15 + .03 + c + .34 + .05 = 1$$

$$\implies c = .23$$

(a) $P_Y(y) = \sum_x p(x,y) \implies$

y	0	1	2
P(y)	.35	.26	.39

$$E(Y) = 0(.35) + 1(.26) + 2(.39) = 1.04$$

(b)
$$P(X=1 \mid Y \leq 1) = \frac{P(X=1, Y=0) + P(X=1, Y=1)}{P(Y=0) + P(Y=1}$$

$$= \frac{0.2 + .03}{.35 + .26} \approx .38$$

(c) $cov(X,Y) = E(XY) - E(X)E(Y)$

x	1	2
P(x)	.57	.43

$\implies E(X) = 1(.57) + 2(.43)$

$= 1.43$

$$E(XY) = \sum_x \sum_y xy\, p(x,y)$$

$$= 0(.2) + 0(.15) + 1(.03) + {}^{2}2(.23) + 2(.34) + 4(.05)$$

$$= 1.37$$

$$\implies cov(X,Y) = 1.37 - (1.43)(1.04) = -0.12$$

$E(X^2) = 1(.57) + 4(.43) = 2.29 \Rightarrow var(X) = 2.29 - (1.43)^2 = .24$

$E(Y^2) = 0(.35) + 1(.26) + 4(.39) = 1.82 \Rightarrow$

$var(Y) = 1.82 - (1.04)^2 = .74$

$$\Rightarrow corr(X,Y) = \frac{-0.12}{\sqrt{(.24)(.74)}} = -0.28$$

(d)

Z	x=1	x=2
y=0	1	2
y=1	0	1
y=2	-1	0

$P(Z=0) = P(X=1, Y=1) + P(X=2, Y=2) = .03 + .05 = .08$

$P(Z=1) = P(X=1, Y=0) + P(X=2, Y=1) = 0.2 + 0.23 = 0.43$

$P(Z=2) = P(X=2, Y=0) = 0.15$

$P(Z=-1) = P(X=1, Y=2) = 0.34$

z	-1	0	1	2
$P(z)$	.34	.08	.43	.15

2. Suppose my neighbour's cat passes by my window according to a Poisson process with an average of twice a day. Let X_i be the number of times she passes by my window in week i. If I keep record of the total number of my cat sightings over the year what is the probability that the number exceeds 800?(3 marks)

$X_i \sim \text{Poisson}(14)$ $\quad E(X_i) = 14 \quad var(X_i) = 14$

$$Y = \sum_{i=1}^{52} X_i = 52\bar{X}$$

$n > 30$

using CLT: $\bar{X} \sim \text{Normal}\left(14, \frac{14}{52}\right)$

$$P(Y > 800) = P(52\bar{X} > 800) = P(\bar{X} > 15.38)$$

where $Z \sim N(0,1)$

$$\approx P\left(Z > \frac{15.38 - 14}{\sqrt{14/52}}\right)$$

$$= P(Z > 2.66)$$

$$= 1 - P(Z \leq 2.66)$$

$$= 1 - \left[P(Z \leq 2.64) + \tfrac{1}{2}\left(P(Z \leq 2.68) - P(Z \leq 2.64)\right)\right]$$

$$= 1 - 0.9961$$

$$= .0039$$

3. Let X and Y have the joint pdf $f(x, y) = \exp(-y)$ for $0 < x < y < \infty$. Obtain the conditional pdf of Y given $X = x$. (3 marks)

Marginal pdf of X:

$$f_X(x) = \int_x^{\infty} e^{-y}\, dy$$

$$= -e^{-y} \Big|_x^{\infty} = e^{-x}$$

$$f_{Y|X=x}(y|x) = \frac{f(x,y)}{f(x)} = \frac{e^{-y}}{e^{-x}} = e^{-(y-x)}$$

$$\Rightarrow \quad f(y|x) = \begin{cases} e^{-(y-x)} & \boxed{x < y < \infty} \\ 0 & \text{o.w.} \end{cases}$$